Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

Nasser M. Abbasi

September 20, 2021 Compiled on September 20, 2021 at 2:34am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [170]. This is test number [11].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (170)	0.00 (0)
Mathematica	100.00 (170)	0.00 (0)
Maple	100.00 (170)	0.00 (0)
Giac	100.00 (170)	0.00 (0)
Maxima	100.00 (170)	0.00 (0)
Fricas	92.94 (158)	7.06 (12)
Sympy	80.00 (136)	% 20.00 (34)
Mupad	75.88 (129)	24.12 (41)
IntegrateAlgebraic	51.76 (88)	48.24 (82)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

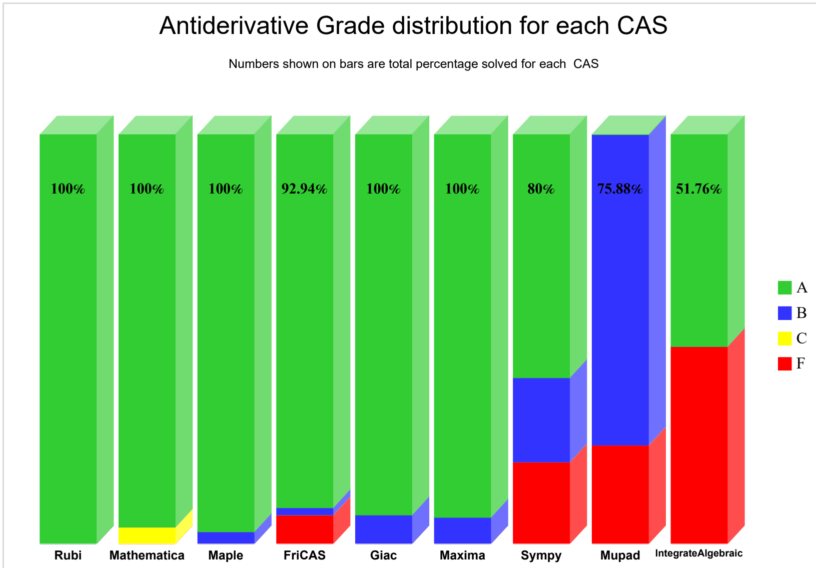
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

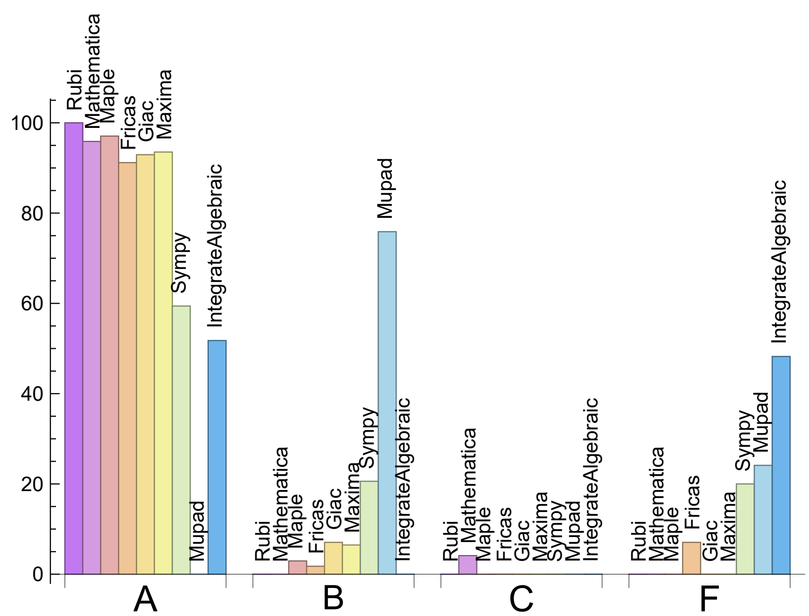
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	97.06	2.94	0.00	0.00
Mathematica	95.88	0.00	4.12	0.00
Maxima	93.53	6.47	0.00	0.00
Giac	92.94	7.06	0.00	0.00
Fricas	91.18	1.76	0.00	7.06
Sympy	59.41	20.59	0.00	20.00
IntegrateAlgebraic	51.76	0.00	0.00	48.24
Mupad	N/A	75.88	0.00	24.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	12	0.00 %	0.00 %	100.00 %
IntegrateAlgebraic	82	100.00 %	0.00 %	0.00 %
Giac	0	0.00 %	0.00 %	0.00 %
Maxima	0	0.00 %	0.00 %	0.00 %
Sympy	34	0.00 %	100.00 %	0.00 %
Mupad	41	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

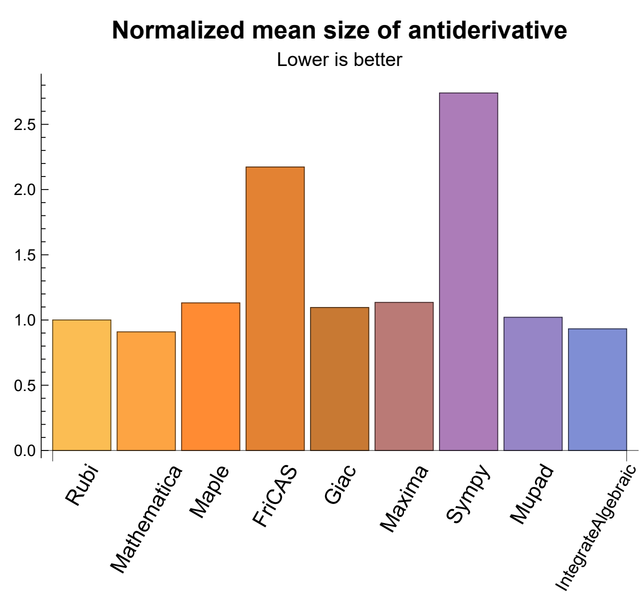
1.3 Performance

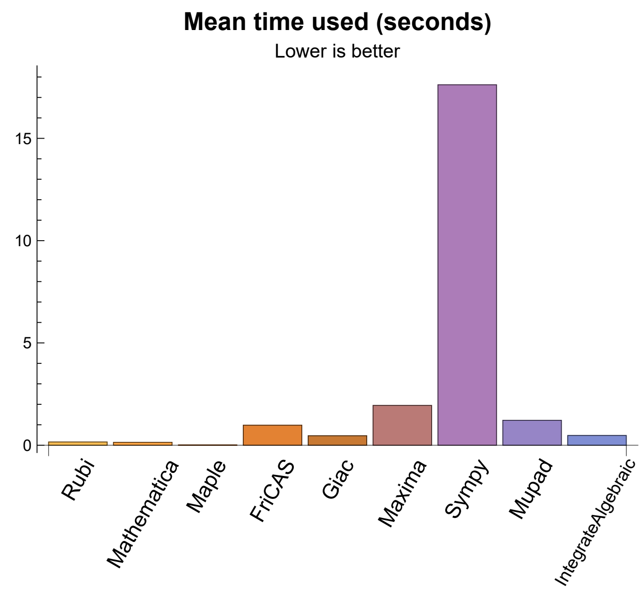
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	130.92	1.00	125.00	1.00
Mathematica	0.14	114.98	0.91	107.00	0.90
Maple	0.01	154.84	1.13	133.50	1.10
Maxima	1.95	164.95	1.14	122.00	0.99
Fricas	0.98	293.15	2.17	242.50	2.14
Sympy	17.62	353.79	2.74	203.50	1.70
Giac	0.46	151.56	1.10	125.50	0.99
Mupad	1.22	129.83	1.02	108.00	0.98
IntegrateAlgebraic	0.48	120.82	0.93	102.00	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

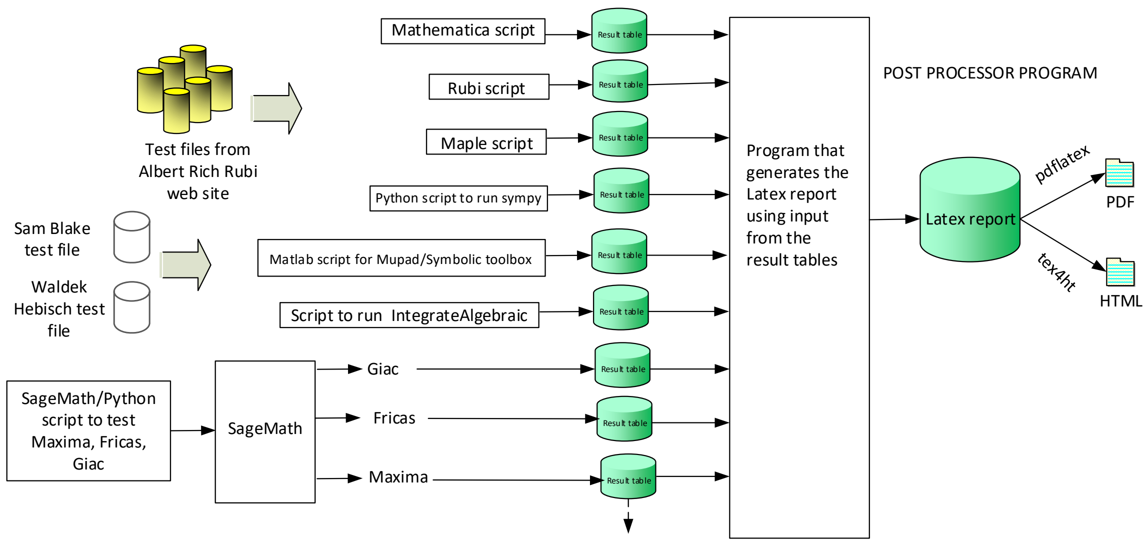
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

B grade: { }

C grade: { 13, 14, 20, 21, 144, 145, 146 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170 }

B grade: { 48, 157, 158, 168, 169 }

C grade: { }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 160, 161, 162, 163, 164, 165, 166, 167 }

B grade: { 47, 48, 49, 155, 156, 157, 158, 159, 168, 169, 170 }

C grade: { }

F grade: { }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

B grade: { 103, 104, 105 }

C grade: { }

F grade: { 58, 59, 60, 61, 66, 67, 68, 69, 74, 75, 76, 77 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 101, 102, 106, 107, 108, 109, 110, 112, 113, 116, 119, 120, 121, 122, 123, 124, 125, 126, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 149, 150, 151, 152, 165, 166, 167 }

B grade: { 25, 33, 34, 37, 39, 40, 41, 42, 52, 54, 55, 56, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 98, 99, 100, 111, 114, 115, 117, 118, 127, 147, 148, 153, 154 }

C grade: { }

F grade: { 47, 48, 49, 50, 51, 57, 87, 88, 89, 95, 96, 97, 103, 104, 105, 128, 137, 138, 144, 145, 146, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 168, 169, 170 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 170 }

B grade: { 7, 14, 28, 35, 45, 146, 152, 153, 154, 162, 163, 164 }

C grade: { }

F grade: { }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 86, 88, 89, 92, 94, 96, 97, 98, 100, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 151, 152, 153, 154, 162, 163, 165, 166, 167 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 15, 16, 17, 22, 29, 36, 47, 48, 82, 83, 84, 85, 87, 90, 91, 93, 95, 99, 101, 103, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 161, 164, 168, 169, 170 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 107, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

B grade: { }

C grade: { }

F grade: { 45, 46, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	107	115	107	206	192	93	-1	101
N.S.	1	1.00	0.84	0.91	0.84	1.62	1.51	0.73	-0.01	0.80
time (sec)	N/A	0.082	0.231	0.010	1.407	0.913	16.250	0.509	0.000	0.258

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	93	94	86	175	165	81	-1	92
N.S.	1	1.00	0.89	0.90	0.83	1.68	1.59	0.78	-0.01	0.88
time (sec)	N/A	0.049	0.192	0.007	1.363	0.997	10.139	0.471	0.000	0.230

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	86	75	67	157	124	68	-1	77
N.S.	1	1.00	1.08	0.94	0.84	1.96	1.55	0.85	-0.01	0.96
time (sec)	N/A	0.026	0.152	0.005	1.319	0.664	14.405	0.440	0.000	0.192

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	53	45	128	70	55	52	68
N.S.	1	1.00	1.00	0.79	0.67	1.91	1.04	0.82	0.78	1.01
time (sec)	N/A	0.019	0.057	0.006	1.318	0.735	6.510	0.414	1.162	0.233

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	100	78	59	341	107	78	68	95
N.S.	1	1.00	1.27	0.99	0.75	4.32	1.35	0.99	0.86	1.20
time (sec)	N/A	0.061	0.253	0.006	1.360	1.017	8.800	0.452	1.236	0.242

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	99	97	59	333	124	102	89	92
N.S.	1	1.00	1.32	1.29	0.79	4.44	1.65	1.36	1.19	1.23
time (sec)	N/A	0.059	0.178	0.010	1.386	0.927	11.972	0.474	1.688	0.249

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	108	121	83	377	107	163	94	96
N.S.	1	1.00	1.35	1.51	1.04	4.71	1.34	2.04	1.18	1.20
time (sec)	N/A	0.060	0.093	0.007	1.362	0.708	5.499	0.482	1.795	0.341

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	126	134	126	254	318	115	-1	125
N.S.	1	1.00	0.84	0.89	0.84	1.69	2.12	0.77	-0.01	0.83
time (sec)	N/A	0.095	0.258	0.012	1.390	0.732	20.789	0.440	0.000	0.350

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	113	113	105	223	287	103	-1	116
N.S.	1	1.00	0.89	0.89	0.83	1.76	2.26	0.81	-0.01	0.91
time (sec)	N/A	0.062	0.240	0.009	1.374	0.910	20.411	0.506	0.000	0.421

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	107	94	86	205	223	89	-1	101
N.S.	1	1.00	1.04	0.91	0.83	1.99	2.17	0.86	-0.01	0.98
time (sec)	N/A	0.033	0.194	0.006	1.351	0.993	22.469	0.473	0.000	0.353

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	88	69	61	176	219	76	54	92
N.S.	1	1.00	1.01	0.79	0.70	2.02	2.52	0.87	0.62	1.06
time (sec)	N/A	0.026	0.078	0.005	1.412	0.925	12.912	0.440	1.182	0.374

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	118	107	88	439	218	100	83	114
N.S.	1	1.00	1.11	1.01	0.83	4.14	2.06	0.94	0.78	1.08
time (sec)	N/A	0.094	0.306	0.009	1.319	0.888	35.492	0.539	1.312	0.401

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	105	126	88	411	184	124	86	115
N.S.	1	1.00	0.97	1.17	0.81	3.81	1.70	1.15	0.80	1.06
time (sec)	N/A	0.089	0.184	0.009	1.386	0.849	13.290	0.602	1.877	0.399

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	90	150	112	425	182	191	91	115
N.S.	1	1.00	0.81	1.35	1.01	3.83	1.64	1.72	0.82	1.04
time (sec)	N/A	0.085	0.063	0.010	1.279	0.978	15.566	0.550	2.120	0.463

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	145	153	145	302	469	140	-1	149
N.S.	1	1.00	0.84	0.88	0.84	1.75	2.71	0.81	-0.01	0.86
time (sec)	N/A	0.105	0.402	0.010	1.358	1.018	37.442	0.529	0.000	0.463

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	131	132	124	271	442	128	-1	140
N.S.	1	1.00	0.87	0.88	0.83	1.81	2.95	0.85	-0.01	0.93
time (sec)	N/A	0.069	0.406	0.007	1.373	0.801	61.217	0.460	0.000	0.432

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	112	113	105	253	354	114	-1	125
N.S.	1	1.00	0.89	0.90	0.83	2.01	2.81	0.90	-0.01	0.99
time (sec)	N/A	0.044	0.550	0.007	1.444	1.018	26.540	0.499	0.000	0.427

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	108	85	77	224	348	101	54	116
N.S.	1	1.00	1.01	0.79	0.72	2.09	3.25	0.94	0.50	1.08
time (sec)	N/A	0.037	0.088	0.005	1.397	0.564	26.046	0.608	1.163	0.421

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	139	138	119	539	323	125	101	138
N.S.	1	1.00	1.05	1.05	0.90	4.08	2.45	0.95	0.77	1.05
time (sec)	N/A	0.159	0.361	0.007	1.378	0.927	40.753	0.585	1.247	0.464

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	117	158	120	519	318	150	104	141
N.S.	1	1.00	0.86	1.16	0.88	3.82	2.34	1.10	0.76	1.04
time (sec)	N/A	0.132	0.232	0.010	1.403	0.946	18.911	0.499	2.164	0.449

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	92	181	143	535	279	219	111	142
N.S.	1	1.00	0.65	1.28	1.01	3.79	1.98	1.55	0.79	1.01
time (sec)	N/A	0.117	0.035	0.012	1.382	0.581	12.983	0.569	2.591	0.511

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	76	96	88	158	150	74	-1	77
N.S.	1	1.00	0.73	0.92	0.85	1.52	1.44	0.71	-0.01	0.74
time (sec)	N/A	0.077	0.051	0.010	1.361	0.758	7.912	0.498	0.000	0.255

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	64	75	67	127	94	61	93	68
N.S.	1	1.00	0.79	0.93	0.83	1.57	1.16	0.75	1.15	0.84
time (sec)	N/A	0.042	0.060	0.008	1.293	0.902	6.228	0.524	1.467	0.293

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	55	47	109	70	50	82	58
N.S.	1	1.00	1.02	0.98	0.84	1.95	1.25	0.89	1.46	1.04
time (sec)	N/A	0.023	0.042	0.004	1.326	0.677	6.255	0.479	1.237	0.247

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	46	37	29	92	102	39	36	46
N.S.	1	1.00	1.07	0.86	0.67	2.14	2.37	0.91	0.84	1.07
time (sec)	N/A	0.015	0.059	0.006	1.355	1.164	2.641	0.547	1.144	0.239

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	52	33	273	99	58	42	70
N.S.	1	1.00	1.00	0.98	0.62	5.15	1.87	1.09	0.79	1.32
time (sec)	N/A	0.040	0.020	0.007	1.377	1.288	5.141	0.499	1.299	0.196

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	49	37	101	41	65	39	61
N.S.	1	1.00	1.00	1.04	0.79	2.15	0.87	1.38	0.83	1.30
time (sec)	N/A	0.033	0.039	0.007	1.306	0.804	2.799	0.511	1.201	0.214

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	63	68	56	123	66	146	58	71
N.S.	1	1.00	0.88	0.94	0.78	1.71	0.92	2.03	0.81	0.99
time (sec)	N/A	0.053	0.136	0.011	1.315	0.752	7.056	0.431	1.350	0.312

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	72	93	85	197	117	70	-1	74
N.S.	1	1.00	0.89	1.15	1.05	2.43	1.44	0.86	-0.01	0.91
time (sec)	N/A	0.043	0.063	0.010	1.352	0.894	10.335	0.512	0.000	0.387

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	67	72	64	164	83	58	61	61
N.S.	1	1.00	1.02	1.09	0.97	2.48	1.26	0.88	0.92	0.92
time (sec)	N/A	0.036	0.059	0.007	1.296	1.047	16.608	0.547	1.342	0.355

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	64	54	46	147	66	48	53	53
N.S.	1	1.00	1.33	1.12	0.96	3.06	1.38	1.00	1.10	1.10
time (sec)	N/A	0.020	0.081	0.007	1.309	0.908	15.185	0.498	1.055	0.346

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	27	26	31	35	46	23	24	27
N.S.	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	0.86	0.96
time (sec)	N/A	0.007	0.032	0.003	1.366	0.848	10.223	0.443	0.909	0.310

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	60	48	146	206	59	50	61
N.S.	1	1.00	1.00	1.28	1.02	3.11	4.38	1.26	1.06	1.30
time (sec)	N/A	0.038	0.048	0.009	1.299	1.002	11.294	0.445	1.291	0.329

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	72	80	68	169	235	96	70	75
N.S.	1	1.00	1.03	1.14	0.97	2.41	3.36	1.37	1.00	1.07
time (sec)	N/A	0.057	0.044	0.009	1.319	0.876	15.826	0.502	1.448	0.345

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	75	101	89	211	124	171	94	87
N.S.	1	1.00	0.79	1.06	0.94	2.22	1.31	1.80	0.99	0.92
time (sec)	N/A	0.078	0.223	0.010	1.353	0.899	10.795	0.497	1.592	0.456

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	69	91	102	239	400	70	-1	72
N.S.	1	1.00	0.87	1.15	1.29	3.03	5.06	0.89	-0.01	0.91
time (sec)	N/A	0.042	0.124	0.010	1.409	0.822	18.323	0.482	0.000	0.509

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	41	70	63	141	36	51	44
N.S.	1	1.00	0.83	0.77	1.32	1.19	2.66	0.68	0.96	0.83
time (sec)	N/A	0.022	0.055	0.006	1.360	0.981	17.325	0.482	0.967	0.440

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	47	32	29	51	49	95	26	34	32
N.S.	1	0.94	0.64	0.58	1.02	0.98	1.90	0.52	0.68	0.64
time (sec)	N/A	0.014	0.023	0.003	1.311	0.924	13.983	0.518	0.924	0.419

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	43	40	48	62	146	37	41	43
N.S.	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	0.80	0.84
time (sec)	N/A	0.010	0.047	0.004	1.367	0.878	13.203	0.478	0.929	0.411

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	69	92	80	239	840	82	80	83
N.S.	1	1.00	0.91	1.21	1.05	3.14	11.05	1.08	1.05	1.09
time (sec)	N/A	0.063	0.098	0.008	1.367	0.753	25.923	0.540	1.375	0.496

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	95	112	100	264	910	119	96	101
N.S.	1	1.00	0.91	1.08	0.96	2.54	8.75	1.14	0.92	0.97
time (sec)	N/A	0.088	0.073	0.010	1.342	1.042	24.183	0.553	1.579	0.538

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	106	134	122	307	1034	197	123	111
N.S.	1	1.00	0.82	1.04	0.95	2.38	8.02	1.53	0.95	0.86
time (sec)	N/A	0.119	0.215	0.011	1.342	0.787	24.756	0.534	1.621	0.633

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	24	29	28	31	24	19	20	37
N.S.	1	1.00	0.89	1.07	1.04	1.15	0.89	0.70	0.74	1.37
time (sec)	N/A	0.008	0.030	0.008	2.969	0.602	0.240	0.471	0.040	0.128

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	24	29	28	31	24	19	20	37
N.S.	1	1.00	0.89	1.07	1.04	1.15	0.89	0.70	0.74	1.37
time (sec)	N/A	0.017	0.020	0.005	2.889	0.624	0.294	0.417	0.033	0.121

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	33	15	22	26	27	30	14	0
N.S.	1	1.00	1.94	0.88	1.29	1.53	1.59	1.76	0.82	0.00
time (sec)	N/A	0.006	0.009	0.003	2.876	0.869	0.186	0.410	0.919	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	6	7	6	6	5	6	6	0
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00	0.00
time (sec)	N/A	0.003	0.005	0.003	2.972	1.003	0.153	0.458	0.039	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	165	265	435	522	0	204	-1	173
N.S.	1	1.00	0.77	1.24	2.04	2.45	0.00	0.96	-0.00	0.81
time (sec)	N/A	0.324	0.649	0.048	1.680	0.860	0.000	0.539	0.000	1.436

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	147	277	447	467	0	138	-1	138
N.S.	1	1.00	0.98	1.85	2.98	3.11	0.00	0.92	-0.01	0.92
time (sec)	N/A	0.165	0.408	0.013	1.635	1.105	0.000	0.527	0.000	1.132

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	89	95	240	137	0	112	196	98
N.S.	1	1.00	0.67	0.72	1.82	1.04	0.00	0.85	1.48	0.74
time (sec)	N/A	0.166	0.114	0.005	1.443	0.676	0.000	0.624	1.274	1.207

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	78	76	253	122	0	81	186	79
N.S.	1	1.00	0.52	0.51	1.70	0.82	0.00	0.54	1.25	0.53
time (sec)	N/A	0.181	0.104	0.008	1.418	0.925	0.000	0.580	1.192	0.930

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	84	85	179	131	0	95	133	88
N.S.	1	1.00	0.60	0.61	1.29	0.94	0.00	0.68	0.96	0.63
time (sec)	N/A	0.151	0.126	0.008	1.380	1.008	0.000	0.586	1.138	1.150

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	87	88	197	134	904	94	133	91
N.S.	1	1.00	0.63	0.63	1.42	0.96	6.50	0.68	0.96	0.65
time (sec)	N/A	0.131	0.142	0.008	1.420	0.987	118.646	0.518	1.094	0.850

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	75	73	123	119	796	82	99	76
N.S.	1	1.00	0.63	0.61	1.03	1.00	6.69	0.69	0.83	0.64
time (sec)	N/A	0.088	0.088	0.005	1.341	0.772	85.296	0.604	1.049	1.030

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	92	96	153	137	1880	112	115	99
N.S.	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91	0.78
time (sec)	N/A	0.069	0.086	0.007	1.334	0.603	94.218	0.504	1.033	0.758

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	120	169	157	465	6613	152	159	146
N.S.	1	1.00	0.87	1.22	1.14	3.37	47.92	1.10	1.15	1.06
time (sec)	N/A	0.162	0.229	0.012	1.448	0.806	107.414	0.573	1.620	1.202

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	158	240	228	525	6922	239	225	190
N.S.	1	1.00	0.84	1.28	1.21	2.79	36.82	1.27	1.20	1.01
time (sec)	N/A	0.381	0.217	0.012	1.412	0.708	165.702	0.511	2.095	1.152

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	178	288	265	688	0	325	279	202
N.S.	1	1.00	0.81	1.32	1.21	3.14	0.00	1.48	1.27	0.92
time (sec)	N/A	0.480	0.534	0.015	1.495	1.171	0.000	0.484	2.518	1.574

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	54	53	0	60	57	57	0
N.S.	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88	0.00
time (sec)	N/A	0.074	0.031	0.003	1.341	0.000	0.101	0.369	1.205	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	54	53	0	60	57	57	0
N.S.	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88	0.00
time (sec)	N/A	0.076	0.016	0.002	1.325	0.000	0.091	0.379	1.184	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	54	53	0	60	57	57	0
N.S.	1	1.00	1.00	0.83	0.82	0.00	0.92	0.88	0.88	0.00
time (sec)	N/A	0.061	0.010	0.003	1.312	0.000	0.136	0.318	1.185	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	51	50	0	56	54	54	0
N.S.	1	1.00	1.00	0.85	0.83	0.00	0.93	0.90	0.90	0.00
time (sec)	N/A	0.041	0.009	0.000	1.322	0.000	0.121	0.361	1.164	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	53	48	48	54	53	52	0
N.S.	1	1.00	1.00	0.95	0.86	0.86	0.96	0.95	0.93	0.00
time (sec)	N/A	0.040	0.016	0.003	1.354	0.825	0.325	0.410	1.170	0.001

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	50	48	55	49	50	49	0
N.S.	1	1.00	1.00	0.93	0.89	1.02	0.91	0.93	0.91	0.00
time (sec)	N/A	0.048	0.055	0.006	1.343	0.815	0.280	0.386	1.145	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	51	48	48	55	51	48	47	0
N.S.	1	1.00	0.94	0.89	0.89	1.02	0.94	0.89	0.87	0.00
time (sec)	N/A	0.048	0.037	0.007	1.316	0.616	0.507	0.375	1.138	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	55	51	49	55	54	50	50	0
N.S.	1	1.00	1.02	0.94	0.91	1.02	1.00	0.93	0.93	0.00
time (sec)	N/A	0.049	0.037	0.004	1.328	0.752	1.010	0.432	1.148	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	98	102	101	0	110	105	108	0
N.S.	1	1.00	0.90	0.94	0.93	0.00	1.01	0.96	0.99	0.00
time (sec)	N/A	0.124	0.057	0.001	1.365	0.000	0.135	0.350	1.130	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	92	102	101	0	110	105	108	0
N.S.	1	1.00	0.84	0.94	0.93	0.00	1.01	0.96	0.99	0.00
time (sec)	N/A	0.112	0.070	0.001	1.305	0.000	0.142	0.369	1.111	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	92	102	101	0	110	105	107	0
N.S.	1	1.00	0.88	0.98	0.97	0.00	1.06	1.01	1.03	0.00
time (sec)	N/A	0.074	0.044	0.002	1.389	0.000	0.088	0.360	1.111	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	88	99	98	0	107	102	105	0
N.S.	1	1.00	0.89	1.00	0.99	0.00	1.08	1.03	1.06	0.00
time (sec)	N/A	0.072	0.056	0.002	1.348	0.000	0.089	0.379	1.106	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	88	100	96	96	104	100	103	0
N.S.	1	1.00	0.96	1.09	1.04	1.04	1.13	1.09	1.12	0.00
time (sec)	N/A	0.069	0.079	0.003	1.384	0.802	0.318	0.464	1.106	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	88	98	96	103	99	98	92	0
N.S.	1	1.00	0.98	1.09	1.07	1.14	1.10	1.09	1.02	0.00
time (sec)	N/A	0.080	0.060	0.007	1.315	0.514	0.351	0.378	1.107	0.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	87	97	96	103	100	97	103	0
N.S.	1	1.00	0.89	0.99	0.98	1.05	1.02	0.99	1.05	0.00
time (sec)	N/A	0.086	0.044	0.007	1.325	0.660	0.578	0.335	1.106	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	83	97	97	103	100	97	106	0
N.S.	1	1.00	0.85	0.99	0.99	1.05	1.02	0.99	1.08	0.00
time (sec)	N/A	0.086	0.056	0.007	1.370	0.881	1.456	0.430	1.275	0.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	149	150	145	0	163	153	153	0
N.S.	1	1.00	1.00	1.01	0.97	0.00	1.09	1.03	1.03	0.00
time (sec)	N/A	0.186	0.026	0.002	1.328	0.000	0.167	0.459	1.299	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	125	150	145	0	165	153	153	0
N.S.	1	1.00	0.84	1.01	0.97	0.00	1.11	1.03	1.03	0.00
time (sec)	N/A	0.141	0.065	0.000	1.361	0.000	0.138	0.461	1.275	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	124	150	145	0	163	153	153	0
N.S.	1	1.00	0.90	1.09	1.05	0.00	1.18	1.11	1.11	0.00
time (sec)	N/A	0.095	0.063	0.002	1.336	0.000	0.136	0.362	1.278	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	121	147	142	0	158	149	149	0
N.S.	1	1.00	0.91	1.11	1.07	0.00	1.19	1.12	1.12	0.00
time (sec)	N/A	0.093	0.050	0.001	1.341	0.000	0.135	0.401	1.259	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	121	148	140	140	158	148	147	0
N.S.	1	1.00	0.94	1.15	1.09	1.09	1.22	1.15	1.14	0.00
time (sec)	N/A	0.090	0.067	0.004	1.321	0.737	0.399	0.327	1.264	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	123	145	139	147	150	145	121	0
N.S.	1	1.00	0.99	1.17	1.12	1.19	1.21	1.17	0.98	0.00
time (sec)	N/A	0.109	0.081	0.007	1.348	0.815	0.456	0.406	1.184	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	124	144	139	147	151	144	143	0
N.S.	1	1.00	0.92	1.07	1.03	1.09	1.12	1.07	1.06	0.00
time (sec)	N/A	0.112	0.062	0.007	1.357	0.842	0.674	0.409	1.257	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	124	146	142	147	155	146	148	0
N.S.	1	1.00	0.89	1.05	1.02	1.06	1.12	1.05	1.06	0.00
time (sec)	N/A	0.114	0.050	0.009	1.317	0.774	1.086	0.396	1.368	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	130	176	145	332	316	161	-1	0
N.S.	1	1.00	0.86	1.17	0.96	2.20	2.09	1.07	-0.01	0.00
time (sec)	N/A	0.145	0.082	0.007	2.962	0.965	1.434	0.431	0.000	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	114	152	127	270	274	137	-1	0
N.S.	1	1.00	0.88	1.17	0.98	2.08	2.11	1.05	-0.01	0.00
time (sec)	N/A	0.124	0.104	0.006	2.932	0.941	1.349	0.350	0.000	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	95	128	98	238	245	112	-1	0
N.S.	1	1.00	0.86	1.15	0.88	2.14	2.21	1.01	-0.01	0.00
time (sec)	N/A	0.113	0.057	0.005	3.002	0.923	1.647	0.399	0.000	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	81	106	82	180	211	88	-1	0
N.S.	1	1.00	0.88	1.15	0.89	1.96	2.29	0.96	-0.01	0.00
time (sec)	N/A	0.085	0.071	0.006	3.007	0.984	0.988	0.423	0.000	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	68	83	64	157	219	66	79	0
N.S.	1	1.00	0.93	1.14	0.88	2.15	3.00	0.90	1.08	0.00
time (sec)	N/A	0.065	0.043	0.003	2.929	0.760	0.881	0.460	1.425	0.001

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	73	80	65	158	0	66	-1	0
N.S.	1	1.00	1.01	1.11	0.90	2.19	0.00	0.92	-0.01	0.00
time (sec)	N/A	0.098	0.056	0.007	2.931	0.920	0.000	0.451	0.000	0.001

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	75	83	67	165	0	68	78	0
N.S.	1	1.00	0.99	1.09	0.88	2.17	0.00	0.89	1.03	0.00
time (sec)	N/A	0.098	0.048	0.008	3.001	0.768	0.000	0.383	1.212	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	84	102	76	205	0	80	97	0
N.S.	1	1.00	0.91	1.11	0.83	2.23	0.00	0.87	1.05	0.00
time (sec)	N/A	0.109	0.084	0.008	3.052	1.044	0.000	0.386	1.304	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	139	201	150	468	335	159	-1	0
N.S.	1	1.00	0.79	1.14	0.85	2.66	1.90	0.90	-0.01	0.00
time (sec)	N/A	0.268	0.134	0.011	2.835	0.584	4.769	0.432	0.000	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	128	177	127	372	289	131	-1	0
N.S.	1	1.00	0.83	1.15	0.82	2.42	1.88	0.85	-0.01	0.00
time (sec)	N/A	0.241	0.083	0.010	2.999	0.855	3.868	0.377	0.000	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	100	154	108	357	284	111	152	0
N.S.	1	1.00	0.75	1.15	0.81	2.66	2.12	0.83	1.13	0.00
time (sec)	N/A	0.226	0.082	0.010	2.963	0.875	4.611	0.390	1.292	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	92	127	84	287	212	81	-1	0
N.S.	1	1.00	0.91	1.26	0.83	2.84	2.10	0.80	-0.01	0.00
time (sec)	N/A	0.118	0.051	0.010	3.016	0.812	5.763	0.431	0.000	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	83	97	89	257	233	88	110	0
N.S.	1	1.00	0.89	1.04	0.96	2.76	2.51	0.95	1.18	0.00
time (sec)	N/A	0.065	0.091	0.008	2.945	0.845	3.094	0.375	1.316	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	85	125	87	296	0	93	-1	0
N.S.	1	1.00	0.89	1.32	0.92	3.12	0.00	0.98	-0.01	0.00
time (sec)	N/A	0.122	0.076	0.013	2.914	0.874	0.000	0.348	0.000	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	110	136	105	336	0	103	133	0
N.S.	1	1.00	1.00	1.24	0.95	3.05	0.00	0.94	1.21	0.00
time (sec)	N/A	0.142	0.074	0.014	2.985	0.876	0.000	0.342	1.410	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	112	169	117	441	0	126	158	0
N.S.	1	1.00	0.83	1.25	0.87	3.27	0.00	0.93	1.17	0.00
time (sec)	N/A	0.204	0.103	0.016	2.945	1.019	0.000	0.422	1.348	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	139	235	165	574	357	157	232	0
N.S.	1	1.00	0.75	1.27	0.89	3.10	1.93	0.85	1.25	0.00
time (sec)	N/A	0.338	0.117	0.013	3.075	0.838	29.587	0.410	1.556	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	126	206	136	480	282	122	-1	0
N.S.	1	1.00	0.81	1.33	0.88	3.10	1.82	0.79	-0.01	0.00
time (sec)	N/A	0.232	0.076	0.012	2.984	0.933	29.728	0.385	0.000	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	122	133	146	447	304	128	195	0
N.S.	1	1.00	0.90	0.98	1.07	3.29	2.24	0.94	1.43	0.00
time (sec)	N/A	0.158	0.095	0.010	2.977	0.865	20.010	0.422	1.391	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	99	110	111	357	178	97	-1	0
N.S.	1	1.00	0.83	0.92	0.93	3.00	1.50	0.82	-0.01	0.00
time (sec)	N/A	0.114	0.134	0.010	2.993	0.768	16.372	0.391	0.000	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	104	111	122	346	184	106	163	0
N.S.	1	1.00	0.90	0.96	1.05	2.98	1.59	0.91	1.41	0.00
time (sec)	N/A	0.068	0.114	0.009	2.998	0.880	11.272	0.386	1.327	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	117	184	133	488	0	128	-1	0
N.S.	1	1.00	0.90	1.42	1.02	3.75	0.00	0.98	-0.01	0.00
time (sec)	N/A	0.135	0.109	0.015	2.904	0.957	0.000	0.479	0.000	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	141	195	152	524	0	141	202	0
N.S.	1	1.00	0.98	1.35	1.06	3.64	0.00	0.98	1.40	0.00
time (sec)	N/A	0.228	0.099	0.016	2.986	0.903	0.000	0.490	1.400	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	147	250	172	696	0	162	229	0
N.S.	1	1.00	0.84	1.44	0.99	4.00	0.00	0.93	1.32	0.00
time (sec)	N/A	0.311	0.163	0.018	2.991	0.827	0.000	0.380	1.456	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	18	24	15	25	20	0
N.S.	1	1.00	1.00	0.95	0.90	1.20	0.75	1.25	1.00	0.00
time (sec)	N/A	0.023	0.007	0.010	1.339	0.804	0.174	0.317	0.912	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	18	15	22	14	22	17	14	18
N.S.	1	1.00	0.78	0.65	0.96	0.61	0.96	0.74	0.61	0.78
time (sec)	N/A	0.021	0.009	0.006	1.344	0.816	0.694	0.401	0.102	0.016

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	20	20	20	20	20	0
N.S.	1	1.00	1.00	0.84	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.024	0.013	0.004	2.920	0.741	0.141	0.355	0.038	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	22	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80	0.00
time (sec)	N/A	0.025	0.007	0.003	2.899	0.829	0.111	0.389	0.035	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	210	278	213	452	384	250	289	0
N.S.	1	1.00	1.00	1.32	1.01	2.15	1.83	1.19	1.38	0.00
time (sec)	N/A	0.161	0.156	0.005	2.897	0.859	1.652	0.457	0.929	0.001

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	162	230	172	368	337	200	243	0
N.S.	1	1.00	0.94	1.34	1.00	2.14	1.96	1.16	1.41	0.00
time (sec)	N/A	0.123	0.121	0.006	2.930	0.668	1.321	0.368	0.944	0.001

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	128	182	133	286	185	152	193	0
N.S.	1	1.00	0.94	1.34	0.98	2.10	1.36	1.12	1.42	0.00
time (sec)	N/A	0.105	0.097	0.003	2.964	0.791	1.128	0.416	0.912	0.001

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	98	135	94	236	160	106	96	0
N.S.	1	1.00	0.98	1.35	0.94	2.36	1.60	1.06	0.96	0.00
time (sec)	N/A	0.062	0.083	0.003	2.972	1.140	1.149	0.448	0.936	0.001

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	83	114	80	211	150	86	76	0
N.S.	1	1.00	0.99	1.36	0.95	2.51	1.79	1.02	0.90	0.00
time (sec)	N/A	0.094	0.066	0.007	2.929	1.252	1.638	0.337	1.067	0.001

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	83	115	79	216	151	81	80	0
N.S.	1	1.00	1.01	1.40	0.96	2.63	1.84	0.99	0.98	0.00
time (sec)	N/A	0.087	0.087	0.009	2.938	1.153	2.266	0.355	0.111	0.001

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	103	142	97	246	167	105	94	0
N.S.	1	1.00	0.99	1.37	0.93	2.37	1.61	1.01	0.90	0.00
time (sec)	N/A	0.102	0.088	0.007	3.090	1.201	6.728	0.367	1.203	0.001

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	139	190	134	292	301	151	127	0
N.S.	1	1.00	1.01	1.39	0.98	2.13	2.20	1.10	0.93	0.00
time (sec)	N/A	0.130	0.124	0.010	3.031	0.849	21.646	0.455	0.981	0.001

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	174	238	175	374	354	201	161	0
N.S.	1	1.00	0.99	1.36	1.00	2.14	2.02	1.15	0.92	0.00
time (sec)	N/A	0.146	0.144	0.010	3.006	0.954	32.721	0.423	1.018	0.001

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	211	286	214	458	398	249	197	0
N.S.	1	1.00	1.00	1.36	1.01	2.17	1.89	1.18	0.93	0.00
time (sec)	N/A	0.175	0.171	0.010	2.952	1.083	84.142	0.356	0.988	0.001

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	227	309	227	572	444	252	413	0
N.S.	1	1.00	0.95	1.29	0.95	2.38	1.85	1.05	1.72	0.00
time (sec)	N/A	0.294	0.128	0.013	3.007	0.875	3.070	0.387	0.099	0.001

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	187	258	183	478	257	201	288	0
N.S.	1	1.00	0.93	1.28	0.91	2.37	1.27	1.00	1.43	0.00
time (sec)	N/A	0.235	0.104	0.012	3.006	1.171	4.762	0.417	0.966	0.001

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	148	212	140	418	221	152	153	0
N.S.	1	1.00	0.91	1.30	0.86	2.56	1.36	0.93	0.94	0.00
time (sec)	N/A	0.229	0.089	0.012	2.957	1.170	3.036	0.373	1.002	0.001

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	122	177	117	364	201	126	113	0
N.S.	1	1.00	1.03	1.50	0.99	3.08	1.70	1.07	0.96	0.00
time (sec)	N/A	0.120	0.098	0.011	2.967	1.044	2.877	0.403	0.100	0.001

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	115	165	117	354	197	122	112	0
N.S.	1	1.00	1.03	1.47	1.04	3.16	1.76	1.09	1.00	0.00
time (sec)	N/A	0.132	0.070	0.013	2.992	1.018	9.465	0.360	0.996	0.001

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	125	182	130	378	212	123	119	0
N.S.	1	1.00	1.03	1.50	1.07	3.12	1.75	1.02	0.98	0.00
time (sec)	N/A	0.158	0.079	0.015	3.023	1.019	25.992	0.483	0.131	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	151	219	151	438	226	151	145	0
N.S.	1	1.00	0.99	1.44	0.99	2.88	1.49	0.99	0.95	0.00
time (sec)	N/A	0.213	0.091	0.015	2.989	0.962	32.024	0.490	0.999	0.001

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	190	268	194	488	394	201	181	0
N.S.	1	1.00	1.01	1.42	1.03	2.58	2.08	1.06	0.96	0.00
time (sec)	N/A	0.293	0.109	0.019	3.047	1.016	99.022	0.433	0.987	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	230	318	238	582	0	252	219	0
N.S.	1	1.00	1.00	1.38	1.03	2.53	0.00	1.10	0.95	0.00
time (sec)	N/A	0.376	0.124	0.016	2.985	0.872	0.000	0.378	1.010	0.001

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	272	394	281	762	503	301	506	0
N.S.	1	1.00	0.95	1.37	0.98	2.66	1.75	1.05	1.76	0.00
time (sec)	N/A	0.492	0.186	0.016	3.104	1.079	16.433	0.400	0.999	0.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	232	343	237	668	316	250	348	0
N.S.	1	1.00	0.94	1.39	0.96	2.70	1.28	1.01	1.41	0.00
time (sec)	N/A	0.411	0.152	0.015	2.991	1.142	18.216	0.470	0.107	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	176	294	193	614	280	200	206	0
N.S.	1	1.00	0.85	1.42	0.93	2.97	1.35	0.97	1.00	0.00
time (sec)	N/A	0.334	0.177	0.014	3.038	0.935	17.840	0.509	0.954	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	156	259	169	555	260	173	163	0
N.S.	1	1.00	0.93	1.55	1.01	3.32	1.56	1.04	0.98	0.00
time (sec)	N/A	0.262	0.134	0.012	3.021	0.971	13.069	0.448	1.018	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	141	234	154	504	243	149	148	0
N.S.	1	1.00	0.96	1.59	1.05	3.43	1.65	1.01	1.01	0.00
time (sec)	N/A	0.150	0.122	0.010	2.996	1.171	10.095	0.457	1.047	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	155	237	161	517	250	153	149	0
N.S.	1	1.00	1.01	1.55	1.05	3.38	1.63	1.00	0.97	0.00
time (sec)	N/A	0.176	0.132	0.013	2.992	1.174	26.563	0.449	1.094	0.001

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	169	264	181	570	270	170	166	0
N.S.	1	1.00	1.01	1.57	1.08	3.39	1.61	1.01	0.99	0.00
time (sec)	N/A	0.242	0.159	0.015	3.073	0.867	70.490	0.449	1.027	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	196	300	202	628	284	198	192	0
N.S.	1	1.00	1.00	1.53	1.03	3.20	1.45	1.01	0.98	0.00
time (sec)	N/A	0.350	0.124	0.018	2.978	1.163	150.751	0.439	1.043	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	234	351	247	678	0	250	230	0
N.S.	1	1.00	1.00	1.50	1.06	2.90	0.00	1.07	0.98	0.00
time (sec)	N/A	0.486	0.147	0.019	3.026	1.080	0.000	0.466	1.046	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	276	401	291	772	0	301	268	0
N.S.	1	1.00	1.00	1.45	1.05	2.79	0.00	1.09	0.97	0.00
time (sec)	N/A	0.603	0.158	0.022	3.090	0.745	0.000	0.403	1.070	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	158	193	347	177	442	264	186	196
N.S.	1	1.00	0.74	0.90	1.62	0.83	2.07	1.23	0.87	0.92
time (sec)	N/A	0.252	0.174	0.008	1.401	1.030	4.676	0.449	1.191	0.113

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	122	145	263	134	340	197	146	148
N.S.	1	1.00	0.73	0.87	1.57	0.80	2.04	1.18	0.87	0.89
time (sec)	N/A	0.194	0.123	0.008	1.449	1.104	2.905	0.508	1.107	0.080

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	89	99	180	94	238	130	103	102
N.S.	1	1.00	0.74	0.82	1.49	0.78	1.97	1.07	0.85	0.84
time (sec)	N/A	0.133	0.085	0.004	1.349	0.976	2.189	0.407	1.064	0.057

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	86	134	122	205	102	127	99	89
N.S.	1	1.00	0.83	1.30	1.18	1.99	0.99	1.23	0.96	0.86
time (sec)	N/A	0.140	0.130	0.009	1.346	1.034	37.870	0.398	1.810	0.094

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	131	127	104	210	138	114	99	92
N.S.	1	1.00	1.31	1.27	1.04	2.10	1.38	1.14	0.99	0.92
time (sec)	N/A	0.203	0.412	0.010	1.327	0.931	127.426	0.554	1.947	0.188

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	141	162	128	221	0	141	133	102
N.S.	1	1.00	1.24	1.42	1.12	1.94	0.00	1.24	1.17	0.89
time (sec)	N/A	0.233	0.374	0.011	1.379	1.080	0.000	0.397	2.191	0.193

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	162	238	193	261	0	232	199	126
N.S.	1	1.00	1.11	1.63	1.32	1.79	0.00	1.59	1.36	0.86
time (sec)	N/A	0.276	1.024	0.013	1.388	1.151	0.000	0.391	2.543	0.246

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	140	320	275	341	0	361	277	172
N.S.	1	1.00	0.72	1.64	1.41	1.75	0.00	1.85	1.42	0.88
time (sec)	N/A	0.350	0.336	0.016	1.346	1.403	0.000	0.397	2.913	0.399

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	184	368	339	414	586	224	-1	215
N.S.	1	1.00	0.75	1.50	1.38	1.69	2.39	0.91	-0.00	0.88
time (sec)	N/A	0.258	0.238	0.024	1.459	1.437	42.122	0.527	0.000	0.444

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	149	284	255	329	444	175	-1	167
N.S.	1	1.00	0.77	1.46	1.31	1.70	2.29	0.90	-0.01	0.86
time (sec)	N/A	0.208	0.167	0.010	1.387	1.068	32.562	0.537	0.000	0.340

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	118	203	174	250	362	129	-1	123
N.S.	1	1.00	0.81	1.40	1.20	1.72	2.50	0.89	-0.01	0.85
time (sec)	N/A	0.119	0.114	0.007	1.350	1.009	13.711	0.564	0.000	0.177

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	103	140	118	216	250	121	-1	105
N.S.	1	1.00	0.88	1.20	1.01	1.85	2.14	1.03	-0.01	0.90
time (sec)	N/A	0.136	0.118	0.010	1.259	0.877	9.052	0.522	0.000	0.224

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	93	117	102	210	197	176	143	95
N.S.	1	1.00	0.85	1.06	0.93	1.91	1.79	1.60	1.30	0.86
time (sec)	N/A	0.127	0.113	0.011	1.333	0.738	4.729	0.559	2.199	0.238

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	95	136	128	221	456	324	105	101
N.S.	1	1.00	0.81	1.15	1.08	1.87	3.86	2.75	0.89	0.86
time (sec)	N/A	0.133	0.113	0.011	1.327	1.024	6.207	0.597	1.724	0.278

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	103	111	193	100	891	554	124	114
N.S.	1	1.00	0.74	0.79	1.38	0.71	6.36	3.96	0.89	0.81
time (sec)	N/A	0.184	0.081	0.006	1.403	1.208	6.712	0.571	1.280	0.258

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	134	157	275	141	1642	667	171	160
N.S.	1	1.00	0.71	0.83	1.46	0.75	8.69	3.53	0.90	0.85
time (sec)	N/A	0.254	0.096	0.007	1.386	1.602	7.689	0.603	1.282	0.389

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	273	517	1221	987	0	342	-1	306
N.S.	1	1.00	0.72	1.36	3.20	2.59	0.00	0.90	-0.00	0.80
time (sec)	N/A	0.662	0.576	0.296	1.888	1.769	0.000	0.642	0.000	1.402

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	229	460	986	816	0	265	-1	241
N.S.	1	1.00	0.82	1.65	3.53	2.92	0.00	0.95	-0.00	0.86
time (sec)	N/A	0.455	0.479	0.011	1.787	1.546	0.000	0.593	0.000	1.122

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	194	405	753	653	0	203	-1	188
N.S.	1	1.00	0.92	1.93	3.59	3.11	0.00	0.97	-0.00	0.90
time (sec)	N/A	0.387	0.497	0.013	1.686	1.091	0.000	0.598	0.000	0.763

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	192	168	363	533	491	0	160	-1	158
N.S.	1	1.07	0.94	2.03	2.98	2.74	0.00	0.89	-0.01	0.88
time (sec)	N/A	0.314	0.456	0.010	1.661	1.425	0.000	0.550	0.000	0.649

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	98	109	335	141	0	131	-1	112
N.S.	1	1.00	0.73	0.81	2.50	1.05	0.00	0.98	-0.01	0.84
time (sec)	N/A	0.210	0.107	0.006	1.423	1.013	0.000	0.534	0.000	0.464

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	179	133	157	313	182	0	211	-1	160
N.S.	1	0.97	0.72	0.85	1.69	0.98	0.00	1.14	-0.01	0.86
time (sec)	N/A	0.246	0.184	0.008	1.471	0.928	0.000	0.575	0.000	0.512

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	165	205	337	225	0	349	-1	208
N.S.	1	1.00	0.68	0.85	1.39	0.93	0.00	1.44	-0.00	0.86
time (sec)	N/A	0.322	0.134	0.010	1.468	1.470	0.000	0.557	0.000	0.569

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	275	202	253	398	270	0	592	405	256
N.S.	1	0.98	0.72	0.90	1.42	0.96	0.00	2.11	1.44	0.91
time (sec)	N/A	0.429	0.158	0.009	1.545	1.829	0.000	0.632	2.397	0.635

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	334	234	301	489	311	0	938	421	304
N.S.	1	1.00	0.70	0.90	1.46	0.93	0.00	2.81	1.26	0.91
time (sec)	N/A	0.480	0.161	0.010	1.518	2.896	0.000	0.712	2.836	0.723

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	392	380	270	349	579	354	0	1162	-1	352
N.S.	1	0.97	0.69	0.89	1.48	0.90	0.00	2.96	-0.00	0.90
time (sec)	N/A	0.546	0.180	0.010	1.574	3.810	0.000	0.736	0.000	0.834

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	158	193	347	177	442	264	186	196
N.S.	1	1.00	0.74	0.90	1.62	0.83	2.07	1.23	0.87	0.92
time (sec)	N/A	0.222	0.185	0.007	1.435	0.956	9.372	0.454	1.203	0.105

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	122	145	263	134	340	197	146	148
N.S.	1	1.00	0.73	0.87	1.57	0.80	2.04	1.18	0.87	0.89
time (sec)	N/A	0.175	0.131	0.007	1.367	1.108	5.409	0.498	1.140	0.084

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	89	99	180	94	238	130	103	102
N.S.	1	1.00	0.74	0.82	1.49	0.78	1.97	1.07	0.85	0.84
time (sec)	N/A	0.150	0.090	0.006	1.355	0.712	3.406	0.398	1.084	0.061

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	257	221	478	826	705	0	224	-1	224
N.S.	1	0.98	0.85	1.83	3.16	2.70	0.00	0.86	-0.00	0.86
time (sec)	N/A	0.716	0.590	0.011	1.747	1.521	0.000	0.559	0.000	0.840

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	250	197	427	597	567	0	204	-1	204
N.S.	1	1.17	0.92	2.00	2.79	2.65	0.00	0.95	-0.00	0.95
time (sec)	N/A	0.410	0.514	0.009	1.743	1.421	0.000	0.590	0.000	0.729

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	138	166	421	187	0	220	-1	169
N.S.	1	1.00	0.72	0.86	2.18	0.97	0.00	1.14	-0.01	0.88
time (sec)	N/A	0.341	0.189	0.007	1.540	0.925	0.000	0.602	0.000	0.572

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [13] had the largest ratio of [.4000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	20	0.250
2	A	5	5	1.00	20	0.250
3	A	4	4	1.00	18	0.222
4	A	4	4	1.00	17	0.235
5	A	7	7	1.00	20	0.350
6	A	7	7	1.00	20	0.350
7	A	7	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	6	5	1.00	20	0.250
10	A	5	4	1.00	18	0.222
11	A	5	4	1.00	17	0.235
12	A	8	7	1.00	20	0.350
13	A	8	8	1.00	20	0.400
14	A	8	7	1.00	20	0.350
15	A	8	5	1.00	20	0.250
16	A	7	5	1.00	20	0.250
17	A	6	4	1.00	18	0.222
18	A	6	4	1.00	17	0.235
19	A	9	7	1.00	20	0.350
20	A	9	8	1.00	20	0.400
21	A	9	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	18	0.167
25	A	3	3	1.00	17	0.176
26	A	6	6	1.00	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	4	4	1.00	20	0.200
28	A	5	5	1.00	20	0.250
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	18	0.167
32	A	1	1	1.00	17	0.059
33	A	5	5	1.00	20	0.250
34	A	5	5	1.00	20	0.250
35	A	6	6	1.00	20	0.300
36	A	4	4	1.00	20	0.200
37	A	2	2	1.00	20	0.100
38	A	2	2	0.94	18	0.111
39	A	2	2	1.00	17	0.118
40	A	6	5	1.00	20	0.250
41	A	6	5	1.00	20	0.250
42	A	7	6	1.00	20	0.300
43	A	2	2	1.00	18	0.111
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	7	5	1.00	25	0.200
48	A	6	5	1.00	25	0.200
49	A	5	4	1.00	25	0.160
50	A	4	4	1.00	25	0.160
51	A	4	4	1.00	25	0.160
52	A	4	4	1.00	25	0.160
53	A	4	4	1.00	23	0.174
54	A	5	4	1.00	22	0.182
55	A	8	6	1.00	25	0.240
56	A	8	5	1.00	25	0.200
57	A	9	6	1.00	25	0.240
58	A	2	1	1.00	26	0.038
59	A	2	1	1.00	26	0.038
60	A	2	1	1.00	24	0.042

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	1	1.00	23	0.043
62	A	2	1	1.00	26	0.038
63	A	2	1	1.00	26	0.038
64	A	2	1	1.00	26	0.038
65	A	2	1	1.00	26	0.038
66	A	2	1	1.00	28	0.036
67	A	2	1	1.00	28	0.036
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	25	0.080
70	A	3	2	1.00	28	0.071
71	A	3	2	1.00	28	0.071
72	A	2	1	1.00	28	0.036
73	A	2	1	1.00	28	0.036
74	A	2	1	1.00	28	0.036
75	A	2	1	1.00	28	0.036
76	A	3	2	1.00	26	0.077
77	A	3	2	1.00	25	0.080
78	A	3	2	1.00	28	0.071
79	A	3	2	1.00	28	0.071
80	A	2	1	1.00	28	0.036
81	A	2	1	1.00	28	0.036
82	A	5	4	1.00	28	0.143
83	A	5	4	1.00	28	0.143
84	A	5	4	1.00	28	0.143
85	A	5	4	1.00	26	0.154
86	A	5	4	1.00	25	0.160
87	A	5	4	1.00	28	0.143
88	A	5	4	1.00	28	0.143
89	A	5	4	1.00	28	0.143
90	A	6	5	1.00	28	0.179
91	A	6	5	1.00	28	0.179
92	A	6	5	1.00	28	0.179
93	A	6	5	1.00	26	0.192
94	A	4	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	5	1.00	28	0.179
96	A	6	5	1.00	28	0.179
97	A	6	5	1.00	28	0.179
98	A	7	5	1.00	28	0.179
99	A	6	5	1.00	28	0.179
100	A	5	4	1.00	28	0.143
101	A	4	4	1.00	26	0.154
102	A	3	3	1.00	25	0.120
103	A	7	6	1.00	28	0.214
104	A	7	5	1.00	28	0.179
105	A	7	5	1.00	28	0.179
106	A	4	3	1.00	17	0.176
107	A	4	3	1.00	17	0.176
108	A	4	4	1.00	21	0.190
109	A	6	5	1.00	15	0.333
110	A	3	2	1.00	30	0.067
111	A	3	2	1.00	30	0.067
112	A	3	2	1.00	30	0.067
113	A	3	2	1.00	27	0.074
114	A	3	2	1.00	30	0.067
115	A	3	2	1.00	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	30	0.067
118	A	3	2	1.00	30	0.067
119	A	3	2	1.00	30	0.067
120	A	5	4	1.00	30	0.133
121	A	5	4	1.00	30	0.133
122	A	5	4	1.00	30	0.133
123	A	4	3	1.00	27	0.111
124	A	4	3	1.00	30	0.100
125	A	4	3	1.00	30	0.100
126	A	4	3	1.00	30	0.100
127	A	4	3	1.00	30	0.100
128	A	4	3	1.00	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	6	5	1.00	30	0.167
130	A	6	5	1.00	30	0.167
131	A	6	5	1.00	30	0.167
132	A	6	5	1.00	30	0.167
133	A	4	4	1.00	27	0.148
134	A	4	4	1.00	30	0.133
135	A	5	4	1.00	30	0.133
136	A	5	3	1.00	30	0.100
137	A	5	3	1.00	30	0.100
138	A	5	3	1.00	30	0.100
139	A	3	2	1.00	32	0.062
140	A	3	2	1.00	32	0.062
141	A	3	2	1.00	30	0.067
142	A	5	4	1.00	32	0.125
143	A	6	5	1.00	32	0.156
144	A	6	6	1.00	32	0.188
145	A	6	6	1.00	32	0.188
146	A	7	7	1.00	32	0.219
147	A	7	6	1.00	32	0.188
148	A	6	6	1.00	32	0.188
149	A	5	5	1.00	29	0.172
150	A	6	6	1.00	32	0.188
151	A	6	6	1.00	32	0.188
152	A	6	6	1.00	32	0.188
153	A	5	3	1.00	32	0.094
154	A	6	4	1.00	32	0.125
155	A	11	9	1.00	32	0.281
156	A	10	9	1.00	32	0.281
157	A	9	9	1.00	32	0.281
158	A	8	8	1.07	32	0.250
159	A	5	4	1.00	29	0.138
160	A	6	5	0.97	32	0.156
161	A	7	5	1.00	32	0.156
162	A	8	4	0.98	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	9	5	1.00	32	0.156
164	A	10	5	0.97	32	0.156
165	A	4	3	1.00	33	0.091
166	A	4	3	1.00	33	0.091
167	A	4	3	1.00	31	0.097
168	A	10	9	0.98	37	0.243
169	A	6	5	1.17	34	0.147
170	A	6	4	1.00	37	0.108

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=127

$$\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{\int x^2(-3aB+6Abx)\sqrt{a+bx^2}\,dx}{6b}$$

$$=\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{\int x(-12aAb-15abBx)\sqrt{a+bx^2}\,dx}{30b^2}$$

$$=\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2}+\frac{\left(a^2B\right)\int\sqrt{a+}}{8b^2}$$

$$=\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}$$

$$=\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}$$

$$=\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/}}{120b^2}$$

Mathematica [A] time = 0.23, size = 107, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-a^2(32A+15Bx)+2abx^2(8A+5Bx)+8b^2x^4(6A+5Bx)\right)\right)}{240b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(8*A + 5*B*x
) - a^2*(32*A + 15*B*x)) + (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[1 + (b*x^2)/a]))/(240*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.26, size = 101, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(-32a^2A-15a^2Bx+16aAbx^2+10abBx^3+48Ab^2x^4+40b^2Bx^5\right)}{240b^2}-\frac{a^3B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-32*a^2*A - 15*a^2*B*x + 16*a*A*b*x^2 + 10*a*b*B*x^3 + 48
*A*b^2*x^4 + 40*b^2*B*x^5))/(240*b^2) - (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(16*b^(5/2))
```

fricas [A] time = 0.91, size = 206, normalized size = 1.62

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b\right)\sqrt{bx^2+a}}{480\,b^3},-\frac{15\,Ba^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b\right)\sqrt{bx^2+a}}{240\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a^2
*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3
+ 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.51, size = 93, normalized size = 0.73

$$-\frac{Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)*x
 - 32*A*a^2/b^2)
```

maple [A] time = 0.01, size = 115, normalized size = 0.91

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B\,x^{3}}{6b}+\frac{B\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} A\,x^{2}}{5b}+\frac{\sqrt{b\,x^{2}+a}\;B\,a^{2}x}{16b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} Bax}{8b^{2}}-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}} Aa}{15b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/8*B*a/b^2*x*(b*x^2+a)^(3/2)+1/16*a^2*B*x*(b*x
^2+a)^(1/2)/b^2+1/16*B*a^3/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A*x^2*
(b*x^2+a)^(3/2)/b-2/15*A*a/b^2*(b*x^2+a)^(3/2)
```

maxima [A] time = 1.41, size = 107, normalized size = 0.84

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bx^{3}}{6\,b}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ax^{2}}{5\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bax}{8\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Ba^{2}x}{16\,b^{2}}+\frac{Ba^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{3}{2}}Aa}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b - 1/8*(b*x^2
+ a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/16*B*a^3*arcsin
h(b*x/sqrt(a*b))/b^(5/2) - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3}\sqrt{b\,x^{2}+a}\;\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 16.25, size = 192, normalized size = 1.51

$$A\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)-\frac{Ba^{\frac{5}{2}}x}{16b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{3}{2}}x^{3}}{48b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5B\sqrt{a}\,x^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}+\frac{Ba^{3}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^{7}}{6\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(
15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) - B*a**
(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)*x**3/(48*b*sqrt(1 + b*x**
2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2 A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^2 A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{\left(a+bx^2\right)^{3/2}(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*A*x*Sqrt[a + b*x^2])/(8*b) + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{\int x(-2aB+5Abx)\sqrt{a+bx^2}\,dx}{5b}$$
$$=\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{(aA)\int\sqrt{a+bx^2}\,dx}{4b}$$
$$=-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\int\frac{1}{\sqrt{a+b}}}{8b}$$
$$=-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)\text{Subst}}{}$$
$$=-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{a^2A\tanh^{-1}\left(\right.}{8b^{3/2}}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-\frac{15a^{3/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-16a^2B+abx(15A+8Bx)+6b^2x^3(5A+4Bx)\right)}{120b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*x
) - (15*a^(3/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(120*b^2)
```

IntegrateAlgebraic [A] time = 0.23, size = 92, normalized size = 0.88

$$\frac{a^2A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(-16a^2B+15aAbx+8abBx^2+30Ab^2x^3+24b^2Bx^4\right)}{120b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 15*a*A*b*x + 8*a*b*B*x^2 + 30*A*b^2*x^3 + 24*
b^2*B*x^4))/(120*b^2) + (a^2*A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3
/2))
```

fricas [A] time = 1.00, size = 175, normalized size = 1.68

$$\left[\frac{15\,Aa^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}}{240\,b^2},\ \frac{15\,Aa^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}}{120\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(
b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2]
```

giac [A] time = 0.47, size = 81, normalized size = 0.78

$$\frac{Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

maple [A] time = 0.01, size = 94, normalized size = 0.90

$$-\frac{A\,a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,Aax}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,x^2}{5b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ax}{4b}-\frac{2\left(b\,x^2+a\right)^{\frac{3}{2}}Ba}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-2/15*B*a/b^2*(b*x^2+a)^(3/2)+1/4*A*x*(b*x^2+a)^
(3/2)/b-1/8*a*A*x*(b*x^2+a)^(1/2)/b-1/8*A*a^2/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.36, size = 86, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^2}{5\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b}-\frac{\sqrt{bx^2+a}\,Aax}{8\,b}-\frac{Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Ba}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^
2 + a)*A*a*x/b - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 + a
)^(3/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{b\,x^2+a}\,\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 10.14, size = 165, normalized size = 1.59

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) + B*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x*
*2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x
**4/4, True))
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*B*x*Sqrt[a + b*x^2])/(8*b) + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int x(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{(aB)\int\sqrt{a+bx^2}\,dx}{4b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 1.08

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(8aA+3aBx+8Abx^2+6bBx^3\right)-\frac{3a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (3*a^
(3/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(24*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 77, normalized size = 0.96

$$\frac{a^2B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{\sqrt{a+bx^2}\left(8aA+3aBx+8Abx^2+6bBx^3\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b) + (a^2*B
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

fricas [A] time = 0.66, size = 157, normalized size = 1.96

$$\left[\frac{3\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab\right)\sqrt{bx^2+a}}{48\,b^2},\ \frac{3\,Ba^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab\right)\sqrt{bx^2+a}}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1/2
4*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 + 8*A
*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.44, size = 68, normalized size = 0.85

$$\frac{Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{bx^2+a}\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
+ a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

maple [A] time = 0.00, size = 75, normalized size = 0.94

$$-\frac{B\,a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,Bax}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bx}{4b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*a*B*x*(b*x^2+a)^(1/2)/b-1/8*B*a^2/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)/b
```

maxima [A] time = 1.32, size = 67, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^2+a}\,Bax}{8\,b}-\frac{Ba^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 1/8*B*a^2*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{b\,x^2+a}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(1/2)*(A + B*x), x)
```

sympy [A] time = 14.41, size = 124, normalized size = 1.55

$$A\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
+ B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x*
*2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a)
*sqrt(1 + b*x**2/a))
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (A+Bx)\sqrt{a+bx^2}\,dx &= \frac{B\left(a+bx^2\right)^{3/2}}{3b}+A\int \sqrt{a+bx^2}\,dx\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\,(2aB+bx(3A+2Bx))+3aA\sqrt{b}\,\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(2*a*B + b*x*(3*A + 2*B*x)) + 3*a*A*Sqrt[b]*Log[b*x + Sqrt
[b]*Sqrt[a + b*x^2]])/(6*b)
```

IntegrateAlgebraic [A] time = 0.23, size = 68, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\left(2aB+3Abx+2bBx^2\right)}{6b}-\frac{aA\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

fricas [A] time = 0.74, size = 128, normalized size = 1.91

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\left(2\,Bbx^2+3\,Abx+2\,Ba\right)\sqrt{bx^2+a}}{12\,b},-\frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\,Bbx^2+3\,Abx+2\,Ba\right)\sqrt{bx^2+a}}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a
))/b]
```

giac [A] time = 0.41, size = 55, normalized size = 0.82

$$-\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{6}\,\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

maple [A] time = 0.01, size = 53, normalized size = 0.79

$$\frac{Aa\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,Ax}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 45, normalized size = 0.67

$$\frac{1}{2}\,\sqrt{bx^2+a}\,Ax+\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x
^2 + a)^(3/2)*B/b
```

mupad [B] time = 1.16, size = 52, normalized size = 0.78

$$\frac{B\left(b\,x^2+a\right)^{3/2}}{3\,b}+\frac{A\,x\,\sqrt{b\,x^2+a}}{2}+\frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)*
x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

sympy [A] time = 6.51, size = 70, normalized size = 1.04

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+B\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True
))
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx)-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx)-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \frac{1}{2}(2A+Bx)\sqrt{a+bx^2} + \frac{\int \frac{2aAb+abBx}{x\sqrt{a+bx^2}}\,dx}{2b}$$

$$= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2} + (aA)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + \frac{1}{2}(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2} + \frac{1}{2}(aA)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + \frac{1}{2}(aB)\,\mathrm{Subst}\left(\int \frac{}{1-\ldots}\right.$$

$$= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2} + \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{(aA)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right.}{b}$$

$$= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2} + \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} - \sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.25, size = 100, normalized size = 1.27

$$\frac{1}{2}\left(\frac{a^{3/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{a+bx^2}} + \sqrt{a+bx^2}\,(2A+Bx) - 2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2] + (a^(3/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]) - 2*Sqrt[a]*A*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/2
```

IntegrateAlgebraic [A] time = 0.24, size = 95, normalized size = 1.20

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx) + 2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{aB\log\left(\sqrt{a+bx^2} - \sqrt{b}\,x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + 2*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a] -
 Sqrt[a + b*x^2]/Sqrt[a]] - (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sq
rt[b])
```

fricas [A] time = 1.02, size = 341, normalized size = 4.32

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b},\frac{4\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sqr
t(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2*A
*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x
+ 2*A*b)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.45, size = 78, normalized size = 0.99

$$\frac{2\,Aa\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{2}\,\sqrt{bx^2+a}\,(Bx+2\,A)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*B*a*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/2*sqrt(b*x^2 + a)*(B*x + 2
*A)
```

maple [A] time = 0.01, size = 78, normalized size = 0.99

$$-A\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{Ba\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,Bx}{2}+\sqrt{b\,x^2+a}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*B*x*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A*a^(
1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+A*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.36, size = 59, normalized size = 0.75

$$\frac{1}{2}\sqrt{bx^2+a}\,Bx + \frac{Ba\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - A\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \sqrt{bx^2+a}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*sqrt(a
)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A
```

mupad [B] time = 1.24, size = 68, normalized size = 0.86

$$A\sqrt{b\,x^2+a} - A\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + \frac{B\,x\,\sqrt{b\,x^2+a}}{2} + \frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) - A*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (B*x*(a
+ b*x^2)^(1/2))/2 + (B*a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

sympy [A] time = 8.80, size = 107, normalized size = 1.35

$$-A\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right) + \frac{Aa}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{Ba\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B
*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a+bx^2}\,(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^2}\,(A-Bx)}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx &= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}-\frac{1}{2}\int \frac{-2aB-2Abx}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+(Ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx+(aB)\int \frac{1}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+(Ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)+\frac{1}{2}(aB)\,\text{Subst}\left(\int \ldots\right.\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{(aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\ldots}\right.}{b}\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.18, size = 99, normalized size = 1.32

$$\frac{\sqrt{a+bx^2}\,(Bx-A)}{x}+\frac{\sqrt{a}\,A\sqrt{b}\,\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + (Sqrt[a]*A*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a + b*x^2] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.25, size = 92, normalized size = 1.23

$$\frac{\sqrt{a+bx^2}\,(Bx-A)}{x}-A\sqrt{b}\,\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)+2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + 2*Sqrt[a]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a] -
Sqrt[a + b*x^2]/Sqrt[a]] - A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.93, size = 333, normalized size = 4.44

$$\left[\frac{A\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+B\sqrt{a}\,x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}\,(Bx-A)}{2\,x},\ -\frac{2\,A\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-B\sqrt{a}\,x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}\,(Bx-A)}{2\,x},\ \frac{2\,B\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+A\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\,(Bx-A)}{2\,x},\ -\frac{A\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,(Bx-A)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt(
a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a
)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B
*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(b*x
^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a))
+ A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^
2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*
sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x - A))/x]
```

giac [A] time = 0.47, size = 102, normalized size = 1.36

$$\frac{2\,Ba\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-A\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\sqrt{bx^2+a}\,B+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 97, normalized size = 1.29

$$A\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)-B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{\sqrt{b\,x^2+a}\,Abx}{a}+\sqrt{b\,x^2+a}\,B-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(3/2)+A/a*b*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))-B*a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/
2)
```

maxima [A] time = 1.39, size = 59, normalized size = 0.79

$$A\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-B\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,B-\frac{\sqrt{bx^2+a}\,A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
+ sqrt(b*x^2 + a)*B - sqrt(b*x^2 + a)*A/x
```

mupad [B] time = 1.69, size = 89, normalized size = 1.19

$$B\,\sqrt{b\,x^2+a}-\frac{A\,\sqrt{b\,x^2+a}}{x}-B\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^2,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2)
^(1/2)/a^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)
*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 11.97, size = 124, normalized size = 1.65

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-B\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B
*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a+bx^2}\,(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a+bx^2}\,(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -((A + 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} - \frac{\int \frac{-2aAb-4abBx}{x\sqrt{a+bx^2}}\,dx}{4a}$$

$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + (bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}(Ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + (bB)\,\text{Subst}\left(\int \frac{1}{1-b\ldots}\right.$$

$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\ldots}\right.$$

$$= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) - \frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 1.35

$$-\frac{\sqrt{a+bx^2}\left(a\sqrt{\frac{bx^2}{a}+1}\,(A+2Bx) + Abx^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) - 2\sqrt{a}\,\sqrt{b}\,Bx^2\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{2ax^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*(a*(A + 2*B*x)*Sqrt[1 + (b*x^2)/a] - 2*Sqrt[a]*Sqrt[b
]*B*x^2*ArcSinh[(Sqrt[b]*x)/Sqrt[a]] + A*b*x^2*ArcTanh[Sqrt[1 + (b*x^2)/a]]
))/(a*x^2*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.34, size = 96, normalized size = 1.20

$$\frac{\sqrt{a+bx^2}\,(-A-2Bx)}{2x^2} + \frac{Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{b}\,B\log\left(\sqrt{a+bx^2} - \sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] ((-A - 2*B*x)*Sqrt[a + b*x^2])/(2*x^2) + (A*b*ArcTanh[(Sqrt[b]*x)/Sqrt[a] -
 Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] - Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]]
```

fricas [A] time = 0.71, size = 377, normalized size = 4.71

$$\left[\frac{2\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},-\frac{4\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},\frac{A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2},-\frac{2\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A
*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*B
*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A*sqrt
(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^
2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))
/(a*x^2)]
```

giac [B] time = 0.48, size = 163, normalized size = 2.04

$$\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
+ a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 121, normalized size = 1.51

$$-\frac{Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}+B\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,Bbx}{a}+\frac{\sqrt{b\,x^2+a}\,Ab}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{ax}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(3/2)-1/2*A/a^(1/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)+1/2*A/a*b*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(3/2)+B/a*b*x*(b*x^2+a)^(
1/2)+B*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 83, normalized size = 1.04

$$B\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\sqrt{a}}+\frac{\sqrt{bx^2+a}\,Ab}{2a}-\frac{\sqrt{bx^2+a}\,B}{x}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sq
rt(a) + 1/2*sqrt(b*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)^(
3/2)*A/(a*x^2)
```

mupad [B] time = 1.79, size = 94, normalized size = 1.18

$$-\frac{A\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\sqrt{b\,x^2+a}}{x}-\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{B\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^3,x)
```

```
[Out] - (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2))/x - (A*b*atanh((a +
 b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2
))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 5.50, size = 107, normalized size = 1.34

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+$$

Rubi [A] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x^2(-3aB+8Abx)\left(a+bx^2\right)^{3/2}\,dx}{8b}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x(-16aAb-21abBx)\left(a+bx^2\right)^{3/2}\,dx}{56b^2}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{\left(a^2B\right)\int \left(a\right.}{1}$$

$$= \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+\right.}{560b^2}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a}{}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a}{}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a}{}$$

Mathematica [A] time = 0.26, size = 126, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{105a^{7/2}B\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-a^3(256A+105Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)+80b^3x^6(8A+7Bx)\right)\right)}{4480b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 3
5*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) + (105*a^(7
/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(4480*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 125, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(-256a^3A-105a^3Bx+128a^2Abx^2+70a^2bBx^3+1024aAb^2x^4+840ab^2Bx^5+640Ab^3x^6+560b^3Bx^7\right)}{4480b^2}-\frac{3a^4B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^3*A - 105*a^3*B*x + 128*a^2*A*b*x^2 + 70*a^2*b*B*x
^3 + 1024*a*A*b^2*x^4 + 840*a*b^2*B*x^5 + 640*A*b^3*x^6 + 560*b^3*B*x^7))/(
4480*b^2) - (3*a^4*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(5/2))
```

fricas [A] time = 0.73, size = 254, normalized size = 1.69

$$\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\left(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+128\,Aa^2b^2x^2-105\,Ba^3bx-256\,Aa^3b\right)\sqrt{bx^2+a}}{8960\,b^3},\ -\frac{105\,Ba^4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+128\,Aa^2b^2x^2-105\,Ba^3bx-256\,Aa^3b\right)\sqrt{bx^2+a}}{4480\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3]
```

giac [A] time = 0.44, size = 115, normalized size = 0.77

$$-\frac{3\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}-\frac{1}{4480}\,\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\,(128\,Aa+5\,(21\,Ba+2\,(7\,Bbx+8\,Ab)x)x)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(b
*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b + 4
*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 134, normalized size = 0.89

$$\frac{3B\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\;B\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,x^{3}}{8b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,x^{2}}{7b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,a^{2}x}{64b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bax}{16b^{2}}-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{2}}Aa}{35b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x*(b*x^2+a)^(5/2)+1/64*a^2*B*x*(b*
x^2+a)^(3/2)/b^2+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2+3/128*B*a^4/b^(5/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))+1/7*A*x^2*(b*x^2+a)^(5/2)/b-2/35*A*a/b^2*(b*x^2+a
)^(5/2)
```

maxima [A] time = 1.39, size = 126, normalized size = 0.84

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bx^{3}}{8\,b}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ax^{2}}{7\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bax}{16\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}x}{64\,b^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Ba^{3}x}{128\,b^{2}}+\frac{3\,Ba^{4}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}Aa}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b - 1/16*(b*x^2
 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x
^2 + a)*B*a^3*x/b^2 + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/35*(b*
x^2 + a)^(5/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3}\left(b\,x^{2}+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 20.79, size = 318, normalized size = 2.12

$$Aa\left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise} \end{cases}\right)+Ab\left(\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise} \end{cases}\right)-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Ba^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + A*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**
2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64
*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**
4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*A*x*Sqrt[a + b*x^2])/(16*b) - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{\int x(-2aB+7Abx)\left(a+bx^2\right)^{3/2}\,dx}{7b}$$
$$= \frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{(aA)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}$$
$$= -\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{\left(a^2A\right)}{}$$
$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a\right.}{210b^2}$$
$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a\right.}{210b^2}$$
$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a\right.}{210b^2}$$

Mathematica [A] time = 0.24, size = 113, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(-\frac{105a^{5/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-96a^3B+3a^2bx(35A+16Bx)+2ab^2x^3(245A+192Bx)+40b^3x^5(7A+6Bx)\right)}{1680b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x) - (105*a^(5/2)*A*Sqrt[b]*ArcSinh[(S
qrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(1680*b^2)
```

IntegrateAlgebraic [A] time = 0.42, size = 116, normalized size = 0.91

$$\frac{a^3 A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{3/2}}+\frac{\sqrt{a+bx^2}\left(-96a^3B+105a^2Abx+48a^2bBx^2+490aAb^2x^3+384ab^2Bx^4+280Ab^3x^5+240b^3Bx^6\right)}{1680b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 105*a^2*A*b*x + 48*a^2*b*B*x^2 + 490*a*A*b^2*
x^3 + 384*a*b^2*B*x^4 + 280*A*b^3*x^5 + 240*b^3*B*x^6))/(1680*b^2) + (a^3*A
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

fricas [A] time = 0.91, size = 223, normalized size = 1.76

$$\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+48\,Ba^2bx^2+105\,Aa^2bx-96\,Ba^3\right)\sqrt{bx^2+a}}{3360\,b^2},\ \frac{105\,Aa^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+48\,Ba^2bx^2+105\,Aa^2bx-96\,Ba^3\right)\sqrt{bx^2+a}}{1680\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48
*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*
A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 280*A*
b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*
x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.51, size = 103, normalized size = 0.81

$$\frac{Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}-\frac{1}{1680}\,\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2}-\left(\frac{105\,Aa^2}{b}+2\left(\frac{24\,Ba^2}{b}+(245\,Aa+4\,(48\,Ba+5\,(6\,Bbx+7\,Ab)x)x)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*x
^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B*a
 + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 113, normalized size = 0.89

$$-\frac{A\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\,A\,a^{2}x}{16b}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Aax}{24b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,x^{2}}{7b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ax}{6b}-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ba}{35b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*B*x^2*(b*x^2+a)^(5/2)/b-2/35*B*a/b^2*(b*x^2+a)^(5/2)+1/6*A*x*(b*x^2+a)^
(5/2)/b-1/24*a*A*x*(b*x^2+a)^(3/2)/b-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b-1/16*A*
a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 105, normalized size = 0.83

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bx^{2}}{7\,b}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ax}{6\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Aax}{24\,b}-\frac{\sqrt{bx^{2}+a}\,Aa^{2}x}{16\,b}-\frac{Aa^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}Ba}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 +
 a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b - 1/16*A*a^3*arcsinh(b*x
/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(b\,x^{2}+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 20.41, size = 287, normalized size = 2.26

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17Aa^{\frac{3}{2}}x^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{11A\sqrt{a}\,bx^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{3}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^{2}x^{7}}{6\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}+Ba\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+Bb\left(\begin{cases}\frac{8a^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4a^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{x^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) +
B*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True))
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2*B*x*Sqrt[a + b*x^2])/(16*b) - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{(aB)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}$$

$$= -\frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^2B\right)\int\sqrt{a+bx^2}\,dx}{8b}$$

$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^3B\right)\int\frac{1}{\sqrt{a+bx^2}}$$

$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{\left(a^3B\right)\text{Subst}\left(\int\right.$$

$$= -\frac{a^2Bx\sqrt{a+bx^2}}{16b} - \frac{aBx\left(a+bx^2\right)^{3/2}}{24b} + \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b} - \frac{a^3B\tanh^{-1}\left(\right.}{16b^{3/2}}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 1.04

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\left(3a^2(16A+5Bx)+2abx^2(48A+35Bx)+8b^2x^4(6A+5Bx)\right)-\frac{15a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{240b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(8*b^2*x^4*(6*A + 5*B*x) + 3*a^2*(16*A + 5*B*x) +
 2*a*b*x^2*(48*A + 35*B*x)) - (15*a^(5/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(240*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 101, normalized size = 0.98

$$\frac{a^3B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{3/2}}+\frac{\sqrt{a+bx^2}\left(48a^2A+15a^2Bx+96aAbx^2+70abBx^3+48Ab^2x^4+40b^2Bx^5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^2*A + 15*a^2*B*x + 96*a*A*b*x^2 + 70*a*b*B*x^3 + 48*
A*b^2*x^4 + 40*b^2*B*x^5))/(240*b) + (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(16*b^(3/2))
```

fricas [A] time = 0.99, size = 205, normalized size = 1.99

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+48\,Aa^2b\right)\sqrt{bx^2+a}}{480\,b^2},\ \frac{15\,Ba^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+48\,Aa^2b\right)\sqrt{bx^2+a}}{240\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a^2
*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 +
 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.47, size = 89, normalized size = 0.86

$$\frac{Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x^
2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A*b
)*x)*x)*x)*x)
```

maple [A] time = 0.01, size = 94, normalized size = 0.91

$$-\frac{B\,a^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\;B\,a^2x}{16b}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\,Bax}{24b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\,Bx}{6b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\,A}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*a*B*x*(b*x^2+a)^(3/2)/b-1/16*a^2*B*x*(b*x^2+
a)^(1/2)/b-1/16*B*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A/b*(b*x^2+
a)^(5/2)
```

maxima [A] time = 1.35, size = 86, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b}-\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*
x^2 + a)*B*a^2*x/b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(3/2)*(A + B*x), x)
```

sympy [A] time = 22.47, size = 223, normalized size = 2.17

$$Aa\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Bb^2x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + A*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x
**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
 B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b
*x**2/a)) + 11*B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt
(b)*x/sqrt(a))/(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{B\left(a+bx^2\right)^{5/2}}{5b} + A\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{4}(3aA)\int \sqrt{a+bx^2}\,dx$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^2A\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^2A\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.$$
$$= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\left(8a^2B+abx(25A+16Bx)+2b^2x^3(5A+4Bx)\right)+15a^2A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x)
) + 15*a^2*A*Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(40*b)
```

IntegrateAlgebraic [A] time = 0.37, size = 92, normalized size = 1.06

$$\frac{\sqrt{a+bx^2}\left(8a^2B+25aAbx+16abBx^2+10Ab^2x^3+8b^2Bx^4\right)}{40b} - \frac{3a^2A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 25*a*A*b*x + 16*a*b*B*x^2 + 10*A*b^2*x^3 + 8*b^
2*B*x^4))/(40*b) - (3*a^2*A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b]
)
```

fricas [A] time = 0.92, size = 176, normalized size = 2.02

$$\left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2\right)\sqrt{bx^2+a}}{80\,b},-\frac{15\,Aa^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2\right)\sqrt{bx^2+a}}{40\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*
x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x
^2 + a))/b]
```

giac [A] time = 0.44, size = 76, normalized size = 0.87

$$-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+\left(25\,Aa+2\,\left(8\,Ba+\left(4\,Bbx+5\,Ab\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^2
 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

maple [A] time = 0.00, size = 69, normalized size = 0.79

$$\frac{3A\,a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\;Aax}{8}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\,Ax}{4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\,B}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+1/4*A*x*(b*x^2+a)^(3/2)+3/8*a*A*x*(b*x^2+a)^(1/2)+3
/8*A*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.41, size = 61, normalized size = 0.70

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\,\sqrt{bx^2+a}\,Aax+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

mupad [B] time = 1.18, size = 54, normalized size = 0.62

$$\frac{B\left(b\,x^2+a\right)^{5/2}}{5\,b}+\frac{A\,x\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

sympy [A] time = 12.91, size = 219, normalized size = 2.52

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a*Piecewise(
(sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*b*Piecew
ise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) +
x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

3.12 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=106

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}\,(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}\,(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{(4aAb+3abBx)\sqrt{a+bx^2}}{x}\,dx}{4b}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x\sqrt{a+bx^2}}\,dx}{8b^2}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \left(a^2A\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{2}\left(a^2A\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a}\right.$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} +$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} -$$

Mathematica [A] time = 0.31, size = 118, normalized size = 1.11

$$\frac{1}{24}\left(-24a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{9a^{5/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx^2}} + \sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + (9*a^(5/2)*B
*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[a + b*x^2]
) - 24*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/24
```

IntegrateAlgebraic [A] time = 0.40, size = 114, normalized size = 1.08

$$2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{3a^2B\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{8\sqrt{b}} + \frac{1}{24}\sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/24 + 2*a^(3/2
)*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]] - (3*a^2*B*Log[-
(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

fricas [A] time = 0.89, size = 439, normalized size = 4.14

$$\left[\frac{9\,Ba^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+15\,Babx+32\,Aab\right)\sqrt{bx^{2}+a}}{48\,b},\ -\frac{9\,Ba^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-12\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+15\,Babx+32\,Aab\right)\sqrt{bx^{2}+a}}{24\,b},\ \frac{48\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+9\,Ba^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+2\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+15\,Babx+32\,Aab\right)\sqrt{bx^{2}+a}}{48\,b},\ -\frac{9\,Ba^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-24\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+15\,Babx+32\,Aab\right)\sqrt{bx^{2}+a}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24
*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b
^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*
B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x^2 +
15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqr
t(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8*A*
b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.54, size = 100, normalized size = 0.94

$$\frac{2\,Aa^{2}\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,Ba^{2}\log\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{24}\,\sqrt{bx^{2}+a}\left(32\,Aa+\left(15\,Ba+2\left(3\,Bbx+4\,Ab\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*B*a^
2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24*sqrt(b*x^2 + a)*(32
*A*a + (15*B*a + 2*(3*B*b*x + 4*A*b)*x)*x)
```

maple [A] time = 0.01, size = 107, normalized size = 1.01

$$-A\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\frac{3B\,a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^{2}+a}\,Bax}{8}+\sqrt{b\,x^{2}+a}\,Aa+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bx}{4}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/4*(b*x^2+a)^(3/2)*B*x+3/8*(b*x^2+a)^(1/2)*B*a*x+3/8*B*a^2/b^(1/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*(b*x^2+a)^
(1/2)*a^(1/2))/x)+A*(b*x^2+a)^(1/2)*a
```

maxima [A] time = 1.32, size = 88, normalized size = 0.83

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx+\frac{3}{8}\sqrt{bx^2+a}\,Bax+\frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}-Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A+\sqrt{bx^2+a}\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/8*B*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^
2 + a)^(3/2)*A + sqrt(b*x^2 + a)*A*a
```

mupad [B] time = 1.31, size = 83, normalized size = 0.78

$$\frac{A\left(b\,x^2+a\right)^{3/2}}{3}-A\,a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+A\,a\,\sqrt{b\,x^2+a}+\frac{B\,x\left(b\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 - A*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*a*
(a + b*x^2)^(1/2) + (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

sympy [A] time = 35.49, size = 218, normalized size = 2.06

$$-Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+Ab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Bb^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*a**(3/2)*x*sqrt(1 + b*x*
*2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(
1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*x**
5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=108

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = -\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-2aB-6Abx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{\int \frac{-4a^2bB-6aAb^2x}{x\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\int \frac{1}{\sqrt{a+bx^2}}\,dx + ($$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,d\right.$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Mathematica [C] time = 0.18, size = 105, normalized size = 0.97

$$-a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{a^2A\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}} + \frac{1}{3}B\sqrt{a+bx^2}\left(4a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]] - (a^2*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*
x^2)/a)])/(x*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.40, size = 115, normalized size = 1.06

$$2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{\sqrt{a+bx^2}\left(-6aA+8aBx+3Abx^2+2bBx^3\right)}{6x} - \frac{3}{2}aA\sqrt{b}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-6*a*A + 8*a*B*x + 3*A*b*x^2 + 2*b*B*x^3))/(6*x) + 2*a^(3
/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]] - (3*a*A*Sqrt[
b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

fricas [A] time = 0.85, size = 411, normalized size = 3.81

$$\left[\frac{9\,Aa\sqrt{b}\,x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{a} + 2\,a}{x^2}\right) + 2\left(2\,Bbx^3 + 3\,Abx^2 + 8\,Bax - 6\,Aa\right)\sqrt{bx^2+a}}{12\,x}, -\frac{9\,Aa\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) - 3\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{a} + 2\,a}{x^2}\right) - \left(2\,Bbx^3 + 3\,Abx^2 + 8\,Bax - 6\,Aa\right)\sqrt{bx^2+a}}{6\,x}, \frac{12\,B\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + 9\,Aa\sqrt{b}\,x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 2\left(2\,Bbx^3 + 3\,Abx^2 + 8\,Bax - 6\,Aa\right)\sqrt{bx^2+a}}{12\,x}, -\frac{9\,Aa\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) - 6\,B\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - \left(2\,Bbx^3 + 3\,Abx^2 + 8\,Bax - 6\,Aa\right)\sqrt{bx^2+a}}{6\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*
B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*
x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(-b)
*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*
sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)
) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
 a))/x]
```

giac [A] time = 0.60, size = 124, normalized size = 1.15

$$\frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}\,x - \sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3}{2}\,Aa\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^2 - a} + \frac{1}{6}\,\sqrt{bx^2+a}\,(8\,Ba + (2\,Bbx + 3\,Ab)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*A*b
)*x)
```

maple [A] time = 0.01, size = 126, normalized size = 1.17

$$\frac{3Aa\sqrt{b}\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{2} - B\,a^{\frac{3}{2}}\ln\left(\frac{2a + 2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right) + \frac{3\sqrt{b\,x^2+a}\,Abx}{2} + \frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Abx}{a} + \sqrt{b\,x^2+a}\,Ba + \frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{3} - \frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -(b*x^2+a)^(5/2)*A/a/x+(b*x^2+a)^(3/2)*A/a*b*x+3/2*(b*x^2+a)^(1/2)*A*b*x+3/
2*A*a*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*B*(b*x^2+a)^(3/2)-B*a^(3/2)
*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a
```

maxima [A] time = 1.39, size = 88, normalized size = 0.81

$$\frac{3}{2}\sqrt{bx^2+a}\,Abx + \frac{3}{2}Aa\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}B + \sqrt{bx^2+a}\,Ba - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*A*b*x + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(3
/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 +
a)*B*a - (b*x^2 + a)^(3/2)*A/x
```

mupad [B] time = 1.88, size = 86, normalized size = 0.80

$$\frac{B\left(b\,x^2+a\right)^{3/2}}{3} - B\,a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + B\,a\,\sqrt{b\,x^2+a} - \frac{A\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a*
(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x
^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

sympy [A] time = 13.29, size = 184, normalized size = 1.70

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} + \frac{A\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2} - \frac{A\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}} + \frac{3Aa\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2} - Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right) + \frac{Ba^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}} + Bb\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
- B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2
, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}\,(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {813, 844, 217, 206, 266, 63, 208}

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}\,(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{(-4aB-4Abx)\sqrt{a+bx^2}}{x^2}\,dx$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{8aAb+8abBx}{x\sqrt{a+bx^2}}\,dx$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{2}(3aAb)\int \frac{1}{x\sqrt{a+bx^2}}\,dx +$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{4}(3aAb)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) -$$

$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) -$$

Mathematica [C] time = 0.06, size = 90, normalized size = 0.81

$$\frac{Ab\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2} - \frac{aB\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] -((a*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7
/2, 1 + (b*x^2)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.46, size = 115, normalized size = 1.04

$$\frac{\sqrt{a+bx^2}\left(-aA-2aBx+2Abx^2+bBx^3\right)}{2x^2} + 3\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{3}{2}a\sqrt{b}\,B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(a*A) - 2*a*B*x + 2*A*b*x^2 + b*B*x^3))/(2*x^2) + 3*Sqrt
[a]*A*b*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]] - (3*a*Sqrt[
b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

fricas [A] time = 0.98, size = 425, normalized size = 3.83

$$\left[\frac{3\,Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+3\,A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\left(Bbx^3+2\,Abx^2-2\,Bax-Aa\right)\sqrt{bx^2+a}}{4\,x^2},-\frac{6\,Ba\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\left(Bbx^3+2\,Abx^2-2\,Bax-Aa\right)\sqrt{bx^2+a}}{4\,x^2},\frac{6\,A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+3\,Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(Bbx^3+2\,Abx^2-2\,Bax-Aa\right)\sqrt{bx^2+a}}{4\,x^2},-\frac{3\,Ba\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(Bbx^3+2\,Abx^2-2\,Bax-Aa\right)\sqrt{bx^2+a}}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3
*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B
*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sqrt(
-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/2*(3
*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a)*b*x^2*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqr
t(b*x^2 + a))/x^2]
```

giac [B] time = 0.55, size = 191, normalized size = 1.72

$$\frac{3\,Aab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Ba\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Aab+2\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aa^2b-2\,Ba^3\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt(b
*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3
*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 150, normalized size = 1.35

$$-\frac{3A\sqrt{a}\;b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{2}+\frac{3Ba\sqrt{b}\;\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\;Bbx}{2}+\frac{3\sqrt{b\,x^2+a}\;Ab}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bbx}{a}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ab}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B}{ax}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(5/2)+1/2*A/a*b*(b*x^2+a)^(3/2)-3/2*A*a^(1/2)*b*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/2)-B/a/x*(b*x^2+a)^(
5/2)+B/a*b*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1/2)+3/2*B*a*b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.28, size = 112, normalized size = 1.01

$$\frac{3}{2}\sqrt{bx^2+a}\,Bbx+\frac{3}{2}Ba\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{3}{2}A\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^2+a}\,Ab+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab}{2a}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{x}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*A*
sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(b*
x^2 + a)^(3/2)*A*b/a - (b*x^2 + a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(a*x
^2)
```

mupad [B] time = 2.12, size = 91, normalized size = 0.82

$$A\,b\,\sqrt{b\,x^2+a}-\frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,A\,\sqrt{a}\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}-\frac{B\,\left(b\,x^2+a\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^3,x)
```

```
[Out] A*b*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*A*a^(1/2)*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(3/2)*hypergeom([-3/2, -
1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

sympy [A] time = 15.57, size = 182, normalized size = 1.64

$$-\frac{3A\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x*
*2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/
sqrt(a))/2
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=173

$$\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+$$

Rubi [A] time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} + \frac{\int x^2(-3aB+10Abx)\left(a+bx^2\right)^{5/2}\,dx}{10b}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} + \frac{\int x(-20aAb-27abBx)\left(a+bx^2\right)^{5/2}\,dx}{90b^2}$$

$$= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{\left(3a^2B\right)}{}$$

$$= \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(\right.}{5040b^2}$$

$$= \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} + \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b} -$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} +$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} +$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2} + \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2} + \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b} +$$

Mathematica [A] time = 0.40, size = 145, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\frac{945a^{9/2}B\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}\left(-5a^4(512A+189Bx)+10a^3bx^2(128A+63Bx)+24a^2b^2x^4(800A+651Bx)+16ab^3x^6(1520A+1323Bx)+896b^4x^8(10A+9Bx)\right)\right)}{80640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
 + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) + 1
6*a*b^3*x^6*(1520*A + 1323*B*x)) + (945*a^(9/2)*B*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[1 + (b*x^2)/a]))/(80640*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.46, size = 149, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(-2560a^4A-945a^4Bx+1280a^3Abx^2+630a^3bBx^3+19200a^2Ab^2x^4+15624a^2b^2Bx^5+24320aAb^3x^6+21168ab^3Bx^7+8960Ab^4x^8+8064b^4Bx^9\right)}{80640b^2}-\frac{3a^5B\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{256b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2560*a^4*A - 945*a^4*B*x + 1280*a^3*A*b*x^2 + 630*a^3*b*
B*x^3 + 19200*a^2*A*b^2*x^4 + 15624*a^2*b^2*B*x^5 + 24320*a*A*b^3*x^6 + 211
68*a*b^3*B*x^7 + 8960*A*b^4*x^8 + 8064*b^4*B*x^9))/(80640*b^2) - (3*a^5*B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(256*b^(5/2))
```

fricas [A] time = 1.02, size = 302, normalized size = 1.75

$$\left[\frac{945\,Ba^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2b^3x^5+19200\,Aa^2b^3x^4+630\,Ba^3b^2x^3+1280\,Aa^3b^2x^2-945\,Ba^4bx-2560\,Aa^4b\right)\sqrt{bx^2+a}}{161280\,b^3},-\frac{945\,Ba^5\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2b^3x^5+19200\,Aa^2b^3x^4+630\,Ba^3b^2x^3+1280\,Aa^3b^2x^2-945\,Ba^4bx-2560\,Aa^4b\right)\sqrt{bx^2+a}}{80640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*
x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*
A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/8064
0*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*x^9
+ 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^2*b^3*
x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 - 945*B*
a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.53, size = 140, normalized size = 0.81

$$-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\left(1953\,Ba^2+2\left(1520\,Aab+7\left(189\,Bab+8\left(9\,Bb^2x+10\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(2560
*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a^2
 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 153, normalized size = 0.88

$$\frac{3Ba^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,B\,a^4x}{256b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B\,x^3}{10b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A\,x^2}{9b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B\,a^2x}{160b^2}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}Bax}{80b^2}-\frac{2\left(b\,x^2+a\right)^{\frac{7}{2}}Aa}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*B*x^3*(b*x^2+a)^(7/2)/b-3/80*B*a/b^2*x*(b*x^2+a)^(7/2)+1/160*a^2*B*x*(
b*x^2+a)^(5/2)/b^2+1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+3/256*a^4*B*x*(b*x^2+a
)^(1/2)/b^2+3/256*B*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/9*A*x^2*(b*
x^2+a)^(7/2)/b-2/63*A*a/b^2*(b*x^2+a)^(7/2)
```

maxima [A] time = 1.36, size = 145, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^3}{10\,b}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax^2}{9\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}Bax}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ba^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}\,Ba^4x}{256\,b^2}+\frac{3\,Ba^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Aa}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b + 1/9*(b*x^2 + a)^(7/2)*A*x^2/b - 3/80*(b*x^
2 + a)^(7/2)*B*a*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2
 + a)^(3/2)*B*a^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 3/256*B*a^5*a
rcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/63*(b*x^2 + a)^(7/2)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 37.44, size = 469, normalized size = 2.71

$$Aa^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29B\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Bb^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x*
*2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
2*A*a*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a
+ b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**
2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + A*b**2*Piecewise((-16*a**4*sqrt(
a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x
**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqr
t(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)) - 3*B*a**(9/2)*x/(256*b
**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a)) + 129*
B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*sqrt(1
 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*a**5*a
sinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}$$

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {833, 780, 195, 217, 206}

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx &= \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{\int x(-2aB+9Abx)\left(a+bx^2\right)^{5/2}\,dx}{9b}\\
&= \frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{(aA)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}\\
&= -\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{\left(5a^2A\right)}{}\\
&= -\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)(}{504b^2}\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 131, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(-\frac{315a^{7/2}A\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-256a^4B+a^3bx(315A+128Bx)+6a^2b^2x^3(413A+320Bx)+8ab^3x^5(357A+304Bx)+112b^4x^7(9A+8Bx)\right)}{8064b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A +
 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*x)
 - (315*a^(7/2)*A*Sqrt[b]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]
))/(8064*b^2)
```

IntegrateAlgebraic [A] time = 0.43, size = 140, normalized size = 0.93

$$\frac{5a^4A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(-256a^4B+315a^3Abx+128a^3bBx^2+2478a^2Ab^2x^3+1920a^2b^2Bx^4+2856aAb^3x^5+2432ab^3Bx^6+1008Ab^4x^7+896b^4Bx^8\right)}{8064b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 315*a^3*A*b*x + 128*a^3*b*B*x^2 + 2478*a^2*A
*b^2*x^3 + 1920*a^2*b^2*B*x^4 + 2856*a*A*b^3*x^5 + 2432*a*b^3*B*x^6 + 1008*
A*b^4*x^7 + 896*b^4*B*x^8))/(8064*b^2) + (5*a^4*A*Log[-(Sqrt[b]*x) + Sqrt[a
 + b*x^2]])/(128*b^(3/2))
```

fricas [A] time = 0.80, size = 271, normalized size = 1.81

$$\left[\frac{315\,Aa^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4+2478\,Aa^2b^2x^3+128\,Ba^3bx^2+315\,Aa^3bx-256\,Ba^4\right)\sqrt{bx^2+a}}{16128\,b^2},\ \frac{315\,Aa^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4+2478\,Aa^2b^2x^3+128\,Ba^3bx^2+315\,Aa^3bx-256\,Ba^4\right)\sqrt{bx^2+a}}{8064\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5
+ 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x
 - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6
 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b
*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.46, size = 128, normalized size = 0.85

$$\frac{5\,Aa^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,Aab+2\left(152\,Bab+7\left(8\,Bb^2x+9\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B*
a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (357*
A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*
x^2 + a)
```

maple [A] time = 0.01, size = 132, normalized size = 0.88

$$-\frac{5A\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^{2}+a}\;A\,a^{3}x}{128b}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,a^{2}x}{192b}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Aax}{48b}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,x^{2}}{9b}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ax}{8b}-\frac{2\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ba}{63b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/9*B*x^2*(b*x^2+a)^(7/2)/b-2/63*B*a/b^2*(b*x^2+a)^(7/2)+1/8*A*x*(b*x^2+a)^
(7/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b-5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-5/128*
a^3*A*x*(b*x^2+a)^(1/2)/b-5/128*A*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 124, normalized size = 0.83

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bx^{2}}{9\,b}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Ax}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Aa^{2}x}{192\,b}-\frac{5\sqrt{bx^{2}+a}\,Aa^{3}x}{128\,b}-\frac{5\,Aa^{4}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}-\frac{2\left(bx^{2}+a\right)^{\frac{7}{2}}Ba}{63\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 +
 a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 +
a)*A*a^3*x/b - 5/128*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2 + a
)^(7/2)*B*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(b\,x^{2}+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 61.22, size = 442, normalized size = 2.95

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{133Aa^{\frac{5}{2}}x^{3}}{384\sqrt{1+\frac{bx^{2}}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^{5}}{192\sqrt{1+\frac{bx^{2}}{a}}}+\frac{23A\sqrt{a}\,b^{2}x^{7}}{48\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5Aa^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^{3}x^{9}}{8\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+Ba^{2}\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+2Bab\left(\begin{cases}\frac{8a^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4a^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{x^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+Bb^{2}\left(\begin{cases}-\frac{16a^{4}\sqrt{a+bx^{2}}}{315b^{4}}+\frac{8a^{3}x^{2}\sqrt{a+bx^{2}}}{315b^{3}}-\frac{2a^{2}x^{4}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{6}\sqrt{a+bx^{2}}}{63b}+\frac{x^{8}\sqrt{a+bx^{2}}}{9} & \text{for}\: b\neq 0\\\frac{\sqrt{a}\,x^{8}}{8} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewis
e((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + 2*B*a*b*Piecewi
se((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*
b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0))
, (sqrt(a)*x**6/6, True)) + B*b**2*Piecewise((-16*a**4*sqrt(a + b*x**2)/(31
5*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*
x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9
, Ne(b, 0)), (sqrt(a)*x**8/8, True))
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {780, 195, 217, 206}

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rubi steps

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}dx=\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(aB)\int\left(a+bx^2\right)^{5/2}dx}{8b}$$
$$=-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^2B\right)\int\left(a+bx^2\right)^{3/2}dx}{48b}$$
$$=-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{\left(5a^3B\right)\int}{6}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+b\right.}{56b}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+b\right.}{56b}$$
$$=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+b\right.}{56b}$$

Mathematica [A] time = 0.55, size = 112, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{7/2}\left(-\frac{7aBx\left(\frac{15a^{7/2}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x}+\left(a+bx^2\right)\left(33a^2+26abx^2+8b^2x^4\right)\right)}{\left(a+bx^2\right)^4}+384A+336Bx\right)}{2688b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(384*A + 336*B*x - (7*a*B*x*((a + b*x^2)*(33*a^2 + 26*a*
b*x^2 + 8*b^2*x^4) + (15*a^(7/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(Sqrt[b]*x)))/(a + b*x^2)^4))/(2688*b)
```

IntegrateAlgebraic [A] time = 0.43, size = 125, normalized size = 0.99

$$\frac{5a^4B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{3/2}}+\frac{\sqrt{a+bx^2}\left(384a^3A+105a^3Bx+1152a^2Abx^2+826a^2bBx^3+1152aAb^2x^4+952ab^2Bx^5+384Ab^3x^6+336b^3Bx^7\right)}{2688b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(384*a^3*A + 105*a^3*B*x + 1152*a^2*A*b*x^2 + 826*a^2*b*B*
x^3 + 1152*a*A*b^2*x^4 + 952*a*b^2*B*x^5 + 384*A*b^3*x^6 + 336*b^3*B*x^7))/
(2688*b) + (5*a^4*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

fricas [A] time = 1.02, size = 253, normalized size = 2.01

$$\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+1152\,Aab^3x^4+826\,Ba^2b^2x^3+1152\,Aa^2b^2x^2+105\,Ba^3bx+384\,Aa^3b\right)\sqrt{bx^2+a}}{5376\,b^2},\ \frac{105\,Ba^4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+1152\,Aab^3x^4+826\,Ba^2b^2x^3+1152\,Aa^2b^2x^2+105\,Ba^3bx+384\,Aa^3b\right)\sqrt{bx^2+a}}{2688\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 + 8
26*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(b
*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(
b*x^2 + a))/b^2]
```

giac [A] time = 0.50, size = 114, normalized size = 0.90

$$\frac{5\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^3}{b}+\left(\frac{105\,Ba^3}{b}+2\left(576\,Aa^2+\left(413\,Ba^2+4\left(144\,Aab+\left(119\,Bab+6\left(7\,Bb^2x+8\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A*
a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*a*
b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 113, normalized size = 0.90

$$-\frac{5B\,a^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^2+a}\,B\,a^3x}{128b}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}B\,a^2x}{192b}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Bax}{48b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}Bx}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b-5/192*a^2*B*x*(b*x^2
+a)^(3/2)/b-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b-5/128*B*a^4/b^(3/2)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))+1/7*A/b*(b*x^2+a)^(7/2)
```

maxima [A] time = 1.44, size = 105, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\sqrt{bx^2+a}\,Ba^3x}{128\,b}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b - 5/192*(b*x^2
 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/128*B*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*A/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(5/2)*(A + B*x), x)
```

sympy [A] time = 26.54, size = 354, normalized size = 2.81

$$Aa^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Ba^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23B\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), Tr
ue)) + 2*A*a*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(
a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, T
rue)) + A*b**2*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*
sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a
+ b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + 5*B*a**(7/2)*x/(128*b*sqr
t(1 + b*x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**
```

```
(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 +
 b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**
9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {641, 195, 217, 206}

$$\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{B\left(a+bx^2\right)^{7/2}}{7b}+A\int\left(a+bx^2\right)^{5/2}\,dx$$

$$=\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{6}(5aA)\int\left(a+bx^2\right)^{3/2}\,dx$$

$$=\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{8}\left(5a^2A\right)\int\sqrt{a+bx^2}\,d$$

$$=\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{16}$$

$$=\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{16}$$

$$=\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{5a}{}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 1.01

$$\frac{105a^3A\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{a+bx^2}\left(48a^3B+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx)+8b^3x^5(7A+6Bx)\right)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 48
*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) + 105*a^3*A*Sqrt[b]*Log[b*x + Sqrt[b]*
Sqrt[a + b*x^2]])/(336*b)
```

IntegrateAlgebraic [A] time = 0.42, size = 116, normalized size = 1.08

$$\frac{\sqrt{a+bx^2}\left(48a^3B+231a^2Abx+144a^2bBx^2+182aAb^2x^3+144ab^2Bx^4+56Ab^3x^5+48b^3Bx^6\right)}{336b}-\frac{5a^3A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 231*a^2*A*b*x + 144*a^2*b*B*x^2 + 182*a*A*b^2*
x^3 + 144*a*b^2*B*x^4 + 56*A*b^3*x^5 + 48*b^3*B*x^6))/(336*b) - (5*a^3*A*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

fricas [A] time = 0.56, size = 224, normalized size = 2.09

$$\left[\frac{105\,Aa^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+231\,Aa^2bx+48\,Ba^3\right)\sqrt{bx^2+a}}{672\,b},\,-\frac{105\,Aa^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+231\,Aa^2bx+48\,Ba^3\right)\sqrt{bx^2+a}}{336\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B
*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A*a^
3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^3*x^
5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 4
8*B*a^3)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.61, size = 101, normalized size = 0.94

$$-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a^
3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7*A
*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 85, normalized size = 0.79

$$\frac{5A\,a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,A\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Aax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Ax}{6}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+1/6*A*x*(b*x^2+a)^(5/2)+5/24*a*A*x*(b*x^2+a)^(3/2)+
5/16*a^2*A*x*(b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))
```

maxima [A] time = 1.40, size = 77, normalized size = 0.72

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^2+a}\,Aa^2x+\frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^
(7/2)*B/b
```

mupad [B] time = 1.16, size = 54, normalized size = 0.50

$$\frac{B\left(b\,x^2+a\right)^{7/2}}{7\,b}+\frac{A\,x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

sympy [A] time = 26.05, size = 348, normalized size = 3.25

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Bab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + 2*B*a*b*Piecewise((-2*a**2
*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a
+ b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b**2*Piecewise((8*a**3*
sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x
**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*
x**6/6, True))
```

3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{16}a^2\sqrt{a+bx^2}\,(16A+5Bx)+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{3}$$

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {815, 844, 217, 206, 266, 63, 208}

$$\frac{1}{16}a^2\sqrt{a+bx^2}\,(16A+5Bx)-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} + \frac{\int \frac{(6aAb+5abBx)\left(a+bx^2\right)^{3/2}}{x}dx}{6b}$$

$$= \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2} + \frac{\int \frac{\left(24a^2Ab^2+15a^2b^2Bx\right)\sqrt{}}{x}}{24b^2}$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+\right.$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2} + \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+\right.$$

Mathematica [A] time = 0.36, size = 139, normalized size = 1.05

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{5a^{7/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}\sqrt{a+bx^2}} + \frac{1}{240}\sqrt{a+bx^2}\left(a^2(368A+165Bx)+2abx^2(88A+65Bx)+8b^2x^4(6A+5Bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2
*(368*A + 165*B*x)))/240 + (5*a^(7/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b
]*x)/Sqrt[a]])/(16*Sqrt[b]*Sqrt[a + b*x^2]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.46, size = 138, normalized size = 1.05

$$2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{5a^3B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16\sqrt{b}} + \frac{1}{240}\sqrt{a+bx^2}\left(368a^2A+165a^2Bx+176aAbx^2+130abBx^3+48Ab^2x^4+40b^2Bx^5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(368*a^2*A + 165*a^2*B*x + 176*a*A*b*x^2 + 130*a*b*B*x^3 +
 48*A*b^2*x^4 + 40*b^2*B*x^5))/240 + 2*a^(5/2)*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
] - Sqrt[a + b*x^2]/Sqrt[a]] - (5*a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(16*Sqrt[b])
```

fricas [A] time = 0.93, size = 539, normalized size = 4.08

$$\left[\frac{75\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+176\,Aab^2x^2+165\,Ba^2bx+368\,Aa^2b\right)\sqrt{bx^2+a}}{480\,b},\ -\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+176\,Aab^2x^2+165\,Ba^2bx+368\,Aa^2b\right)\sqrt{bx^2+a}}{240\,b},\ \frac{480\,A\sqrt{-a}\,a^2b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+75\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+176\,Aab^2x^2+165\,Ba^2bx+368\,Aa^2b\right)\sqrt{bx^2+a}}{480\,b},\ -\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-240\,A\sqrt{-a}\,a^2b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+176\,Aab^2x^2+165\,Ba^2bx+368\,Aa^2b\right)\sqrt{bx^2+a}}{240\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(40
*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b
*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A
*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*(480*A*
sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 +
130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2
 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24
0*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*
b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.58, size = 125, normalized size = 0.95

$$\frac{2\,Aa^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{240}\left(368\,Aa^2+\left(165\,Ba^2+2\left(88\,Aab+\left(65\,Bab+4\left(5\,Bb^2x+6\,Ab^2\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 5/16*B*a
^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/240*(368*A*a^2 + (165
*B*a^2 + 2*(88*A*a*b + (65*B*a*b + 4*(5*B*b^2*x + 6*A*b^2)*x)*x)*x)*x)*sqrt
(b*x^2 + a)
```

maple [A] time = 0.01, size = 138, normalized size = 1.05

$$-A\,a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{5B\,a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,B\,a^2x}{16}+\sqrt{b\,x^2+a}\,A\,a^2+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Bax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Aa}{3}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Bx}{6}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(b*x^2+a)^(
1/2)+5/16*B*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/5*A*(b*x^2+a)^(5/2)
+1/3*A*a*(b*x^2+a)^(3/2)-A*a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+A*
(b*x^2+a)^(1/2)*a^2
```

maxima [A] time = 1.38, size = 119, normalized size = 0.90

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Bx+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Bax+\frac{5}{16}\sqrt{bx^2+a}\,Ba^2x+\frac{5\,Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-Aa^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}A+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Aa+\sqrt{bx^2+a}\,Aa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*a^(5/2)*arcsin
h(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b*x^2 + a)^(3/2)*A
*a + sqrt(b*x^2 + a)*A*a^2
```

mupad [B] time = 1.25, size = 101, normalized size = 0.77

$$\frac{A\left(b\,x^2+a\right)^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{A\,a\left(b\,x^2+a\right)^{3/2}}{3}+\frac{B\,x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + A*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)^(3/2))/3 + (B*x*(a + b*x^2)^(
5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

sympy [A] time = 40.75, size = 323, normalized size = 2.45

$$-Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^3}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa^2\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17B\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Bb^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((sqrt(a)*
```

```
x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + A*b**2*Piecewise((-
2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*s
qrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x*sqrt(1
 + b*x**2/a)/2 + 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x
**3/(48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)
*sqrt(1 + b*x**2/a))
```

3.20 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=136

$$a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}\,(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-B$$

Rubi [A] time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$\frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{8}a\sqrt{a+bx^2}\,(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{1}{2}\int \frac{(-2aB-10Abx)\left(a+bx^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{\int \frac{\left(-8a^2bB-30aAb^2x\right)\sqrt{a+bx}}{x}}{8b}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx\right.}{5x}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx\right.}{5x}$$

Mathematica [C] time = 0.23, size = 117, normalized size = 0.86

$$-a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{a^3A\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{a+bx^2}} + \frac{1}{15}B\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (B*Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]] - (a^3*A*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-
5/2, -1/2, 1/2, -((b*x^2)/a)])/(x*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.45, size = 141, normalized size = 1.04

$$2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{\sqrt{a+bx^2}\left(-120a^2A+184a^2Bx+135aAbx^2+88abBx^3+30Ab^2x^4+24b^2Bx^5\right)}{120x} - \frac{15}{8}a^2A\sqrt{b}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-120*a^2*A + 184*a^2*B*x + 135*a*A*b*x^2 + 88*a*b*B*x^3 +
 30*A*b^2*x^4 + 24*b^2*B*x^5))/(120*x) + 2*a^(5/2)*B*ArcTanh[(Sqrt[b]*x)/Sq
rt[a] - Sqrt[a + b*x^2]/Sqrt[a]] - (15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/8
```

fricas [A] time = 0.95, size = 519, normalized size = 3.82

$$\left[\frac{225\,Aa^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2\right)\sqrt{bx^2+a}}{240\,x},\ -\frac{225\,Aa^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2\right)\sqrt{bx^2+a}}{120\,x},\ \frac{240\,B\sqrt{-a}\,a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+225\,Aa^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2\right)\sqrt{bx^2+a}}{240\,x},\ -\frac{225\,Aa^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,B\sqrt{-a}\,a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2\right)\sqrt{bx^2+a}}{120\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x -
 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^
2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-a)*a^2*
x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x
^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(2
25*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*sqrt(-a)*a^2
*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a
*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x]
```

giac [A] time = 0.50, size = 150, normalized size = 1.10

$$\frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Aa^2\sqrt{b}\,\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{120}\left(184\,Ba^2+\left(135\,Aab+2\left(44\,Bab+3\left(4\,Bb^2x+5\,Ab^2\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*a
^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqrt(
b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B*a*
b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

maple [A] time = 0.01, size = 158, normalized size = 1.16

$$\frac{15A\,a^2\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}-B\,a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\frac{15\sqrt{b\,x^2+a}\,Aabx}{8}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Abx}{4}+\sqrt{b\,x^2+a}\,B\,a^2+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Abx}{a}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ba}{3}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B}{5}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -A/a/x*(b*x^2+a)^(7/2)+A/a*b*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^(3/2)+15
/8*A*a*b*x*(b*x^2+a)^(1/2)+15/8*A*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
+1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2)-B*a^(5/2)*ln((2*a+2*(b*x^2+a
)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a^2
```

maxima [A] time = 1.40, size = 120, normalized size = 0.88

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx+\frac{15}{8}\sqrt{bx^2+a}\,Aabx+\frac{15}{8}Aa^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-Ba^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}B+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Ba+\sqrt{bx^2+a}\,Ba^2-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x + 15/8*A*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5
*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(b*x^2 + a)*B*a^2 -
(b*x^2 + a)^(5/2)*A/x
```

mupad [B] time = 2.16, size = 104, normalized size = 0.76

$$\frac{B\left(b\,x^2+a\right)^{5/2}}{5}+B\,a^2\,\sqrt{b\,x^2+a}+\frac{B\,a\left(b\,x^2+a\right)^{3/2}}{3}-\frac{A\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (B*a*(a + b*x^2)^(3/2))/3 - (A*(a + b*x^2)^(5/
2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(5/2))
```

sympy [A] time = 18.91, size = 318, normalized size = 2.34

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba^3}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba^2\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+2Bab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
```

```
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3
/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) +
 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b),
True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt
(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4,
True))
```

3.21 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^3} dx$

Optimal. Leaf size=141

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}(2A-Bx)}{4x^2}-\frac{5(a+bx^2)^{3/2}(3aB-2A$$

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {813, 815, 844, 217, 206, 266, 63, 208}

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}(2A-Bx)}{4x^2}-\frac{5(a+bx^2)^{3/2}(3aB-2Abx)}{12x}+\frac{5}{8}ab\sqrt{a+bx^2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = -\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} - \frac{5}{16}\int \frac{(-4aB-8Abx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{5}{32}\int \frac{(16aAb+24abB}{x}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}$$

Mathematica [C] time = 0.04, size = 92, normalized size = 0.65

$$\frac{Ab\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2} - \frac{a^2B\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] -((a^2*B*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/
(x*Sqrt[1 + (b*x^2)/a])) + (A*b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2,
 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.51, size = 142, normalized size = 1.01

$$5a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{\sqrt{a+bx^2}\left(-12a^2A-24a^2Bx+56aAbx^2+27abBx^3+8Ab^2x^4+6b^2Bx^5\right)}{24x^2}-\frac{15}{8}a^2\sqrt{b}\,B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-12*a^2*A - 24*a^2*B*x + 56*a*A*b*x^2 + 27*a*b*B*x^3 + 8*
A*b^2*x^4 + 6*b^2*B*x^5))/(24*x^2) + 5*a^(3/2)*A*b*ArcTanh[(Sqrt[b]*x)/Sqrt
[a] - Sqrt[a + b*x^2]/Sqrt[a]] - (15*a^2*Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/8
```

fricas [A] time = 0.58, size = 535, normalized size = 3.79

$$\left[\frac{45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+60\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,Aabx^2-24\,Ba^2x-12\,Aa^2\right)\sqrt{bx^2+a}}{48\,x^2},\ -\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-30\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,Aabx^2-24\,Ba^2x-12\,Aa^2\right)\sqrt{bx^2+a}}{24\,x^2},\ \frac{120\,A\sqrt{-a}abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,Aabx^2-24\,Ba^2x-12\,Aa^2\right)\sqrt{bx^2+a}}{48\,x^2},\ -\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,A\sqrt{-a}abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,Aabx^2-24\,Ba^2x-12\,Aa^2\right)\sqrt{bx^2+a}}{24\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x -
12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x
^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*
x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45
*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*
b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 0.57, size = 219, normalized size = 1.55

$$\frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{15}{8}\,Ba^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,Ab^2\right)x\right)x\right)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aa^2b+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^3b-2\,Ba^4\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*B
*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (27*
B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (
sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^2
```

maple [A] time = 0.01, size = 181, normalized size = 1.28

$$-\frac{5A\,a^{\frac{3}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{15B\,a^2\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}+\frac{15\sqrt{b\,x^2+a}\,Babx}{8}+\frac{5\sqrt{b\,x^2+a}\,Aab}{2}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Bbx}{4}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Ab}{6}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Bbx}{a}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Ab}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B}{ax}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2*A/a/x^2*(b*x^2+a)^(7/2)+1/2*A/a*b*(b*x^2+a)^(5/2)+5/6*A*b*(b*x^2+a)^(3
/2)-5/2*A*a^(3/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*A*a*b*(b*x^2+
a)^(1/2)-B/a/x*(b*x^2+a)^(7/2)+B/a*b*x*(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^
(3/2)+15/8*B*a*b*x*(b*x^2+a)^(1/2)+15/8*B*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.38, size = 143, normalized size = 1.01

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx+\frac{15}{8}\sqrt{bx^2+a}\,Babx+\frac{15}{8}Ba^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{5}{2}Aa^{\frac{3}{2}}b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^2+a\right)^{\frac{3}{2}}Ab+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab}{2a}+\frac{5}{2}\sqrt{bx^2+a}\,Aab-\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{x}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 15/8*B*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))
 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a + 5/2*sqrt(b*x^2
 + a)*A*a*b - (b*x^2 + a)^(5/2)*B/x - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

mupad [B] time = 2.59, size = 111, normalized size = 0.79

$$\frac{A\,b\,\left(b\,x^2+a\right)^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\,\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^3,x)
```

```
[Out] (A*b*(a + b*x^2)^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^
(1/2))/(2*x^2) + (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 -
(B*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(5/2))
```

sympy [A] time = 12.98, size = 279, normalized size = 1.98

$$-\frac{5Aa^{\frac{3}{2}}b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Ba^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Ba^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Ba^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Bb^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3
/2)*b*x*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*
sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*
x/sqrt(a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}\,(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 217, 206}

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}\,(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^3\sqrt{a+bx^2}}{4b} + \frac{\int \frac{x^2(-3aB+4Abx)}{\sqrt{a+bx^2}}\,dx}{4b} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} + \frac{\int \frac{x(-8aAb-9abBx)}{\sqrt{a+bx^2}}\,dx}{12b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} + \frac{(3a^2B)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} + \frac{(3a^2B)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{8b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.73

$$\frac{9a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\sqrt{b}\,\sqrt{a+bx^2}\,\left(-16aA-9aBx+8Abx^2+6bBx^3\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) + 9*a^
2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(24*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.25, size = 77, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,\left(-16aA-9aBx+8Abx^2+6bBx^3\right)}{24b^2}-\frac{3a^2B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b^2) - (3
*a^2*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.76, size = 158, normalized size = 1.52

$$\left[\frac{9\,Ba^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}-9\,Babx-16\,Aab\right)\sqrt{bx^{2}+a}}{48\,b^{3}},-\frac{9\,Ba^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}-9\,Babx-16\,Aab\right)\sqrt{bx^{2}+a}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3, -1
/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8
*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.50, size = 74, normalized size = 0.71

$$\frac{1}{24}\,\sqrt{bx^{2}+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^{2}}\right)x-\frac{16\,Aa}{b^{2}}\right)-\frac{3\,Ba^{2}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2) -
 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 96, normalized size = 0.92

$$\frac{\sqrt{b\,x^{2}+a}\;B\,x^{3}}{4b}+\frac{\sqrt{b\,x^{2}+a}\;A\,x^{2}}{3b}+\frac{3B\,a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^{2}+a}\;Bax}{8b^{2}}-\frac{2\sqrt{b\,x^{2}+a}\;Aa}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B*a/b^2*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(5/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*A*x^2*(b*x^2+a)^(1/2)/b-2/3*A*a/b^2*(b*x
^2+a)^(1/2)
```

maxima [A] time = 1.36, size = 88, normalized size = 0.85

$$\frac{\sqrt{bx^2+a}\,Bx^3}{4\,b}+\frac{\sqrt{bx^2+a}\,Ax^2}{3\,b}-\frac{3\,\sqrt{bx^2+a}\,Bax}{8\,b^2}+\frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{2\,\sqrt{bx^2+a}\,Aa}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b + 1/3*sqrt(b*x^2 + a)*A*x^2/b - 3/8*sqrt(b*x^2
+ a)*B*a*x/b^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^2
+ a)*A*a/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 7.91, size = 150, normalized size = 1.44

$$A\left(\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}\right)-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b),
Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x*
*2/a)) - B*sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x
/sqrt(a))/(8*b**(5/2)) + B*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=81

$$-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}\,(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 780, 217, 206}

$$-\frac{\sqrt{a+bx^2}\,(4aB-3Abx)}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^2\sqrt{a+bx^2}}{3b}+\frac{\int \frac{x(-2aB+3Abx)}{\sqrt{a+bx^2}}\,dx}{3b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\,(bx(3A+2Bx)-4aB)-3aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + b*x*(3*A + 2*B*x)) - 3*a*A*Sqrt[b]*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(6*b^2)
```

IntegrateAlgebraic [A] time = 0.29, size = 68, normalized size = 0.84

$$\frac{aA\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}}+\frac{\sqrt{a+bx^2}\left(-4aB+3Abx+2bBx^2\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b^2) + (a*A*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

fricas [A] time = 0.90, size = 127, normalized size = 1.57

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}}{12\,b^2},\ \frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(2\,Bbx^2+3\,Abx-4\,Ba\right)\sqrt{bx^2+a}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 +
a))/b^2]
```

giac [A] time = 0.52, size = 61, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 75, normalized size = 0.93

$$\frac{\sqrt{b\,x^2+a}\;B\,x^2}{3b}-\frac{Aa\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\;Ax}{2b}-\frac{2\sqrt{b\,x^2+a}\;Ba}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*B*x^2*(b*x^2+a)^(1/2)/b-2/3*B*a/b^2*(b*x^2+a)^(1/2)+1/2*A*x/b*(b*x^2+a)
^(1/2)-1/2*A*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.29, size = 67, normalized size = 0.83

$$\frac{\sqrt{bx^2+a}\,Bx^2}{3\,b}+\frac{\sqrt{bx^2+a}\,Ax}{2\,b}-\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}-\frac{2\,\sqrt{bx^2+a}\,Ba}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x/b - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

mupad [B] time = 1.47, size = 93, normalized size = 1.15

$$\begin{cases} \frac{3\,B\,x^4+4\,A\,x^3}{12\,\sqrt{a}} & \text{if } b=0\\ \frac{A\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{A\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (4*A*x^3 + 3*B*x^4)/(12*a^(1/2)), b ~= 0, - (A*a*log(2*b^
(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (A*x*(a + b*x^2)^(1/2))/(2*b)
 - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2))
```

sympy [A] time = 6.23, size = 94, normalized size = 1.16

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+B\left(\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + B*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)
/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {780, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{a+bx^2}\,(2A+Bx) - aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*(2*A + B*x)*Sqrt[a + b*x^2] - a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.25, size = 58, normalized size = 1.04

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b} + \frac{aB\log\left(\sqrt{a+bx^2} - \sqrt{b}\,x\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) + (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(2*b^(3/2))
```

fricas [A] time = 0.68, size = 109, normalized size = 1.95

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 2\,(Bbx + 2\,Ab)\sqrt{bx^2+a}}{4\,b^2}, \frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) + (Bbx + 2\,Ab)\sqrt{bx^2+a}}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*
x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.48, size = 50, normalized size = 0.89

$$\frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2A}{b}\right)+\frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 55, normalized size = 0.98

$$-\frac{Ba\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\,Bx}{2b}+\frac{\sqrt{b\,x^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*x/b*(b*x^2+a)^(1/2)-1/2*B*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+A/b
*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 47, normalized size = 0.84

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}-\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(b
*x^2 + a)*A/b
```

mupad [B] time = 1.24, size = 82, normalized size = 1.46

$$\begin{cases} \frac{2\,B\,x^3+3\,A\,x^2}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\sqrt{b\,x^2+a}}{b}-\frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x^2 + 2*B*x^3)/(6*a^(1/2)), b ~= 0, (A*(a + b*x^2)^(
1/2))/b - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (B*x*(
a + b*x^2)^(1/2))/(2*b))
```

sympy [A] time = 6.26, size = 70, normalized size = 1.25

$$A\left(\begin{cases}\frac{x^{2}}{2\sqrt{a}} & \text{for}\: b = 0\\\frac{\sqrt{a + bx^{2}}}{b} & \text{otherwise}\end{cases}\right) + \frac{B\sqrt{a}\,x\sqrt{1 + \frac{bx^{2}}{a}}}{2b} - \frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + B*s
qrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {641, 217, 206}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{a+bx^2}}{b} + A\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{B\sqrt{a+bx^2}}{b} + A\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{B\sqrt{a+bx^2}}{b} + \frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 1.07

$$\frac{A\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}} + \frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
[Out] (B*Sqrt[a + b*x^2])/b + (A*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.24, size = 46, normalized size = 1.07

$$\frac{B\sqrt{a+bx^2}}{b} - \frac{A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/Sqrt[a + b*x^2],x]
[Out] (B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

fricas [A] time = 1.16, size = 92, normalized size = 2.14

$$\left[\frac{A\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\,B}{2\,b}, -\frac{A\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,B}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*
x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2
 + a)*B)/b]
```

giac [A] time = 0.55, size = 39, normalized size = 0.91

$$-\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

maple [A] time = 0.01, size = 37, normalized size = 0.86

$$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] B*(b*x^2+a)^(1/2)/b+A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.36, size = 29, normalized size = 0.67

$$\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

mupad [B] time = 1.14, size = 36, normalized size = 0.84

$$\frac{B\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(1/2),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

sympy [B] time = 2.64, size = 102, normalized size = 2.37

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*Piecewise((x**2/(2*sqrt(a)),
 Eq(b, 0)), (sqrt(a + b*x**2)/b, True))
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {844, 217, 206, 266, 63, 208}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx &= A\int \frac{1}{x\sqrt{a+bx^2}}\,dx + B\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$
\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.20, size = 70, normalized size = 1.32

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] - (B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

fricas [A] time = 1.29, size = 273, normalized size = 5.15

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},-\frac{2\,Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},\frac{2\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,ab},-\frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt(
a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*B*
a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqr
t(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

giac [A] time = 0.50, size = 58, normalized size = 1.09

$$\frac{2\,A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.98

$$-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)
*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 33, normalized size = 0.62

$$\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 1.30, size = 42, normalized size = 0.79

$$\frac{B\;\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/a^(1/2)
```

sympy [A] time = 5.14, size = 99, normalized size = 1.87

$$-\frac{A\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((sqrt(-a/b)*asin(x*sqrt
(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a),
 (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a
 < 0)))
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {807, 266, 63, 208}

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= -\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.21, size = 61, normalized size = 1.30

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a +
b*x^2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.80, size = 101, normalized size = 2.15

$$\left[\frac{B\sqrt{a}\,x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}\,A}{2\,ax},\ \frac{B\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,A}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*s
qrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - s
qrt(b*x^2 + a)*A)/(a*x)]
```

giac [A] time = 0.51, size = 65, normalized size = 1.38

$$\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,A\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.01, size = 49, normalized size = 1.04

$$-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}-\frac{\sqrt{b\,x^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -A*(b*x^2+a)^(1/2)/a/x-B/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.31, size = 37, normalized size = 0.79

$$-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}-\frac{\sqrt{bx^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*A/(a*x)
```

mupad [B] time = 1.20, size = 39, normalized size = 0.83

$$-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

sympy [A] time = 2.80, size = 41, normalized size = 0.87

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{a}-\frac{B\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {835, 807, 266, 63, 208}

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{\int \frac{-2aB+Abx}{x^2\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 63, normalized size = 0.88

$$
\frac{\sqrt{a+bx^2}\left(\frac{Ab\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-\frac{a(A+2Bx)}{x^2}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((a*(A + 2*B*x))/x^2) + (A*b*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a]))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.31, size = 71, normalized size = 0.99

$$\frac{\sqrt{a+bx^2}\,(-A-2Bx)}{2ax^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((-A - 2*B*x)*Sqrt[a + b*x^2])/(2*a*x^2) - (A*b*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
 - Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.75, size = 123, normalized size = 1.71

$$\left[\frac{A\sqrt{a}\,bx^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,a^2x^2},-\frac{A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)]
```

giac [B] time = 0.43, size = 146, normalized size = 2.03

$$-\frac{Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(
b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2*a)
```

maple [A] time = 0.01, size = 68, normalized size = 0.94

$$\frac{Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,B}{ax}-\frac{\sqrt{b\,x^2+a}\,A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-B*(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.31, size = 56, normalized size = 0.78

$$\frac{Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,B}{ax}-\frac{\sqrt{bx^2+a}\,A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x) - 1
/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

mupad [B] time = 1.35, size = 58, normalized size = 0.81

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{a\,x}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2))/
(a*x) - (A*(a + b*x^2)^(1/2))/(2*a*x^2)
```

sympy [A] time = 7.06, size = 66, normalized size = 0.92

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{a+bx^2}\,(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 780, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}} + \frac{\int \frac{x(2aA+3aBx)}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}} + \frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2} - \frac{(3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}} + \frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2} - \frac{(3aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}} + \frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2} - \frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.89

$$\frac{a(4A+3Bx)+bx^2(2A+Bx)}{2b^2\sqrt{a+bx^2}} - \frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (b*x^2*(2*A + B*x) + a*(4*A + 3*B*x))/(2*b^2*Sqrt[a + b*x^2]) - (3*a*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.39, size = 74, normalized size = 0.91

$$\frac{4aA+3aBx+2Abx^2+bBx^3}{2b^2\sqrt{a+bx^2}} + \frac{3aB\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*A + 3*a*B*x + 2*A*b*x^2 + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*B*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

fricas [A] time = 0.89, size = 197, normalized size = 2.43

$$\left[\frac{3\left(Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)+2\left(Bb^2x^3+2\,Ab^2x^2+3\,Babx+4\,Aab\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)},\ \frac{3\left(Babx^2+Ba^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(Bb^2x^3+2\,Ab^2x^2+3\,Babx+4\,Aab\right)\sqrt{bx^2+a}}{2\left(b^4x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*
x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.51, size = 70, normalized size = 0.86

$$\frac{\left(\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{4\,Aa}{b^2}}{2\,\sqrt{bx^2+a}}+\frac{3\,Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*B
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 93, normalized size = 1.15

$$\frac{B\,x^3}{2\sqrt{b\,x^2+a}\;b}+\frac{A\,x^2}{\sqrt{b\,x^2+a}\;b}+\frac{3Bax}{2\sqrt{b\,x^2+a}\;b^2}-\frac{3Ba\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}+\frac{2Aa}{\sqrt{b\,x^2+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B*a/b^2*x/(b*x^2+a)^(1/2)-3/2*B*a/b^(5/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))+A*x^2/b/(b*x^2+a)^(1/2)+2*A*a/b^2/(b*x^2+a)^(1
/2)
```

maxima [A] time = 1.35, size = 85, normalized size = 1.05

$$\frac{Bx^{3}}{2\sqrt{bx^{2}+a}\,b}+\frac{Ax^{2}}{\sqrt{bx^{2}+a}\,b}+\frac{3\,Bax}{2\sqrt{bx^{2}+a}\,b^{2}}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{2\,Aa}{\sqrt{bx^{2}+a}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + A*x^2/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt
(b*x^2 + a)*b^2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*A*a/(sqrt(b*x
^2 + a)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\left(A+B\,x\right)}{{\left(b\,x^{2}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 10.34, size = 117, normalized size = 1.44

$$A\left(\begin{cases}\frac{2a}{b^{2}\sqrt{a+bx^{2}}}+\frac{x^{2}}{b\sqrt{a+bx^{2}}} & \text{for}\; b\neq 0\\ \frac{x^{4}}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3\sqrt{a}\,x}{2b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^{3}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a
)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 +
 b*x**2/a)))
```

3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{A \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 641, 217, 206}

$$\frac{A \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{\int \frac{aA+2aBx}{\sqrt{a+bx^2}}\,dx}{ab} \\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b} \\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b} \\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}} + \frac{2B\sqrt{a+bx^2}}{b^2} + \frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.02

$$\frac{A\sqrt{b}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+2aB+bx(Bx-A)}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B + b*x*(-A + B*x) + A*Sqrt[b]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(b^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.36, size = 61, normalized size = 0.92

$$\frac{2aB-Abx+bBx^2}{b^2\sqrt{a+bx^2}}-\frac{A\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B - A*b*x + b*B*x^2)/(b^2*Sqrt[a + b*x^2]) - (A*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/b^(3/2)
```

fricas [A] time = 1.05, size = 164, normalized size = 2.48

$$\left[\frac{\left(Abx^2 + Aa\right)\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\left(Bbx^2 - Abx + 2\,Ba\right)\sqrt{bx^2 + a}}{2\left(b^3x^2 + ab^2\right)}, -\frac{\left(Abx^2 + Aa\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) - \left(Bbx^2 - Abx + 2\,Ba\right)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A*b
*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b*x
+ 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.55, size = 58, normalized size = 0.88

$$\frac{\left(\frac{Bx}{b} - \frac{A}{b}\right)x + \frac{2\,Ba}{b^2}}{\sqrt{bx^2 + a}} - \frac{A\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 72, normalized size = 1.09

$$\frac{B\,x^2}{\sqrt{b\,x^2 + a}\;b} - \frac{Ax}{\sqrt{b\,x^2 + a}\;b} + \frac{A\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2 + a}\right)}{b^{\frac{3}{2}}} + \frac{2Ba}{\sqrt{b\,x^2 + a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x^2/b/(b*x^2+a)^(1/2)+2*B*a/b^2/(b*x^2+a)^(1/2)-A*x/b/(b*x^2+a)^(1/2)+A/b
^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.30, size = 64, normalized size = 0.97

$$\frac{Bx^2}{\sqrt{bx^2 + a}\,b} - \frac{Ax}{\sqrt{bx^2 + a}\,b} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Ba}{\sqrt{bx^2 + a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x^2/(sqrt(b*x^2 + a)*b) - A*x/(sqrt(b*x^2 + a)*b) + A*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

mupad [B] time = 1.34, size = 61, normalized size = 0.92

$$\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A\,x}{b\,\sqrt{b\,x^2+a}}+\frac{B\,\left(b\,x^2+2\,a\right)}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (A*x)/(b*(a + b*x^2)^(1/2)
) + (B*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 16.61, size = 83, normalized size = 1.26

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True))
```

3.31 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=48

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{A+Bx}{b\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {778, 217, 206}

$$\frac{B \tanh^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{A+Bx}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [**A**]   time = 0.08, size = 64, normalized size = 1.33

$$\frac{\sqrt{a}\,B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - \sqrt{b}\,(A+Bx)}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*(A + B*x)) + Sqrt[a]*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/S
qrt[a]])/(b^(3/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [**A**]   time = 0.35, size = 53, normalized size = 1.10

$$\frac{-A-Bx}{b\sqrt{a+bx^2}} - \frac{B\log\left(\sqrt{a+bx^2} - \sqrt{b}\,x\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-A - B*x)/(b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^
(3/2)
```

fricas [**A**]   time = 0.91, size = 147, normalized size = 3.06

$$\left[\frac{\left(Bbx^2+Ba\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(Bbx+Ab)\sqrt{bx^2+a}}{2\left(b^3x^2+ab^2\right)}, -\frac{\left(Bbx^2+Ba\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(Bbx+Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 + a)
)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.50, size = 48, normalized size = 1.00

$$-\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(3/2)
```

maple [A] time = 0.01, size = 54, normalized size = 1.12

$$-\frac{Bx}{\sqrt{b\,x^2+a}\;b}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{b\,x^2+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A/b/(b*x^2+a
)^(1/2)
```

maxima [A] time = 1.31, size = 46, normalized size = 0.96

$$-\frac{Bx}{\sqrt{bx^2+a}\,b}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A/(sqrt(b*x^2
 + a)*b)
```

mupad [B] time = 1.06, size = 53, normalized size = 1.10

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A}{b\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - A/(b*(a + b*x^2)^(1/2)) -
(B*x)/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 15.18, size = 66, normalized size = 1.38

$$A\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.32 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {637}

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.96

$$\frac{Abx-aB}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.31, size = 27, normalized size = 0.96

$$\frac{Abx - aB}{ab\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(a + b*x^2)^(3/2),x]
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

fricas [A] time = 0.85, size = 35, normalized size = 1.25

$$\frac{(Abx - Ba)\sqrt{bx^2 + a}}{ab^2x^2 + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.44, size = 23, normalized size = 0.82

$$\frac{\frac{Ax}{a} - \frac{B}{b}}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$\frac{Abx - Ba}{\sqrt{b\,x^2 + a}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(3/2),x)
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 31, normalized size = 1.11

$$\frac{Ax}{\sqrt{bx^2 + a}\,a} - \frac{B}{\sqrt{bx^2 + a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

mupad [B] time = 0.91, size = 24, normalized size = 0.86

$$-\frac{\frac{B}{b}-\frac{A\,x}{a}}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(3/2),x)
```

```
[Out] -(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

sympy [A] time = 10.22, size = 46, normalized size = 1.64

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(3/2)), True))
```

3.33 $\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 12, 266, 63, 208}

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{\int \frac{aAb}{x\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 1.00

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

IntegrateAlgebraic [A] time = 0.33, size = 61, normalized size = 1.30

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a +
 b*x^2]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 1.00, size = 146, normalized size = 3.11

$$\left[\frac{\left(Abx^2+Aa\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Bax+Aa)\sqrt{bx^2+a}}{2\left(a^2bx^2+a^3\right)},\ \frac{\left(Abx^2+Aa\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(Bax+Aa)\sqrt{bx^2+a}}{a^2bx^2+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 + A*
a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2 + a
))/(a^2*b*x^2 + a^3)]
```

giac [A] time = 0.44, size = 59, normalized size = 1.26

$$\frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a)
```

maple [A] time = 0.01, size = 60, normalized size = 1.28

$$\frac{Bx}{\sqrt{b\,x^2+a}\;a}-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{b\,x^2+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] B*x/a/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/
2)*a^(1/2))/x)
```

maxima [A] time = 1.30, size = 48, normalized size = 1.02

$$\frac{Bx}{\sqrt{bx^2+a}\,a}-\frac{A\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x/(sqrt(b*x^2 + a)*a) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt
(b*x^2 + a)*a)
```

mupad [B] time = 1.29, size = 50, normalized size = 1.06

$$\frac{A}{a\,\sqrt{b\,x^2+a}}-\frac{A\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{B\,x}{a\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) + (B
*x)/(a*(a + b*x^2)^(1/2))
```

sympy [B] time = 11.29, size = 206, normalized size = 4.38

$$A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*
x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/
2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.34 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 807, 266, 63, 208}

$$-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{\int \frac{-2aAb-abBx}{x^2\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 72, normalized size = 1.03

$$
-\frac{a(A-Bx)+\sqrt{a}\,Bx\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+2Abx^2}{a^2x\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((2*A*b*x^2 + a*(A - B*x) + Sqrt[a]*B*x*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(a^2*x*Sqrt[a + b*x^2]))
```

IntegrateAlgebraic [A] time = 0.35, size = 75, normalized size = 1.07

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{-aA+aBx-2Abx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) + a*B*x - 2*A*b*x^2)/(a^2*x*Sqrt[a + b*x^2]) + (2*B*ArcTanh[(Sqrt[b
]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.88, size = 169, normalized size = 2.41

$$\left[\frac{\left(Bbx^3+Bax\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^3+a^3x\right)},\ \frac{\left(Bbx^3+Bax\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}}{a^2bx^3+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)
, ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*x^2
 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.50, size = 96, normalized size = 1.37

$$-\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a)
```

maple [A] time = 0.01, size = 80, normalized size = 1.14

$$-\frac{2Abx}{\sqrt{b\,x^2+a}\;a^2}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{b\,x^2+a}\;a}-\frac{A}{\sqrt{b\,x^2+a}\;ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(1/2)-2*A*b/a^2*x/(b*x^2+a)^(1/2)+B/a/(b*x^2+a)^(1/2)-B/a^
(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.32, size = 68, normalized size = 0.97

$$-\frac{2\,Abx}{\sqrt{bx^2+a}\,a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a}-\frac{A}{\sqrt{bx^2+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
B/(sqrt(b*x^2 + a)*a) - A/(sqrt(b*x^2 + a)*a*x)
```

mupad [B] time = 1.45, size = 70, normalized size = 1.00

$$\frac{B}{a\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A}{a\,x\,\sqrt{b\,x^2+a}}-\frac{2\,A\,b\,x}{a^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - A/
(a*x*(a + b*x^2)^(1/2)) - (2*A*b*x)/(a^2*(a + b*x^2)^(1/2))
```

sympy [B] time = 15.83, size = 235, normalized size = 3.36

$$A\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 +
b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)
/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

3.35 $\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {823, 835, 807, 266, 63, 208}

$$-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{\int \frac{-3aAb-2abBx}{x^3\sqrt{a+bx^2}}\,dx}{a^2b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} + \frac{\int \frac{4a^2bB-3aAb^2x}{x^2\sqrt{a+bx^2}}\,dx}{2a^3b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3A)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} + \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.22, size = 75, normalized size = 0.79

$$\frac{3Ab\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) - \frac{a(A+2Bx)}{x^2} - b(3A+4Bx)}{2a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-((a*(A + 2*B*x))/x^2) - b*(3*A + 4*B*x) + 3*A*b*Sqrt[1 + (b*x^2)/a]*ArcTa
nh[Sqrt[1 + (b*x^2)/a]])/(2*a^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.46, size = 87, normalized size = 0.92

$$\frac{-aA-2aBx-3Abx^2-4bBx^3}{2a^2x^2\sqrt{a+bx^2}} - \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}} - \frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 2*a*B*x - 3*A*b*x^2 - 4*b*B*x^3)/(2*a^2*x^2*Sqrt[a + b*x^2]) - (3
*A*b*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.90, size = 211, normalized size = 2.22

$$\left[\frac{3\left(Ab^{2}x^{4}+Aabx^{2}\right)\sqrt{a}\log\left(-\frac{bx^{2}+2\sqrt{bx^{2}+a}\sqrt{a}+2a}{x^{2}}\right)-2\left(4Babx^{3}+3Aabx^{2}+2Ba^{2}x+Aa^{2}\right)\sqrt{bx^{2}+a}}{4\left(a^{3}bx^{4}+a^{4}x^{2}\right)},-\frac{3\left(Ab^{2}x^{4}+Aabx^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(4Babx^{3}+3Aabx^{2}+2Ba^{2}x+Aa^{2}\right)\sqrt{bx^{2}+a}}{2\left(a^{3}bx^{4}+a^{4}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt(b
*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x +
 A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

giac [B] time = 0.50, size = 171, normalized size = 1.80

$$-\frac{\frac{Bbx}{a^{2}}+\frac{Ab}{a^{2}}}{\sqrt{bx^{2}+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{3}Ab+2\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)Aab-2\,Ba^{2}\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b +
 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 +
a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

maple [A] time = 0.01, size = 101, normalized size = 1.06

$$-\frac{2Bbx}{\sqrt{b\,x^{2}+a}\;a^{2}}+\frac{3Ab\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}-\frac{3Ab}{2\sqrt{b\,x^{2}+a}\;a^{2}}-\frac{B}{\sqrt{b\,x^{2}+a}\;ax}-\frac{A}{2\sqrt{b\,x^{2}+a}\;a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A/a^2*b/(b*x^2+a)^(1/2)+3/2*A/a^(5/2)*b*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^2+a)^(1/2)-2*B*b/a^2*x/(b*x^
2+a)^(1/2)
```

maxima [A] time = 1.35, size = 89, normalized size = 0.94

$$-\frac{2\,Bbx}{\sqrt{bx^2+a}\,a^2}+\frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}\,a^2}-\frac{B}{\sqrt{bx^2+a}\,ax}-\frac{A}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5
/2) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)*a*x) - 1/2*A/(sqrt
(b*x^2 + a)*a*x^2)
```

mupad [B] time = 1.59, size = 94, normalized size = 0.99

$$\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{3\,A\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{A}{2\,a\,x^2\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - (3*A*b)/(2*a^2*(a +
b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a + (
2*B*b*x^2)/a^2))/(a*x + b*x^3)
```

sympy [A] time = 10.79, size = 124, normalized size = 1.31

$$A\left(-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

3.36 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}} dx$

Optimal. Leaf size=79

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 778, 217, 206}

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(A + B*x))/(3*b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a + b*
x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{x(2aA+3aBx)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.87

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{a(2A+3Bx)+bx^2(3A+4Bx)}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*(2*A + 3*B*x) + b*x^2*(3*A + 4*B*x))/(b^2*(a + b*x^2)^(3/2)) + (B*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.51, size = 72, normalized size = 0.91

$$\frac{-2aA-3aBx-3Abx^2-4bBx^3}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a*A - 3*a*B*x - 3*A*b*x^2 - 4*b*B*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - (B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

fricas [A] time = 0.82, size = 239, normalized size = 3.03

$$\left[\frac{3\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(4\,Bb^2x^3+3\,Ab^2x^2+3\,Babx+2\,Aab\right)\sqrt{bx^2+a}}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)},-\frac{3\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(4\,Bb^2x^3+3\,Ab^2x^2+3\,Babx+2\,Aab\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b
)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4 +
2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B*b^2
*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b
^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.48, size = 70, normalized size = 0.89

$$-\frac{\left(\left(\frac{4\,Bx}{b}+\frac{3\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{2\,Aa}{b^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 91, normalized size = 1.15

$$-\frac{B\,x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b}-\frac{A\,x^2}{\left(b\,x^2+a\right)^{\frac{3}{2}}\,b}-\frac{2Aa}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b^2}-\frac{Bx}{\sqrt{b\,x^2+a}\;b^2}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B*x^3/b/(b*x^2+a)^(3/2)-B/b^2*x/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))-A*x^2/b/(b*x^2+a)^(3/2)-2/3*A*a/b^2/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.41, size = 102, normalized size = 1.29

$$-\frac{1}{3}Bx\left(\frac{3x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)-\frac{Ax^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{Bx}{3\sqrt{bx^2+a}\,b^2}+\frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2Aa}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - A*x^
2/((b*x^2 + a)^(3/2)*b) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) + B*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x\right)}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(5/2), x)
```

sympy [A] time = 18.32, size = 400, normalized size = 5.06

$$A\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a
**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
```

```
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b
*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.37 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=53

$$-\frac{x^2(aB - Abx)}{3ab\left(a + bx^2\right)^{3/2}} - \frac{2B}{3b^2\sqrt{a + bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {805, 261}

$$-\frac{x^2(aB - Abx)}{3ab\left(a + bx^2\right)^{3/2}} - \frac{2B}{3b^2\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(x^2*(a*B - A*b*x))/(3*a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^
2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}$$
$$= -\frac{x^2(aB-Abx)}{3ab\left(a+bx^2\right)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}$$

<u>Mathematica</u> [A] time = 0.05, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

<u>IntegrateAlgebraic</u> [A] time = 0.44, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

<u>fricas</u> [A] time = 0.98, size = 63, normalized size = 1.19

$$\frac{\left(Ab^2x^3-3\,Babx^2-2\,Ba^2\right)\sqrt{bx^2+a}}{3\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2*
b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.48, size = 36, normalized size = 0.68

$$\frac{\left(\frac{Ax}{a} - \frac{3B}{b}\right)x^2 - \frac{2Ba}{b^2}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.01, size = 41, normalized size = 0.77

$$\frac{A\,x^3b^2 - 3Bab\,x^2 - 2B\,a^2}{3\left(b\,x^2 + a\right)^{\frac{3}{2}}a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x)
[Out] 1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

maxima [A] time = 1.36, size = 70, normalized size = 1.32

$$-\frac{Bx^2}{\left(bx^2 + a\right)^{\frac{3}{2}}b} - \frac{Ax}{3\left(bx^2 + a\right)^{\frac{3}{2}}b} + \frac{Ax}{3\sqrt{bx^2 + a}\,ab} - \frac{2Ba}{3\left(bx^2 + a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqr
t(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [B] time = 0.97, size = 51, normalized size = 0.96

$$\frac{B\,a^2 - 3\,B\,a\,\left(b\,x^2 + a\right) + A\,b\,x\,\left(b\,x^2 + a\right) - A\,a\,b\,x}{3\,a\,b^2\,{\left(b\,x^2 + a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*a^2 - 3*B*a*(a + b*x^2) + A*b*x*(a + b*x^2) - A*a*b*x)/(3*a*b^2*(a + b*x
^2)^(3/2))
```

sympy [B] time = 17.32, size = 141, normalized size = 2.66

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*
x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

3.38 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=50

$$\frac{-A-Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778, 191}

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A + B*x)/(3*b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}} \, dx &= -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx}{3b}\\
&= -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.64

$$\frac{bBx^3 - aA}{3ab\left(a + bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.42, size = 32, normalized size = 0.64

$$\frac{bBx^3 - aA}{3ab\left(a + bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.92, size = 49, normalized size = 0.98

$$\frac{\left(Bbx^3 - Aa\right)\sqrt{bx^2 + a}}{3\left(ab^3x^4 + 2\,a^2b^2x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.52, size = 26, normalized size = 0.52

$$\frac{\frac{Bx^3}{a} - \frac{A}{b}}{3\left(bx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.58

$$-\frac{-Bb\,x^{3}+Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(5/2),x)
[Out] -1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

maxima [A] time = 1.31, size = 51, normalized size = 1.02

$$-\frac{Bx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{Bx}{3\sqrt{bx^{2}+a}\,ab}-\frac{A}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

mupad [B] time = 0.92, size = 34, normalized size = 0.68

$$\frac{B\,x^{3}}{3\,a\,\left(b\,x^{2}+a\right)^{3/2}}-\frac{A}{3\,b\,\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(5/2),x)
[Out] (B*x^3)/(3*a*(a + b*x^2)^(3/2)) - A/(3*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 13.98, size = 95, normalized size = 1.90

$$A\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^{2}}+3b^{2}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{2}}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Bx^{3}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{3}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a)
 + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

3.39 $\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{Abx-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {639, 191}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*B - A*b*x)/(3*a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.84

$$\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 43, normalized size = 0.84

$$\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(a + b*x^2)^(5/2),x]
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.88, size = 62, normalized size = 1.22

$$\frac{\left(2\,Ab^2x^3+3\,Aabx-Ba^2\right)\sqrt{bx^2+a}}{3\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3*
b^2*x^2 + a^4*b)
```

giac [A] time = 0.48, size = 37, normalized size = 0.73

$$\frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 40, normalized size = 0.78

$$\frac{2A\,x^{3}b^{2}+3Axab-B\,a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

maxima [A] time = 1.37, size = 48, normalized size = 0.94

$$\frac{2\,Ax}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{Ax}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x
^2 + a)^(3/2)*b)
```

mupad [B] time = 0.93, size = 41, normalized size = 0.80

$$\frac{2\,A\,b\,x\,\left(b\,x^{2}+a\right)-B\,a^{2}+A\,a\,b\,x}{3\,a^{2}\,b\,\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

sympy [B] time = 13.20, size = 146, normalized size = 2.86

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^{2}}+3b^{2}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{2}}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a
 + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

3.40 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 12, 266, 63, 208}

$$\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-3aAb-2abBx}{x\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{\int \frac{3a^2Ab^2}{x\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 69, normalized size = 0.91

$$\frac{a(4A+3Bx)+bx^2(3A+2Bx)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) - (A*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [A] time = 0.50, size = 83, normalized size = 1.09

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{4aA+3aBx+3Abx^2+2bBx^3}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (4*a*A + 3*a*B*x + 3*A*b*x^2 + 2*b*B*x^3)/(3*a^2*(a + b*x^2)^(3/2)) + (2*A*
ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.75, size = 239, normalized size = 3.14

$$\left[\frac{3\left(Ab^2x^4+2\,Aabx^2+Aa^2\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Babx^3+3\,Aabx^2+3\,Ba^2x+4\,Aa^2\right)\sqrt{bx^2+a}}{6\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)},\frac{3\left(Ab^2x^4+2\,Aabx^2+Aa^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,Babx^3+3\,Aabx^2+3\,Ba^2x+4\,Aa^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A
*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*x^4
 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*
b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 +
2*a^4*b*x^2 + a^5)]
```

giac [A] time = 0.54, size = 82, normalized size = 1.08

$$\frac{\left(\left(\frac{2\,Bbx}{a^2}+\frac{3\,Ab}{a^2}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) + 2
*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

maple [A] time = 0.01, size = 92, normalized size = 1.21

$$\frac{Bx}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}+\frac{A}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}+\frac{2Bx}{3\sqrt{b\,x^{2}+a}\,a^{2}}-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{b\,x^{2}+a}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*B*x/a/(b*x^2+a)^(3/2)+2/3*B/a^2*x/(b*x^2+a)^(1/2)+1/3*A/a/(b*x^2+a)^(3/
2)+A/a^2/(b*x^2+a)^(1/2)-A/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.37, size = 80, normalized size = 1.05

$$\frac{2\,Bx}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{Bx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{A\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a^{2}}+\frac{A}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - A*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 + a)
^(3/2)*a)
```

mupad [B] time = 1.38, size = 80, normalized size = 1.05

$$\frac{\frac{A}{3\,a}+\frac{A\left(b\,x^{2}+a\right)}{a^{2}}}{\left(b\,x^{2}+a\right)^{3/2}}+\frac{2\,B\,x\,\left(b\,x^{2}+a\right)+B\,a\,x}{3\,a^{2}\left(b\,x^{2}+a\right)^{3/2}}-\frac{A\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*B*x*(a + b*x^2) + B*
a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/
2)
```

sympy [B] time = 25.92, size = 840, normalized size = 11.05

$$A\left(\frac{8a^{7}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{7}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{14a^{6}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{6}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{6}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{6a^{5}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{5}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{5}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{4}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{4}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15
/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2)
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log
(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(1
9/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x*
*3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

3.41 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {823, 807, 266, 63, 208}

$$\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rubi steps

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-4aAb-3abBx}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x^2\sqrt{a+bx^2}}\,dx}{3a^4b^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2b}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.91

$$\frac{a^2(4Bx-3A)+3abx^2(Bx-4A)-3\sqrt{a}\,Bx\left(a+bx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x) - 3*Sqrt[a]*B*x
*(a + b*x^2)^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(3*a^3*x*(a + b*x^2)^(
3/2))
```

IntegrateAlgebraic [A] time = 0.54, size = 101, normalized size = 0.97

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{-3a^2A+4a^2Bx-12aAbx^2+3abBx^3-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A + 4*a^2*B*x - 12*a*A*b*x^2 + 3*a*b*B*x^3 - 8*A*b^2*x^4)/(3*a^3*x*
(a + b*x^2)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqr
t[a]])/a^(5/2)
```

fricas [A] time = 1.04, size = 264, normalized size = 2.54

$$\left[\frac{3\left(Bb^2x^5+2\,Babx^3+Ba^2x\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(8\,Ab^2x^4-3\,Babx^3+12\,Aabx^2-4\,Ba^2x+3\,Aa^2\right)\sqrt{bx^2+a}}{6\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)},\ \frac{3\left(Bb^2x^5+2\,Babx^3+Ba^2x\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(8\,Ab^2x^4-3\,Babx^3+12\,Aabx^2-4\,Ba^2x+3\,Aa^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2
 - 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
, 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a
^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

giac [A] time = 0.55, size = 119, normalized size = 1.14

$$-\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(3
/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

maple [A] time = 0.01, size = 112, normalized size = 1.08

$$-\frac{4Abx}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{8Abx}{3\sqrt{b\,x^{2}+a}\;a^{3}}+\frac{B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}-\frac{A}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}ax}+\frac{B}{\sqrt{b\,x^{2}+a}\;a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -A/a/x/(b*x^2+a)^(3/2)-4/3*A/a^2*b*x/(b*x^2+a)^(3/2)-8/3*A/a^3*b*x/(b*x^2+a
)^(1/2)+1/3*B/a/(b*x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*ln((2*a+2*(
b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.34, size = 100, normalized size = 0.96

$$-\frac{8\,Abx}{3\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{4\,Abx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{B}{\sqrt{bx^{2}+a}\,a^{2}}+\frac{B}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{A}{\left(bx^{2}+a\right)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*A*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*ar
csinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x
^2 + a)^(3/2)*a) - A/((b*x^2 + a)^(3/2)*a*x)
```

mupad [B] time = 1.58, size = 96, normalized size = 0.92

$$\frac{\frac{B}{3\,a}+\frac{B\left(b\,x^{2}+a\right)}{a^{2}}}{\left(b\,x^{2}+a\right)^{3/2}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A\,a^{2}-8\,A\left(b\,x^{2}+a\right)^{2}+4\,A\,a\,\left(b\,x^{2}+a\right)}{3\,a^{3}\,x\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(3*
a^3*x*(a + b*x^2)^(3/2))
```

sympy [B] time = 24.18, size = 910, normalized size = 8.75

$$A\left(-\frac{3a^{2}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{12ab^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{8b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}\right)+B\left(\frac{8a^{7}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{7}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{14a^{6}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{6}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{6}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{6a^{5}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{5}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{5}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{4}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{4}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1
+ b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6
*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1
+ b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) +
9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/
2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*
b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a
) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(
13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x
**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

3.42 $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {823, 835, 807, 266, 63, 208}

$$\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}+\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-5aAb-4abBx}{x^3\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} + \frac{\int \frac{15a^2Ab^2+8a^2b^2Bx}{x^3\sqrt{a+bx^2}}\,dx}{3a^4b^2} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{\int \frac{-16a^3b^2B+15a^2Ab^3x}{x^2\sqrt{a+bx^2}}\,dx}{6a^5b^2} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\int \frac{1}{x\sqrt{a+bx^2}}}{2a^3} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\,\mathrm{Subst}\left(\int \right.}{4a} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5A)\,\mathrm{Subst}\left(\int \right.}{} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} + \frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{2a^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 106, normalized size = 0.82

$$
\frac{-\frac{3a^3(A+2Bx)}{x^2} - 4a^2b(5A+6Bx) - ab^2x^2(15A+16Bx) + \frac{15Ab\left(a+bx^2\right)^2\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}}{6a^4\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] ((-3*a^3*(A + 2*B*x))/x^2 - 4*a^2*b*(5*A + 6*B*x) - a*b^2*x^2*(15*A + 16*B*
x) + (15*A*b*(a + b*x^2)^2*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a
])/(6*a^4*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.63, size = 111, normalized size = 0.86

$$\frac{-3a^2A-6a^2Bx-20aAbx^2-24abBx^3-15Ab^2x^4-16b^2Bx^5}{6a^3x^2\left(a+bx^2\right)^{3/2}}-\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 6*a^2*B*x - 20*a*A*b*x^2 - 24*a*b*B*x^3 - 15*A*b^2*x^4 - 16*b^2
*B*x^5)/(6*a^3*x^2*(a + b*x^2)^(3/2)) - (5*A*b*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
- Sqrt[a + b*x^2]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.79, size = 307, normalized size = 2.38

$$\left[\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(16\,Bab^2x^5+15\,Aab^2x^4+24\,Ba^2bx^3+20\,Aa^2bx^2+6\,Ba^3x+3\,Aa^3\right)\sqrt{bx^2+a}}{12\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)},-\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(16\,Bab^2x^5+15\,Aab^2x^4+24\,Ba^2bx^3+20\,Aa^2bx^2+6\,Ba^3x+3\,Aa^3\right)\sqrt{bx^2+a}}{6\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 +
 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))/(a
^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 +
A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x^5 +
15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqr
t(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

giac [A] time = 0.53, size = 197, normalized size = 1.53

$$-\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2 +
 a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/
(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

maple [A] time = 0.01, size = 134, normalized size = 1.04

$$-\frac{4Bbx}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{5Ab}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{8Bbx}{3\sqrt{b\,x^{2}+a}\;a^{3}}+\frac{5Ab\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}-\frac{5Ab}{2\sqrt{b\,x^{2}+a}\;a^{3}}-\frac{B}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}ax}-\frac{A}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A/a^2*b/(b*x^2+a)^(3/2)-5/2*A/a^3*b/(b*x^2
+a)^(1/2)+5/2*A/a^(7/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^
2+a)^(3/2)-4/3*B/a^2*b*x/(b*x^2+a)^(3/2)-8/3*B/a^3*b*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 122, normalized size = 0.95

$$-\frac{8\,Bbx}{3\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{4\,Bbx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}+\frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,Ab}{2\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{5\,Ab}{6\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{B}{\left(bx^{2}+a\right)^{\frac{3}{2}}ax}-\frac{A}{2\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 5/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*A*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

mupad [B] time = 1.62, size = 123, normalized size = 0.95

$$\frac{B\,a^{2}-8\,B\left(b\,x^{2}+a\right)^{2}+4\,B\,a\,\left(b\,x^{2}+a\right)}{3\,a^{3}\,x\left(b\,x^{2}+a\right)^{3/2}}-\frac{10\,A\,b}{3\,a^{2}\left(b\,x^{2}+a\right)^{3/2}}-\frac{A}{2\,a\,x^{2}\left(b\,x^{2}+a\right)^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b^{2}\,x^{2}}{2\,a^{3}\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B*a^2 - 8*B*(a + b*x^2)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
 - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*
A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5*A*b^2*x^2)/(2*a^3*(a
 + b*x^2)^(3/2))
```

sympy [B] time = 24.76, size = 1034, normalized size = 8.02

$$A\left(-\frac{6a^{17}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{46a^{16}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{15a^{16}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{30a^{16}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{70a^{15}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{45a^{15}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{90a^{15}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{30a^{14}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{45a^{14}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{90a^{14}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{15a^{13}b^{4}x^{8}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{30a^{13}b^{4}x^{8}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}\right)+B\left(-\frac{3a^{2}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{12ab^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{8b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

IntegrateAlgebraic [A] time = 0.13, size = 37, normalized size = 1.37

$$\frac{1}{2}\sqrt{1-x^2}\,(x-2)+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x)*x)/Sqrt[1 - x^2],x]
[Out] ((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

fricas [A] time = 0.60, size = 31, normalized size = 1.15

$$\frac{1}{2}\,\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.47, size = 19, normalized size = 0.70

$$\frac{1}{2}\,\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

maxima [A] time = 2.97, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2} - \frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/(1 - x^2)^(1/2),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [A] time = 0.24, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 780, 216}

$$-\frac{1}{2}\sqrt{1-x^2}\,(2-x)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -((2 - x)*Sqrt[1 - x^2])/2 - ArcSin[x]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.89

$$\frac{1}{2}\left((x-2)\sqrt{1-x^2}-\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)/Sqrt[1 - x^2],x]
[Out] ((-2 + x)*Sqrt[1 - x^2] - ArcSin[x])/2
```

IntegrateAlgebraic [A] time = 0.12, size = 37, normalized size = 1.37

$$\frac{1}{2}\sqrt{1-x^2}\,(x-2)+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - x^2)/Sqrt[1 - x^2],x]
[Out] ((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

fricas [A] time = 0.62, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [A] time = 0.42, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.07

$$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

maxima [A] time = 2.89, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

sympy [A] time = 0.29, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.45 $\int \frac{3+x^2}{-3+x^2} \, dx$

Optimal. Leaf size=17

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 207}

$$x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{3+x^2}{-3+x^2} \, dx &= x+6 \int \frac{1}{-3+x^2} \, dx \\ &= x-2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.94

$$x+\sqrt{3}\log\left(\sqrt{3}-x\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+x^2}{-3+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] IntegrateAlgebraic[(3 + x^2)/(-3 + x^2), x]
```

fricas [A] time = 0.87, size = 26, normalized size = 1.53

$$\sqrt{3}\,\log\left(\frac{x^2-2\,\sqrt{3}\,x+3}{x^2-3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

giac [B] time = 0.41, size = 30, normalized size = 1.76

$$\sqrt{3}\,\log\left(\frac{\left|2\,x-2\,\sqrt{3}\right|}{\left|2\,x+2\,\sqrt{3}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="giac")
```

```
[Out] sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

maple [A] time = 0.00, size = 15, normalized size = 0.88

$$x-2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2-3),x)
```

[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)

maxima [A] time = 2.88, size = 22, normalized size = 1.29

$$\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3)/(x^2-3),x, algorithm="maxima")

[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x

mupad [B] time = 0.92, size = 14, normalized size = 0.82

$$x-2\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3)/(x^2 - 3),x)

[Out] x - 2*3^(1/2)*atanh((3^(1/2)*x)/3)

sympy [A] time = 0.19, size = 27, normalized size = 1.59

$$x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+3)/(x**2-3),x)

[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))

3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$

Optimal. Leaf size=6

$$x-2\tan^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {388, 203}

$$x-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{1+x^2}\,dx &= x-2\int \frac{1}{1+x^2}\,dx\\ &= x-2\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$x-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
[Out] x - 2*ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(1 + x^2),x]
[Out] IntegrateAlgebraic[(-1 + x^2)/(1 + x^2), x]
```

fricas [A] time = 1.00, size = 6, normalized size = 1.00

$$x - 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
[Out] x - 2*arctan(x)
```

giac [A] time = 0.46, size = 6, normalized size = 1.00

$$x - 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="giac")
[Out] x - 2*arctan(x)
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$x - 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1),x)
[Out] x-2*arctan(x)
```

maxima [A] time = 2.97, size = 6, normalized size = 1.00

$$x - 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(x)
```

mupad [B] time = 0.04, size = 6, normalized size = 1.00

$$x - 2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 + 1),x)
```

```
[Out] x - 2*atan(x)
```

sympy [A] time = 0.15, size = 5, normalized size = 0.83

$$x - 2\operatorname{atan}{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1),x)
```

```
[Out] x - 2*atan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{16\sqrt{a+bx^2}\,(Ab-8aC)}{35ab^5}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^7(a}{7a}$$

Rubi [A] time = 0.32, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1804, 819, 641, 217, 206}

$$-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{16\sqrt{a+bx^2}\,(Ab-8aC)}{35ab^5}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^7*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A*b
 - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*C)
*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35*a
*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (B*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7 \left(A + Bx + Cx^2\right)}{\left(a + bx^2\right)^{9/2}} \, dx = -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{x^6(-7aB + (Ab - 8aC)x)}{\left(a + bx^2\right)^{7/2}} \, dx}{7ab}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{\int \frac{x^4\left(-35a^2B + 6a(Ab - 8aC)x\right)}{\left(a + bx^2\right)^{5/2}} \, dx}{35a^2b^2}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{x^3(35aB - 6(Ab - 8aC)x)}{105ab^3\left(a + bx^2\right)^{3/2}} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{x^3(35aB - 6(Ab - 8aC)x)}{105ab^3\left(a + bx^2\right)^{3/2}} - \frac{x(\cdots}{\cdots}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{x^3(35aB - 6(Ab - 8aC)x)}{105ab^3\left(a + bx^2\right)^{3/2}} - \frac{x(\cdots}{\cdots}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{x^3(35aB - 6(Ab - 8aC)x)}{105ab^3\left(a + bx^2\right)^{3/2}} - \frac{x(\cdots}{\cdots}$$

$$= -\frac{x^7(aB - (Ab - aC)x)}{7ab\left(a + bx^2\right)^{7/2}} - \frac{x^5(7aB - (Ab - 8aC)x)}{35ab^2\left(a + bx^2\right)^{5/2}} - \frac{x^3(35aB - 6(Ab - 8aC)x)}{105ab^3\left(a + bx^2\right)^{3/2}} - \frac{x(\cdots}{\cdots}$$

Mathematica [A] time = 0.65, size = 165, normalized size = 0.77

$$\frac{384a^4C - 3a^3b(16A + 7x(5B - 64Cx)) + 14a^2b^2x^2(5x(24Cx - 5B) - 12A) + 14ab^3x^4(x(60Cx - 29B) - 15A) + 105\sqrt{a}\sqrt{b}B\left(a + bx^2\right)^3\sqrt{\frac{bx^2}{a} + 1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + b^4x^6(x(105Cx - 176B) - 105A)}{105b^5\left(a + bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^6*
(-105*A + x*(-176*B + 105*C*x)) + 105*Sqrt[a]*Sqrt[b]*B*(a + b*x^2)^3*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(105*b^5*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 1.44, size = 173, normalized size = 0.81

$$\frac{384a^4C - 48a^3Ab - 105a^3bBx + 1344a^3bCx^2 - 168a^2Ab^2x^2 - 350a^2b^2Bx^3 + 1680a^2b^2Cx^4 - 210aAb^3x^4 - 406ab^3Bx^5 + 840ab^3Cx^6 - 105Ab^4x^6 - 176b^4Bx^7 + 105b^4Cx^8}{105b^5\left(a+bx^2\right)^{7/2}} - \frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^3*A*b + 384*a^4*C - 105*a^3*b*B*x - 168*a^2*A*b^2*x^2 + 1344*a^3*b*C
*x^2 - 350*a^2*b^2*B*x^3 - 210*a*A*b^3*x^4 + 1680*a^2*b^2*C*x^4 - 406*a*b^3
*B*x^5 - 105*A*b^4*x^6 + 840*a*b^3*C*x^6 - 176*b^4*B*x^7 + 105*b^4*C*x^8)/(
105*b^5*(a + b*x^2)^(7/2)) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2
)
```

fricas [A] time = 0.86, size = 522, normalized size = 2.45

$$\left[\frac{105\left(Bb^4x^8+4Bab^3x^6+6Ba^2b^2x^4+4Ba^3bx^2+Ba^4\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(105Cb^4x^8-176Bb^4x^7-406Bab^3x^5-350Ba^2b^2x^3+105\left(8Cab^3-Ab^4\right)x^6-105Ba^3bx+384Ca^4-48Aa^3b+210\left(8Ca^2b^2-Aab^3\right)x^4+168\left(8Ca^3b-Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{210\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)}, -\frac{105\left(Bb^4x^8+4Bab^3x^6+6Ba^2b^2x^4+4Ba^3bx^2+Ba^4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(105Cb^4x^8-176Bb^4x^7-406Bab^3x^5-350Ba^2b^2x^3+105\left(8Cab^3-Ab^4\right)x^6-105Ba^3bx+384Ca^4-48Aa^3b+210\left(8Ca^2b^2-Aab^3\right)x^4+168\left(8Ca^3b-Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(b^9x^8+4ab^8x^6+6a^2b^7x^4+4a^3b^6x^2+a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C*b
^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b
^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2
 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(B*
b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*
B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b
*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^
3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x
^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.54, size = 204, normalized size = 0.96

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\left(8\,Ca^4b^7-Aa^3b^8\right)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\left(8\,Ca^5b^6-Aa^4b^7\right)}{a^3b^9}\right)x-\frac{350\,Ba^2}{b^3}\right)x+\frac{168\left(8\,Ca^6b^5-Aa^5b^6\right)}{a^3b^9}\right)x-\frac{105\,Ba^3}{b^4}\right)x+\frac{48\left(8\,Ca^7b^4-Aa^6b^5\right)}{a^3b^9}}{105\left(bx^2+a\right)^{\frac{7}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*b
^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 350*
```

```
B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^4)*
x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.05, size = 265, normalized size = 1.24

$$\frac{C x^8}{\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{B x^7}{7\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{A x^6}{\left(b x^2+a\right)^{\frac{7}{2}} b}+\frac{8 C a x^6}{\left(b x^2+a\right)^{\frac{7}{2}} b^2}-\frac{2 A a x^4}{\left(b x^2+a\right)^{\frac{7}{2}} b^2}-\frac{B x^5}{5\left(b x^2+a\right)^{\frac{5}{2}} b^2}+\frac{16 C a^2 x^4}{\left(b x^2+a\right)^{\frac{7}{2}} b^3}-\frac{8 A a^2 x^2}{5\left(b x^2+a\right)^{\frac{7}{2}} b^3}+\frac{64 C a^3 x^2}{5\left(b x^2+a\right)^{\frac{7}{2}} b^4}-\frac{B x^3}{3\left(b x^2+a\right)^{\frac{3}{2}} b^3}-\frac{16 A a^3}{35\left(b x^2+a\right)^{\frac{7}{2}} b^4}+\frac{128 C a^4}{35\left(b x^2+a\right)^{\frac{7}{2}} b^5}-\frac{B x}{\sqrt{b x^2+a}\, b^4}+\frac{B \ln \left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] C*x^8/b/(b*x^2+a)^(7/2)+8*C*a/b^2*x^6/(b*x^2+a)^(7/2)+16*C*a^2/b^3*x^4/(b*x
^2+a)^(7/2)+64/5*C*a^3/b^4*x^2/(b*x^2+a)^(7/2)+128/35*C*a^4/b^5/(b*x^2+a)^(
7/2)-1/7*B*x^7/b/(b*x^2+a)^(7/2)-1/5*B/b^2*x^5/(b*x^2+a)^(5/2)-1/3*B/b^3*x^
3/(b*x^2+a)^(3/2)-B/b^4*x/(b*x^2+a)^(1/2)+B/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^
(1/2))-A*x^6/b/(b*x^2+a)^(7/2)-2*A*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*A*a^2/b^3*
x^2/(b*x^2+a)^(7/2)-16/35*A*a^3/b^4/(b*x^2+a)^(7/2)
```

maxima [B] time = 1.68, size = 435, normalized size = 2.04

$$\frac{Cx^8}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{1}{35}\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)Bx+\frac{8\,Cax^6}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{Bx\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{3\,b^2}+\frac{16\,Ca^2x^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{2\,Aax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Bax^3}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}+\frac{64\,Ca^3x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^4}-\frac{8\,Aa^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{139\,Bx}{105\,\sqrt{bx^2+a}\,b^4}+\frac{17\,Bax}{105\left(bx^2+a\right)^{\frac{3}{2}}b^4}-\frac{29\,Ba^2x}{35\left(bx^2+a\right)^{\frac{5}{2}}b^4}+\frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}+\frac{128\,Ca^4}{35\left(bx^2+a\right)^{\frac{7}{2}}b^5}-\frac{16\,Aa^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4
/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*
x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b*x^
2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/((b*x^
2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b*x^2 +
 a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2 + a)^(
5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/((b*x^2
+ a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsinh(b*x/sqr
t(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A*a^3/((b*x^
2 + a)^(7/2)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
[Out] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}$$

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1804, 819, 778, 217, 206}

$$-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^6*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*x)
)/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)^(
3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^5(-6aB-7aCx)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^3\left(-24a^2B-35a^2Cx\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x\left(-48a^3B-105a^3C\right.}{\left(a+bx^2\right)^{3/2}}}{105a^3b^3}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} +$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} +$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} +$$

Mathematica [A] time = 0.41, size = 147, normalized size = 0.98

$$\frac{\sqrt{a}\,C\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}} - \frac{3a^4(16B+35Cx)+14a^3bx^2(12B+25Cx)+14a^2b^2x^4(15B+29Cx)+ab^3x^6(105B+176Cx)-15Ab^4x^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/105*(-15*A*b^4*x^7 + 14*a^3*b*x^2*(12*B + 25*C*x) + 14*a^2*b^2*x^4*(15*B
 + 29*C*x) + 3*a^4*(16*B + 35*C*x) + a*b^3*x^6*(105*B + 176*C*x))/(a*b^4*(a
 + b*x^2)^(7/2)) + (Sqrt[a]*C*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 1.13, size = 138, normalized size = 0.92

$$\frac{-48a^4B-105a^4Cx-168a^3bBx^2-350a^3bCx^3-210a^2b^2Bx^4-406a^2b^2Cx^5-105ab^3Bx^6-176ab^3Cx^7+15Ab^4x^7}{105ab^4\left(a+bx^2\right)^{7/2}} - \frac{C\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*B - 105*a^4*C*x - 168*a^3*b*B*x^2 - 350*a^3*b*C*x^3 - 210*a^2*b^2*
B*x^4 - 406*a^2*b^2*C*x^5 - 105*a*b^3*B*x^6 + 15*A*b^4*x^7 - 176*a*b^3*C*x^
7)/(105*a*b^4*(a + b*x^2)^(7/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
b^(9/2)
```

fricas [A] time = 1.11, size = 467, normalized size = 3.11

$$\left[\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(105\,Bab^4x^6+406\,Ca^2b^3x^5+210\,Ba^2b^3x^4+350\,Ca^3b^2x^3+168\,Ba^3b^2x^2+\left(176\,Cab^4-15\,Ab^5\right)x^7+105\,Ca^4bx+48\,Ba^4b\right)\sqrt{bx^2+a}}{210\left(ab^9x^8+4\,a^2b^8x^6+6\,a^3b^7x^4+4\,a^4b^6x^2+a^5b^5\right)},\ -\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(105\,Bab^4x^6+406\,Ca^2b^3x^5+210\,Ba^2b^3x^4+350\,Ca^3b^2x^3+168\,Ba^3b^2x^2+\left(176\,Cab^4-15\,Ab^5\right)x^7+105\,Ca^4bx+48\,Ba^4b\right)\sqrt{bx^2+a}}{105\left(ab^9x^8+4\,a^2b^8x^6+6\,a^3b^7x^4+4\,a^4b^6x^2+a^5b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^
2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105
*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3 +
168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*a^4
*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6
*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x
^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (
105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*
a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*
b^6*x^2 + a^5*b^5)]
```

giac [A] time = 0.53, size = 138, normalized size = 0.92

$$-\frac{\left(\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{\left(176\,Ca^3b^7-15\,Aa^2b^8\right)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{C\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 406
*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*C*
a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 277, normalized size = 1.85

$$-\frac{C\,x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{B\,x^6}{\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{A\,x^5}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{2Ba\,x^4}{\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{C\,x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}-\frac{5Aa\,x^3}{8\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}-\frac{8B\,a^2x^2}{5\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}-\frac{15A\,a^2x}{56\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}-\frac{C\,x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}+\frac{3Aax}{56\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}-\frac{16B\,a^3}{35\left(b\,x^2+a\right)^{\frac{7}{2}}b^4}+\frac{Ax}{14\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}+\frac{Ax}{7\sqrt{b\,x^2+a}\,a\,b^3}-\frac{Cx}{\sqrt{b\,x^2+a}\,b^4}+\frac{C\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C*x^7/b/(b*x^2+a)^(7/2)-1/5*C/b^2*x^5/(b*x^2+a)^(5/2)-1/3*C/b^3*x^3/(b
*x^2+a)^(3/2)-C/b^4*x/(b*x^2+a)^(1/2)+C/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-B*x^6/b/(b*x^2+a)^(7/2)-2*B*a/b^2*x^4/(b*x^2+a)^(7/2)-8/5*B*a^2/b^3*x^2/
(b*x^2+a)^(7/2)-16/35*B*a^3/b^4/(b*x^2+a)^(7/2)-1/2*A*x^5/b/(b*x^2+a)^(7/2)
-5/8*A*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*A*a^2/b^3*x/(b*x^2+a)^(7/2)+3/56*A*a
/b^3*x/(b*x^2+a)^(5/2)+1/14*A/b^3*x/(b*x^2+a)^(3/2)+1/7*A/a/b^3*x/(b*x^2+a)
^(1/2)
```

maxima [B] time = 1.63, size = 447, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Cx-\frac{Bx^6}{(bx^2+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{Ax^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{2\,Bax^4}{(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Cax^3}{(bx^2+a)^{\frac{5}{2}}b^3}-\frac{5\,Aax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{8\,Ba^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{139\,Cx}{105\,\sqrt{bx^2+a}\,b^4}+\frac{17\,Cax}{105\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{29\,Ca^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4}+\frac{Ax}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Ax}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Aax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Aa^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}-\frac{16\,Ba^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x - B*
x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x
^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x^5/(
(b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 +
 a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b*x^2 +
 a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2/((b*x^
2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*x/((b*x^
2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^
2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsinh(b*x/sqrt(
a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} \, dx$

Optimal. Leaf size=132

$$-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 805, 266, 43}

$$-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^5*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*C
- 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a +
b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^4(-5aB-(Ab+6aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4(Ab+6aC))\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(2(Ab+6aC))\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,d\right.}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(2(Ab+6aC))\,\mathrm{Subst}\left(\int \left(-\frac{a}{b(a+bx)^5}\right.\right.}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{4(Ab+6aC)}{105b^4\left(a+bx^2\right)^{3/2}} - \frac{4(Ab+6aC}{35ab^4\sqrt{a+b}}$$

Mathematica [A] time = 0.11, size = 89, normalized size = 0.67

$$\frac{-48a^4C-8a^3b\left(A+21Cx^2\right)-14a^2b^2x^2\left(2A+15Cx^2\right)-35ab^3x^4\left(A+3Cx^2\right)+15b^4Bx^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2*
A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 1.21, size = 98, normalized size = 0.74

$$\frac{-48a^4C-8a^3Ab-168a^3bCx^2-28a^2Ab^2x^2-210a^2b^2Cx^4-35aAb^3x^4-105ab^3Cx^6+15b^4Bx^7}{105ab^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^3*A*b - 48*a^4*C - 28*a^2*A*b^2*x^2 - 168*a^3*b*C*x^2 - 35*a*A*b^3*x^
4 - 210*a^2*b^2*C*x^4 - 105*a*b^3*C*x^6 + 15*b^4*B*x^7)/(105*a*b^4*(a + b*x
^2)^(7/2))
```

fricas [A] time = 0.68, size = 137, normalized size = 1.04

$$\frac{\left(15\,Bb^4x^7-105\,Cab^3x^6-48\,Ca^4-8\,Aa^3b-35\left(6\,Ca^2b^2+Aab^3\right)x^4-28\left(6\,Ca^3b+Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(ab^8x^8+4\,a^2b^7x^6+6\,a^3b^6x^4+4\,a^4b^5x^2+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2*
b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

giac [A] time = 0.62, size = 112, normalized size = 0.85

$$\frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7\,C}{b}\right)x^2-\frac{7\left(6\,Ca^4b^2+Aa^3b^3\right)}{a^3b^4}\right)x^2-\frac{28\left(6\,Ca^5b+Aa^4b^2\right)}{a^3b^4}\right)x^2-\frac{8\left(6\,Ca^6+Aa^5b\right)}{a^3b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*x
^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a^3
*b^4))/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.00, size = 95, normalized size = 0.72

$$-\frac{-15B\,x^7b^4+105C\,x^6a\,b^3+35Aa\,b^3x^4+210C\,a^2b^2x^4+28A\,a^2b^2x^2+168C\,a^3b\,x^2+8A\,a^3b+48C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*A
*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

maxima [B] time = 1.44, size = 240, normalized size = 1.82

$$-\frac{Cx^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,Cax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Bax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,Ca^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{4\,Aax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{Bx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Bx}{7\sqrt{bx^2+a}\,ab^3}+\frac{3\,Bax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,Ba^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{16\,Ca^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}-\frac{8\,Aa^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4/
((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b
*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*x^2
/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt
(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b
*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((
b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 1.27, size = 196, normalized size = 1.48

$$\frac{\frac{a\left(\frac{C}{3\,b^3}-\frac{7\,A\,b-14\,C\,a}{21\,a\,b^3}\right)}{b}-\frac{3\,B\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{a^2\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b^2}+\frac{B\,a^2\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{C}{b^4}-\frac{B\,x}{7\,a\,b^3}}{\sqrt{b\,x^2+a}}-\frac{\frac{a\left(\frac{7\,C\,a^2-7\,A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)}{b}-\frac{3\,B\,a\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b*
x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a +
 b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*a^2
 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)))/b
))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=149

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 819, 778, 191}

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^4*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2*A
*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a +
b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^3(-4aB-(2Ab+5aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x\left(-8a^2B-3a(2Ab+5aC)x\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2Ab}{\ldots}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2Ab}{35a^2b}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.52

$$\frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+3ab^3x^5\left(7A+5Cx^2\right)+6Ab^4x^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 6*A*b^4*x^7 + 3*a*b^3*x^5*(
7*A + 5*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.93, size = 79, normalized size = 0.53

$$\frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+21aAb^3x^5+15ab^3Cx^7+6Ab^4x^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 21*a*A*b^3*x^5 + 6*A*b^4*x^
7 + 15*a*b^3*C*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.93, size = 122, normalized size = 0.82

$$\frac{\left(21\,Aab^3x^5-35\,Ba^2b^2x^4+3\left(5\,Cab^3+2\,Ab^4\right)x^7-28\,Ba^3bx^2-8\,Ba^4\right)\sqrt{bx^2+a}}{105\left(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 28
*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^
4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

giac [A] time = 0.58, size = 81, normalized size = 0.54

$$\frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{\left(5\,Ca^2b^3+2\,Aab^4\right)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x^
2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 76, normalized size = 0.51

$$\frac{6A\,b^4x^7+15Ca\,b^3x^7+21A\,x^5a\,b^3-35B\,a^2b^2x^4-28B\,a^3b\,x^2-8B\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3*
b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

maxima [A] time = 1.42, size = 253, normalized size = 1.70

$$-\frac{Cx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Cax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,Bax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{Cx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Cx}{7\sqrt{bx^2+a}\,ab^3}+\frac{3\,Cax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,Ca^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{3\,Ax}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Ax}{35\sqrt{bx^2+a}\,a^2b^2}+\frac{Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Aax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,Ba^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C*
a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(s
qrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/
((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqr
t(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b
*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 1.19, size = 186, normalized size = 1.25

$$\frac{x\left(\frac{C\,a^2-A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^3}}{\left(b\,x^2+a\right)^{5/2}}-\frac{\frac{B}{3\,b^3}+x\left(\frac{C}{3\,b^3}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^3}\right)}{\left(b\,x^2+a\right)^{3/2}}-\frac{\frac{B\,a^2}{7\,b^3}-\frac{a\,x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}}{\left(b\,x^2+a\right)^{7/2}}+\frac{x\,\left(2\,A\,b+5\,C\,a\right)}{35\,a^2\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*
b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3)
- (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) - (a*
x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))/(35
*a^2*b^3*(a + b*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 819, 639, 191}

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^3*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A*b
 + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x)/
(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2(-3aB-(3Ab+4aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-3a^2B-2a(3Ab+4aC)x}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int}{3}$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{2}{35a^2b^2}$$

Mathematica [A] time = 0.13, size = 84, normalized size = 0.60

$$\frac{-8a^4C-2a^3b\left(3A+14Cx^2\right)-7a^2b^2x^2\left(3A+5Cx^2\right)+21ab^3Bx^5+6b^4Bx^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 1.15, size = 88, normalized size = 0.63

$$\frac{-8a^4C-6a^3Ab-28a^3bCx^2-21a^2Ab^2x^2-35a^2b^2Cx^4+21ab^3Bx^5+6b^4Bx^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^3*A*b - 8*a^4*C - 21*a^2*A*b^2*x^2 - 28*a^3*b*C*x^2 - 35*a^2*b^2*C*x^
4 + 21*a*b^3*B*x^5 + 6*b^4*B*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.01, size = 131, normalized size = 0.94

$$\frac{\left(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\left(4\,Ca^3b+3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3*
b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b
^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

giac [A] time = 0.59, size = 95, normalized size = 0.68

$$\frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\left(4\,Ca^4b+3\,Aa^3b^2\right)}{a^3b^3}\right)x^2-\frac{2\left(4\,Ca^5+3\,Aa^4b\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^3
*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 85, normalized size = 0.61

$$-\frac{-6B\,x^7b^4-21B\,x^5a\,b^3+35C\,a^2b^2x^4+21A\,a^2b^2x^2+28C\,a^3b\,x^2+6A\,a^3b+8C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C*
a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

maxima [A] time = 1.38, size = 179, normalized size = 1.29

$$-\frac{Cx^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,Cax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Bx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Bx}{35\sqrt{bx^2+a}\,a^2b^2}+\frac{Bx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Bax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,Ca^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{2\,Aa}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*C
*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2
/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 1.14, size = 133, normalized size = 0.96

$$\frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{\left(b\,x^2+a\right)^{3/2}}+\frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{\left(b\,x^2+a\right)^{5/2}}+\frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C/
(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b - 7
*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35*a^
2*b^2*(a + b*x^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} \, dx$

Optimal. Leaf size=139

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1804, 778, 192, 191}

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x^2*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b +
3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2*(
a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x(-2aB-(4Ab+3aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{(2(4Ab+3\ldots}{1\ldots}$$

$$= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{2(4Ab+3\ldots}{105a^3b^2\sqrt{a\ldots}}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.63

$$\frac{-6a^4B-21a^3bBx^2+7a^2b^2x^3\left(5A+3Cx^2\right)+2ab^3x^5\left(14A+3Cx^2\right)+8Ab^4x^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.85, size = 91, normalized size = 0.65

$$\frac{-6a^4B-21a^3bBx^2+35a^2Ab^2x^3+21a^2b^2Cx^5+28aAb^3x^5+6ab^3Cx^7+8Ab^4x^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 35*a^2*A*b^2*x^3 + 28*a*A*b^3*x^5 + 21*a^2*b^2
*C*x^5 + 8*A*b^4*x^7 + 6*a*b^3*C*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.99, size = 134, normalized size = 0.96

$$\frac{\left(35\,Aa^2b^2x^3+2\left(3\,Cab^3+4\,Ab^4\right)x^7-21\,Ba^3bx^2+7\left(3\,Ca^2b^2+4\,Aab^3\right)x^5-6\,Ba^4\right)\sqrt{bx^2+a}}{105\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7*
(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

giac [A] time = 0.52, size = 94, normalized size = 0.68

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,Cab^4+4\,Ab^5\right)x^2}{a^3b^3}+\frac{7\left(3\,Ca^2b^3+4\,Aab^4\right)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A*
a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 88, normalized size = 0.63

$$\frac{8A\,b^4x^7+6Ca\,b^3x^7+28A\,x^5a\,b^3+21C\,a^2b^2x^5+35A\,x^3a^2b^2-21B\,a^3b\,x^2-6B\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*b
^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

maxima [A] time = 1.42, size = 197, normalized size = 1.42

$$-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,Cx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Ax}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Ax}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{2\,Ba}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x
/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/(
(b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^
2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 +
 a)^(7/2)*b^2)
```

mupad [B] time = 1.09, size = 133, normalized size = 0.96

$$\frac{x\,\left(4\,A\,b+3\,C\,a\right)}{105\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{B}{5\,b^2}+x\,\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{{\left(b\,x^2+a\right)}^{5/2}}-\frac{x\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(8\,A\,b+6\,C\,a\right)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5*
b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(7*
b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*b^2
*(a + b*x^2)^(1/2))
```

sympy [B] time = 118.65, size = 904, normalized size = 6.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
```

```
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 1
05*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*
b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105
*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b*
*5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + C*(7*a*x
**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*
b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*
a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

3.53 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=119

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1804, 639, 192, 191}

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -(x*(a*B - (A*b - a*C)*x))/(7*a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b*B
*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2)) +
(8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-aB-(5Ab+2aC)x}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{(4B)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}} + \frac{(8B)\int \frac{1}{\left(a+bx^2\right)^{3/2}}}{105a^2b}$$

$$= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}} + \frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}} + \frac{8Bx}{105a^3b\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.63

$$\frac{-6a^4C-3a^3b\left(5A+7Cx^2\right)+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*C + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5 + 8*b^4*B*x^7 - 3*a^3*b*(5*A
+ 7*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 1.03, size = 76, normalized size = 0.64

$$\frac{-6a^4C-15a^3Ab-21a^3bCx^2+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^3*A*b - 6*a^4*C - 21*a^3*b*C*x^2 + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5
 + 8*b^4*B*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.77, size = 119, normalized size = 1.00

$$\frac{\left(8\,Bb^{4}x^{7}+28\,Bab^{3}x^{5}+35\,Ba^{2}b^{2}x^{3}-21\,Ca^{3}bx^{2}-6\,Ca^{4}-15\,Aa^{3}b\right)\sqrt{bx^{2}+a}}{105\left(a^{3}b^{6}x^{8}+4\,a^{4}b^{5}x^{6}+6\,a^{5}b^{4}x^{4}+4\,a^{6}b^{3}x^{2}+a^{7}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 - 6
*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b
^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

giac [A] time = 0.60, size = 82, normalized size = 0.69

$$\frac{\left(\left(4\left(\frac{2\,Bb^{2}x^{2}}{a^{3}}+\frac{7\,Bb}{a^{2}}\right)x^{2}+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^{2}-\frac{3\left(2\,Ca^{4}b+5\,Aa^{3}b^{2}\right)}{a^{3}b^{3}}}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3*
(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.00, size = 73, normalized size = 0.61

$$-\frac{-8B\,x^{7}b^{4}-28B\,x^{5}a\,b^{3}-35B\,x^{3}a^{2}b^{2}+21C\,a^{3}b\,x^{2}+15A\,a^{3}b+6C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a^
3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

maxima [A] time = 1.34, size = 123, normalized size = 1.03

$$-\frac{Cx^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Bx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{2\,Ca}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{A}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*
x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/
((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x^2
+ a)^(7/2)*b)
```

mupad [B] time = 1.05, size = 99, normalized size = 0.83

$$\frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}-\frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{\left(b\,x^2+a\right)^{5/2}}+\frac{4\,B\,x}{105\,a^2\,b\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(7
*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) + (
4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 85.30, size = 796, normalized size = 6.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x
**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1
+ b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*
a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqr
t(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19
/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
```

```
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**
2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*
a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 63
0*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 +
 b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) + C*Piecewise((-2
*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 1
05*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2
/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105
*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)),
(x**4/(4*a**(9/2)), True))
```

3.54 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}} dx$

Optimal. Leaf size=127

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*b - a*C)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*a^
2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2))
+ (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{b}}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{(4(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{(8(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)}}{105a^3b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 92, normalized size = 0.72

$$
\frac{-15a^4B+35a^3bx\left(3A+Cx^2\right)+14a^2b^2x^3\left(15A+2Cx^2\right)+8ab^3x^5\left(21A+Cx^2\right)+48Ab^4x^7}{105a^4b\left(a+bx^2\right)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.76, size = 99, normalized size = 0.78

$$
\frac{-15a^4B+105a^3Abx+35a^3bCx^3+210a^2Ab^2x^3+28a^2b^2Cx^5+168aAb^3x^5+8ab^3Cx^7+48Ab^4x^7}{105a^4b\left(a+bx^2\right)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 105*a^3*A*b*x + 210*a^2*A*b^2*x^3 + 35*a^3*b*C*x^3 + 168*a*A*b
^3*x^5 + 28*a^2*b^2*C*x^5 + 48*A*b^4*x^7 + 8*a*b^3*C*x^7)/(105*a^4*b*(a + b
*x^2)^(7/2))
```

fricas [A] time = 0.60, size = 137, normalized size = 1.08

$$\frac{\left(8\left(Cab^{3}+6\,Ab^{4}\right)x^{7}+105\,Aa^{3}bx+28\left(Ca^{2}b^{2}+6\,Aab^{3}\right)x^{5}-15\,Ba^{4}+35\left(Ca^{3}b+6\,Aa^{2}b^{2}\right)x^{3}\right)\sqrt{bx^{2}+a}}{105\left(a^{4}b^{5}x^{8}+4\,a^{5}b^{4}x^{6}+6\,a^{6}b^{3}x^{4}+4\,a^{7}b^{2}x^{2}+a^{8}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*b^
5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

giac [A] time = 0.50, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^{2}\left(\frac{2\left(Cab^{5}+6\,Ab^{6}\right)x^{2}}{a^{4}b^{3}}+\frac{7\left(Ca^{2}b^{4}+6\,Aab^{5}\right)}{a^{4}b^{3}}\right)+\frac{35\left(Ca^{3}b^{3}+6\,Aa^{2}b^{4}\right)}{a^{4}b^{3}}\right)x^{2}+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
- 15*B/b)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{48A\,b^{4}x^{7}+8Ca\,b^{3}x^{7}+168A\,x^{5}a\,b^{3}+28C\,a^{2}b^{2}x^{5}+210A\,x^{3}a^{2}b^{2}+35C\,a^{3}b\,x^{3}+105Ax\,a^{3}b-15B\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a^
2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

maxima [A] time = 1.33, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\,\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Cx}{105\,\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Cx}{35\,\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{B}{7\,\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^
2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

mupad [B] time = 1.03, size = 115, normalized size = 0.91

$$\frac{x\,\left(6\,A\,b+C\,a\right)}{35\,a^2\,b\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{7\,b}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(24\,A\,b+4\,C\,a\right)}{105\,a^3\,b\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{x\,\left(48\,A\,b+8\,C\,a\right)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C/
(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(3/
2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

sympy [B] time = 94.22, size = 1880, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
```

```
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqr
t(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(
19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a*
*(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) +
 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x
**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**
2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)))
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 823, 12, 266, 63, 208}

$$\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-6Bx}{x\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35aAb+24abBx}{x\left(a+bx^2\right)^{5/2}}\,dx}{35a^3b}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105a^2Ab^2-48a^2b^2Bx}{x\left(a+bx^2\right)^{3/2}}\,dx}{105a^5b^2}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{\int \frac{105a^3Ab^3}{x\sqrt{a+bx^2}}}{105a^7b^3}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\int \frac{1}{x\sqrt{a+bx}}}{a^4}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\Big(\int}{}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\mathrm{Subst}\Big(\int}{}$$

$$= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\Big(}{a^{9/2}}$$

Mathematica [A] time = 0.23, size = 120, normalized size = 0.87

$$\frac{-15a^4C+a^3b(176A+105Bx)+14a^2b^2x^2(29A+15Bx)+14ab^3x^4(25A+12Bx)+3b^4x^6(35A+16Bx)}{105a^4b\left(a+bx^2\right)^{7/2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
+ 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*x^
2)^(7/2)) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

<u>IntegrateAlgebraic</u> [A] time = 1.20, size = 146, normalized size = 1.06

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-15a^4C+176a^3Ab+105a^3bBx+406a^2Ab^2x^2+210a^2b^2Bx^3+350aAb^3x^4+168ab^3Bx^5+105Ab^4x^6+48b^4Bx^7}{105a^4b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (176*a^3*A*b - 15*a^4*C + 105*a^3*b*B*x + 406*a^2*A*b^2*x^2 + 210*a^2*b^2*B
*x^3 + 350*a*A*b^3*x^4 + 168*a*b^3*B*x^5 + 105*A*b^4*x^6 + 48*b^4*B*x^7)/(1
05*a^4*b*(a + b*x^2)^(7/2)) + (2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b
*x^2]/Sqrt[a]])/a^(9/2)
```

<u>fricas</u> [A] time = 0.81, size = 465, normalized size = 3.37

$$\left[\frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(48\,Bab^4x^7+105\,Aab^4x^6+168\,Ba^2b^3x^5+350\,Aa^2b^3x^4+210\,Ba^3b^2x^3+406\,Aa^3b^2x^2+105\,Ba^4bx-15\,Ca^5+176\,Aa^4b\right)\sqrt{bx^2+a}}{210\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4\,a^8b^2x^2+a^9b\right)},\ \frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(48\,Bab^4x^7+105\,Aab^4x^6+168\,Ba^2b^3x^5+350\,Aa^2b^3x^4+210\,Ba^3b^2x^3+406\,Aa^3b^2x^2+105\,Ba^4bx-15\,Ca^5+176\,Aa^4b\right)\sqrt{bx^2+a}}{105\left(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4\,a^8b^2x^2+a^9b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
+ A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 +
 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a
^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8
*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4
+ 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4
8*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 + 2
10*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a^4
*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b
^2*x^2 + a^9*b)]
```

<u>giac</u> [A] time = 0.57, size = 152, normalized size = 1.10

$$\frac{\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)x-\frac{15\,Ca^{14}b^2-176\,Aa^{13}b^3}{a^{14}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*b
^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2 -
 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x - s
qrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

maple [A] time = 0.01, size = 169, normalized size = 1.22

$$\frac{Bx}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}a}+\frac{A}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}a}+\frac{6Bx}{35\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{2}}-\frac{C}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}}b}+\frac{A}{5\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{8Bx}{35\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{A}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{16Bx}{35\sqrt{b\,x^{2}+a}\,a^{4}}-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{9}{2}}}+\frac{A}{\sqrt{b\,x^{2}+a}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*C/b/(b*x^2+a)^(7/2)+1/7*B*x/a/(b*x^2+a)^(7/2)+6/35*B/a^2*x/(b*x^2+a)^(
5/2)+8/35*B/a^3*x/(b*x^2+a)^(3/2)+16/35*B/a^4*x/(b*x^2+a)^(1/2)+1/7*A/a/(b*
x^2+a)^(7/2)+1/5*A/a^2/(b*x^2+a)^(5/2)+1/3*A/a^3/(b*x^2+a)^(3/2)+A/a^4/(b*x
^2+a)^(1/2)-A/a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.45, size = 157, normalized size = 1.14

$$\frac{16\,Bx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Bx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{6\,Bx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{Bx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a^{4}}+\frac{A}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{A}{5\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{A}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}-\frac{C}{7\left(bx^{2}+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)^(3
/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a) - 1/
7*C/((b*x^2 + a)^(7/2)*b)
```

mupad [B] time = 1.62, size = 159, normalized size = 1.15

$$\frac{\frac{A}{7\,a}+\frac{A\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{A\left(b\,x^2+a\right)^3}{a^4}+\frac{A\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{C}{7\,b\left(b\,x^2+a\right)^{7/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,B\,x}{35\,a^3\left(b\,x^2+a\right)^{3/2}}+\frac{6\,B\,x}{35\,a^2\left(b\,x^2+a\right)^{5/2}}+\frac{B\,x}{7\,a\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] (A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a +
```

```
b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*B
*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (B*x)
/(7*a*(a + b*x^2)^(7/2))
```

sympy [B] time = 107.41, size = 6613, normalized size = 47.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x*
*2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**
4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)
*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 945
0*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x*
*20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)
*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(
65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b*
*9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(21
0*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(
67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b*
*8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a*
*31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 472
5*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
```

```
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23
630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210
*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(6
7/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4
4100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**
8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a*
*29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a
) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**
5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a*
*(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
 + 31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/
a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 2520
0*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x*
*10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57
/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5
2920*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**
(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 4410
0*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x
```

```
**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55
/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1
 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*l
og(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
+ 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x
**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**
16*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100
*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b*
*6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a*
*(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log
(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(
69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5
2920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b*
*7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(
53/2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
```

```
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x*
*18*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10
*x**20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2
)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920
*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x
**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2
)*b**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + B*(35*a**14
*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/
a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*s
qrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27
/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2
/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*
x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*
a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*
x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x*
*2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*s
qrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29
/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2
/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35
*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 5
25*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1
+ b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b*
*5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**
2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**
(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x*
*2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**
12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/
a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)
*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
```

```
 + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a
**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*
x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x
**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) + C*
Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{B-x\left(\right.}{7a\left(a+\right.}$$

Rubi [A] time = 0.38, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1805, 807, 266, 63, 208}

$$\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+6\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-4\left(\frac{13Ab}{a}-6C\right)x^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+6\left(\frac{29Ab}{a}-8\right.}{x^2\left(a+bx^2\right)^{3/2}}}{105a^3}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

$$= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.22, size = 158, normalized size = 0.84

$$\frac{a^4(x(176B+105Cx)-105A)+14a^3bx^2(x(29B+15Cx)-60A)+14a^2b^2x^4(x(25B+12Cx)-120A)+3ab^3x^6(x(35B+16Cx)-448A)-105\sqrt{a}\,Bx\left(a+bx^2\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x^2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x)) + a^4*(-105*A + x*(176*B + 105*C*x)) - 105*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2))

IntegrateAlgebraic [A] time = 1.15, size = 190, normalized size = 1.01

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{-105a^4A+176a^4Bx+105a^4Cx^2-840a^3Abx^2+406a^3bBx^3+210a^3bCx^4-1680a^2Ab^2x^4+350a^2b^2Bx^5+168a^2b^2Cx^6-1344aAb^3x^6+105ab^3Bx^7+48ab^3Cx^8-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]

[Out] (-105*a^4*A + 176*a^4*B*x - 840*a^3*A*b*x^2 + 105*a^4*C*x^2 + 406*a^3*b*B*x^3 - 1680*a^2*A*b^2*x^4 + 210*a^3*b*C*x^4 + 350*a^2*b^2*B*x^5 - 1344*a*A*b^3*x^6 + 168*a^2*b^2*C*x^6 + 105*a*b^3*B*x^7 - 384*A*b^4*x^8 + 48*a*b^3*C*x^8)/(105*a^5*x*(a + b*x^2)^(7/2)) + (2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a] - Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)

fricas [A] time = 0.71, size = 525, normalized size = 2.79

$$\left[\frac{105\left(Bb^4x^9+4Bab^3x^7+6Ba^2b^2x^5+4Ba^3bx^3+Ba^4x\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(105Bab^3x^7+350Ba^2b^2x^5+48\left(Cab^3-8Ab^4\right)x^8+406Ba^3bx^3+168\left(Ca^2b^2-8Aab^3\right)x^6+176Ba^4x-105Aa^4+210\left(Ca^3b-8Aa^2b^2\right)x^4+105\left(Ca^4-8Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{210\left(a^5b^4x^9+4a^6b^3x^7+6a^7b^2x^5+4a^8bx^3+a^9x\right)},\ \frac{105\left(Bb^4x^9+4Bab^3x^7+6Ba^2b^2x^5+4Ba^3bx^3+Ba^4x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(105Bab^3x^7+350Ba^2b^2x^5+48\left(Cab^3-8Ab^4\right)x^8+406Ba^3bx^3+168\left(Ca^2b^2-8Aab^3\right)x^6+176Ba^4x-105Aa^4+210\left(Ca^3b-8Aa^2b^2\right)x^4+105\left(Ca^4-8Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^5b^4x^9+4a^6b^3x^7+6a^7b^2x^5+4a^8bx^3+a^9x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")

[Out] [1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]

giac [A] time = 0.51, size = 239, normalized size = 1.27

$$\frac{\left(\left(\left(\left(3\left(x\left(\frac{35Bb^3}{a^4}+\frac{\left(16Ca^{20}b^6-93Aa^{19}b^7\right)x}{a^{24}b^3}\right)+\frac{28\left(2Ca^{21}b^5-11Aa^{20}b^6\right)}{a^{24}b^3}\right)x+\frac{350Bb^2}{a^3}\right)x+\frac{210\left(Ca^{22}b^4-5Aa^{21}b^5\right)}{a^{24}b^3}\right)x+\frac{406Bb}{a^2}\right)x+\frac{105\left(Ca^{23}b^3-4Aa^{22}b^4\right)}{a^{24}b^3}\right)x+\frac{176B}{a}}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{2A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^3
)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x + 2
10*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a^23
*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arcta
n(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 240, normalized size = 1.28

$$-\frac{8Abx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}+\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{48Abx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}+\frac{B}{7\left(bx^2+a\right)^{\frac{7}{2}}a}+\frac{6Cx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}-\frac{A}{\left(bx^2+a\right)^{\frac{7}{2}}ax}-\frac{64Abx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}+\frac{B}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{8Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}-\frac{128Abx}{35\sqrt{bx^2+a}\,a^5}+\frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{16Cx}{35\sqrt{bx^2+a}\,a^4}-\frac{B\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{9}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*C*x/a/(b*x^2+a)^(7/2)+6/35*C/a^2*x/(b*x^2+a)^(5/2)+8/35*C/a^3*x/(b*x^2+
a)^(3/2)+16/35*C/a^4*x/(b*x^2+a)^(1/2)-A/a/x/(b*x^2+a)^(7/2)-8/7*A/a^2*b*x/
(b*x^2+a)^(7/2)-48/35*A/a^3*b*x/(b*x^2+a)^(5/2)-64/35*A/a^4*b*x/(b*x^2+a)^(
3/2)-128/35*A/a^5*b*x/(b*x^2+a)^(1/2)+1/7*B/a/(b*x^2+a)^(7/2)+1/5*B/a^2/(b*
x^2+a)^(5/2)+1/3*B/a^3/(b*x^2+a)^(3/2)+B/a^4/(b*x^2+a)^(1/2)-B/a^(9/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.41, size = 228, normalized size = 1.21

$$\frac{16\,Cx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{128\,Abx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Abx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,Abx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,Abx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^4}+\frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{B}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{B}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{A}{\left(bx^2+a\right)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a)^(3/
2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a) - A/(
(b*x^2 + a)^(7/2)*a*x)
```

mupad [B] time = 2.10, size = 225, normalized size = 1.20

$$\frac{\frac{B}{7\,a}+\frac{B\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{B\left(b\,x^2+a\right)^3}{a^4}+\frac{B\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\left(b\,x^2+a\right)^{3/2}}+\frac{6\,C\,x}{35\,a^2\left(b\,x^2+a\right)^{5/2}}+\frac{C\,x}{7\,a\left(b\,x^2+a\right)^{7/2}}-\frac{29\,A\,b\,x}{35\,a^4\left(b\,x^2+a\right)^{3/2}}-\frac{13\,A\,b\,x}{35\,a^3\left(b\,x^2+a\right)^{5/2}}-\frac{A\,b\,x}{7\,a^2\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] (B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a + b
*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(35*
a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(35*a
^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35*a^4*
(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/(7*a^2
*(a + b*x^2)^(7/2))
```

sympy [B] time = 165.70, size = 6922, normalized size = 36.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(352*a*
*32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 29
24*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
+ 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a**(73/
2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**
```

```
3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**
(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
+ 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a**30*b
**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630*a**29
*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a**(73/2
) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3
*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(
61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29*b**3*
x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)/(210*
a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67
/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44
100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8
*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 22050*a**
28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a*
*(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 +
 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*
b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a*
*(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
```

```
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 31442*
a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/(210*a
**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 52920*a**2
7*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b
**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*
a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**1
8 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 + b*x**2
/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) -
25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
```

4 + 25200*a(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
9*x18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sqrt(1 +
 b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**
2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(
63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14
+ 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**
10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
(71/2)*b*x2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18*log(b
*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
5*x10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
6*x12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
(55/2)*b9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x**20*lo
g(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x*
*4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2
)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 94
50*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x
20) - 210*a22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + C*(35*a**14*x/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b
*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*
x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 17
5*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)
*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a)
+ 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqr
t(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 2
10*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b

```
*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*
x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*
a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2
)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*
sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 286*a**1
0*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b*
*3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 2
10*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*
a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**
8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**
(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)))
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=219

$$\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2\ldots}{3\ldots}$$

Rubi [A] time = 0.48, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1805, 1807, 807, 266, 63, 208}

$$-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}+\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(a*((A*b)/a - C) + b*B*x)/(7*a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) + 1
3*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*a^
4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x^2
]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*A*
b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+7\left(\frac{Ab}{a}-C\right)x^2+\frac{6bBx^3}{a}}{x^3\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-35\left(\frac{2Ab}{a}-C\right)x^2-\frac{52bBx^3}{a}}{x^3\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+1\cdots}{x^3(\cdots}}{\cdots}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+\cdots}{35a^5\sqrt{a+b\cdots}}$$

Mathematica [A] time = 0.53, size = 178, normalized size = 0.81

$$\frac{\frac{a^5(-105A-210Bx+352Cx^2)}{x^2}-4a^4b(396A+7x(60B-29Cx))+14a^3b^2x^2(10x(5Cx-24B)-261A)+42a^2b^3x^4(x(5Cx-64B)-75A)-3ab^4x^6(315A+256Bx)+\frac{105(a+bx^2)^4(9Ab-2aC)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}}{210a^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3*a*b^4*x^6*(315*A + 256*B*x) + (a^5*(-105*A - 210*B*x + 352*C*x^2))/x^2
- 4*a^4*b*(396*A + 7*x*(60*B - 29*C*x)) + 42*a^2*b^3*x^4*(-75*A + x*(-64*B
+ 5*C*x)) + 14*a^3*b^2*x^2*(-261*A + 10*x*(-24*B + 5*C*x)) + (105*(9*A*b -
2*a*C)*(a + b*x^2)^4*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a])/(21
0*a^6*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 1.57, size = 202, normalized size = 0.92

$$\frac{(2aC-9Ab)\tanh^{-1}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{11/2}}+\frac{-105a^4A-210a^4Bx+352a^4Cx^2-1584a^3Abx^2-1680a^3bBx^3+812a^3bCx^4-3654a^2Ab^2x^4-3360a^2b^2Bx^5+700a^2b^2Cx^6-3150aAb^3x^6-2688ab^3Bx^7+210ab^3Cx^8-945Ab^4x^8-768b^4Bx^9}{210a^5x^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-105*a^4*A - 210*a^4*B*x - 1584*a^3*A*b*x^2 + 352*a^4*C*x^2 - 1680*a^3*b*B
*x^3 - 3654*a^2*A*b^2*x^4 + 812*a^3*b*C*x^4 - 3360*a^2*b^2*B*x^5 - 3150*a*A
*b^3*x^6 + 700*a^2*b^2*C*x^6 - 2688*a*b^3*B*x^7 - 945*A*b^4*x^8 + 210*a*b^3
*C*x^8 - 768*b^4*B*x^9)/(210*a^5*x^2*(a + b*x^2)^(7/2)) + ((-9*A*b + 2*a*C)
*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(11/2)
```

fricas [A] time = 1.17, size = 688, normalized size = 3.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
+ 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*
C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1
680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*
C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^
4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b
^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*a*b^4 -
9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*
```

```
b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688*B*a^2*b^3*
x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)
*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*
(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 +
 a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2
)]
```

giac [A] time = 0.48, size = 325, normalized size = 1.48

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\left(Ca^{24}b^6-4\,Aa^{23}b^7\right)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\left(10\,Ca^{25}b^5-39\,Aa^{24}b^6\right)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\left(29\,Ca^{26}b^4-108\,Aa^{25}b^5\right)}{a^{28}b^3}\right)x+\frac{420\,Bb}{a^2}\right)x-\frac{2\left(88\,Ca^{27}b^3-291\,Aa^{26}b^4\right)}{a^{28}b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{(2\,Ca-9\,Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^5}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3))
*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x +
1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x + 420
*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7
/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt
(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

maple [A] time = 0.02, size = 288, normalized size = 1.32

$$-\frac{8Bbx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{9Ab}{14\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{48Bbx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}+\frac{C}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{9Ab}{10\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{B}{\left(bx^2+a\right)^{\frac{7}{2}}ax}-\frac{64Bbx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}+\frac{C}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2}-\frac{A}{2\left(bx^2+a\right)^{\frac{7}{2}}ax^2}-\frac{3Ab}{2\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{128Bbx}{35\sqrt{bx^2+a}\,a^5}+\frac{C}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{9Ab\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{11}{2}}}-\frac{C\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{9}{2}}}-\frac{9Ab}{2\sqrt{bx^2+a}\,a^5}+\frac{C}{\sqrt{bx^2+a}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(7/2)-9/14*A/a^2*b/(b*x^2+a)^(7/2)-9/10*A/a^3*b/(b*x
^2+a)^(5/2)-3/2*A/a^4*b/(b*x^2+a)^(3/2)-9/2*A/a^5*b/(b*x^2+a)^(1/2)+9/2*A/a
^(11/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-B/a/x/(b*x^2+a)^(7/2)-8/7*B
/a^2*b*x/(b*x^2+a)^(7/2)-48/35*B/a^3*b*x/(b*x^2+a)^(5/2)-64/35*B/a^4*b*x/(b
*x^2+a)^(3/2)-128/35*B/a^5*b*x/(b*x^2+a)^(1/2)+1/7*C/a/(b*x^2+a)^(7/2)+1/5*
C/a^2/(b*x^2+a)^(5/2)+1/3*C/a^3/(b*x^2+a)^(3/2)+C/a^4/(b*x^2+a)^(1/2)-C/a^(
9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.49, size = 265, normalized size = 1.21

$$-\frac{128\,Bbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Bbx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,Bbx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,Bbx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}}+\frac{9\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{11}{2}}}+\frac{C}{\sqrt{bx^2+a}\,a^4}+\frac{C}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{C}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{C}{7\left(bx^2+a\right)^{\frac{7}{2}}a}-\frac{9\,Ab}{2\,\sqrt{bx^2+a}\,a^5}-\frac{3\,Ab}{2\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{9\,Ab}{10\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{9\,Ab}{14\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{B}{\left(bx^2+a\right)^{\frac{7}{2}}ax}-\frac{A}{2\left(bx^2+a\right)^{\frac{7}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) -
C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*abs(
x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3) + 1
/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(sqrt(
b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2 + a)^(
5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*a*x) -
1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

mupad [**B**] time = 2.52, size = 279, normalized size = 1.27

$$\frac{\frac{C}{7a}+\frac{C\left(bx^2+a\right)^2}{3a^3}+\frac{C\left(bx^2+a\right)^3}{a^4}+\frac{C\left(bx^2+a\right)}{5a^2}}{\left(bx^2+a\right)^{7/2}}-\frac{\frac{Ab}{7a}+\frac{9Ab\left(bx^2+a\right)}{35a^2}+\frac{3Ab\left(bx^2+a\right)^2}{5a^3}+\frac{3Ab\left(bx^2+a\right)^3}{a^4}-\frac{9Ab\left(bx^2+a\right)^4}{2a^5}}{a\left(bx^2+a\right)^{7/2}-\left(bx^2+a\right)^{9/2}}-\frac{\frac{B}{a^4}+\frac{128Bbx^2}{35a^5}}{x\sqrt{bx^2+a}}-\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{9Ab\,\mathrm{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{29Bbx}{35a^4\left(bx^2+a\right)^{3/2}}-\frac{13Bbx}{35a^3\left(bx^2+a\right)^{5/2}}-\frac{Bbx}{7a^2\left(bx^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a^2
) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(a +
 b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4 + (1
28*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/2)/a^
(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (
29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2)^(5/2)
) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.58 $\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right) dx &= \int \left(aAx^3+aBx^4+(Ab+aC)x^5+(bB+aD)x^6+bCx^7+bDx^8\right) dx \\ &= \frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bD\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.37, size = 57, normalized size = 0.88

$$\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,Dax^7+\frac{1}{7}\,Bbx^7+\frac{1}{6}\,Cax^6+\frac{1}{6}\,Abx^6+\frac{1}{5}\,Bax^5+\frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*A
*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^9}{9}+\frac{Cb\,x^8}{8}+\frac{Ba\,x^5}{5}+\frac{(bB+aD)\,x^7}{7}+\frac{Aa\,x^4}{4}+\frac{(Ab+aC)\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

maxima [A] time = 1.34, size = 53, normalized size = 0.82

$$\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7+\frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
A*b)*x^6 + 1/4*A*a*x^4
```

mupad [B] time = 1.20, size = 57, normalized size = 0.88

$$\frac{a\,x^7\,D}{7}+\frac{b\,x^9\,D}{9}+\frac{A\,a\,x^4}{4}+\frac{B\,a\,x^5}{5}+\frac{A\,b\,x^6}{6}+\frac{C\,a\,x^6}{6}+\frac{B\,b\,x^7}{7}+\frac{C\,b\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a*
x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

sympy [A] time = 0.10, size = 60, normalized size = 0.92

$$\frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9}+x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
x**6*(A*b/6 + C*a/6)
```

3.59 $\int x^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(aAx^2+aBx^3+(Ab+aC)x^4+(bB+aD)x^5+bCx^6+bDx^7\right) d$$

$$= \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}b$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right) \left(A + Bx + Cx^2 + Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.38, size = 57, normalized size = 0.88

$$\frac{1}{8} Dbx^8 + \frac{1}{7} Cbx^7 + \frac{1}{6} Dax^6 + \frac{1}{6} Bbx^6 + \frac{1}{5} Cax^5 + \frac{1}{5} Abx^5 + \frac{1}{4} Bax^4 + \frac{1}{3} Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A
*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^8}{8} + \frac{Cb\,x^7}{7} + \frac{Ba\,x^4}{4} + \frac{(bB + aD)\,x^6}{6} + \frac{Aa\,x^3}{3} + \frac{(Ab + aC)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
[Out] 1/3*A*a*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

maxima [A] time = 1.33, size = 53, normalized size = 0.82

$$\frac{1}{8} Dbx^8 + \frac{1}{7} Cbx^7 + \frac{1}{6} (Da + Bb)x^6 + \frac{1}{4} Bax^4 + \frac{1}{5} (Ca + Ab)x^5 + \frac{1}{3} Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

mupad [B] time = 1.18, size = 57, normalized size = 0.88

$$\frac{a\,x^6\,D}{6}+\frac{b\,x^8\,D}{8}+\frac{A\,a\,x^3}{3}+\frac{B\,a\,x^4}{4}+\frac{A\,b\,x^5}{5}+\frac{C\,a\,x^5}{5}+\frac{B\,b\,x^6}{6}+\frac{C\,b\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a*
x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

sympy [A] time = 0.09, size = 60, normalized size = 0.92

$$\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Cbx^7}{7}+\frac{Dbx^8}{8}+x^6\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
x**5*(A*b/5 + C*a/5)
```

3.60 $\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1802}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+(bB+aD)x^4+bCx^5+bDx^6\right)\,dx\\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.32, size = 57, normalized size = 0.88

$$\frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,Dax^5+\frac{1}{5}\,Bbx^5+\frac{1}{4}\,Cax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Bax^3+\frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*A
*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{Db\,x^7}{7}+\frac{Cb\,x^6}{6}+\frac{Ba\,x^3}{3}+\frac{(bB+aD)\,x^5}{5}+\frac{Aa\,x^2}{2}+\frac{(Ab+aC)\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

maxima [A] time = 1.31, size = 53, normalized size = 0.82

$$\frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
A*b)*x^4 + 1/2*A*a*x^2
```

mupad [B] time = 1.19, size = 57, normalized size = 0.88

$$\frac{a\,x^5\,D}{5}+\frac{b\,x^7\,D}{7}+\frac{A\,a\,x^2}{2}+\frac{B\,a\,x^3}{3}+\frac{A\,b\,x^4}{4}+\frac{C\,a\,x^4}{4}+\frac{B\,b\,x^5}{5}+\frac{C\,b\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a*
x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

sympy [A] time = 0.14, size = 60, normalized size = 0.92

$$\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Cbx^6}{6}+\frac{Dbx^7}{7}+x^5\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
x**4*(A*b/4 + C*a/4)
```

3.61 $\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1810}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aA+aBx+(Ab+aC)x^2+(bB+aD)x^3+bCx^4+bDx^5\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right)\left(A + Bx + Cx^2 + Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 54, normalized size = 0.90

$$\frac{1}{6}\,Dbx^6 + \frac{1}{5}\,Cbx^5 + \frac{1}{4}\,Dax^4 + \frac{1}{4}\,Bbx^4 + \frac{1}{3}\,Cax^3 + \frac{1}{3}\,Abx^3 + \frac{1}{2}\,Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A
*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

maple [A] time = 0.00, size = 51, normalized size = 0.85

$$\frac{Db\,x^6}{6} + \frac{Cb\,x^5}{5} + \frac{Ba\,x^2}{2} + \frac{(bB + aD)\,x^4}{4} + Aax + \frac{(Ab + aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] A*a*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

maxima [A] time = 1.32, size = 50, normalized size = 0.83

$$\frac{1}{6}\,Dbx^6 + \frac{1}{5}\,Cbx^5 + \frac{1}{4}\,(Da + Bb)x^4 + \frac{1}{2}\,Bax^2 + \frac{1}{3}\,(Ca + Ab)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

mupad [B] time = 1.16, size = 54, normalized size = 0.90

$$\frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/3
 + (B*b*x^4)/4 + (C*b*x^5)/5
```

sympy [A] time = 0.12, size = 56, normalized size = 0.93

$$Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3*
(A*b/3 + C*a/3)
```

3.62 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} dx$

Optimal. Leaf size=56

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} dx &= \int \left(aB + \frac{aA}{x} + (Ab+aC)x + (bB+aD)x^2 + bCx^3 + bDx^4\right) dx \\ &= aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5 + aA\log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 1.00

$$\frac{1}{2}x^2(aC + Ab) + aA\log(x) + \frac{1}{3}x^3(aD + bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x, x]
```

fricas [A] time = 0.82, size = 48, normalized size = 0.86

$$\frac{1}{5}Dbx^5+\frac{1}{4}Cbx^4+\frac{1}{3}(Da+Bb)x^3+Bax+\frac{1}{2}(Ca+Ab)x^2+Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

giac [A] time = 0.41, size = 53, normalized size = 0.95

$$\frac{1}{5}Dbx^5+\frac{1}{4}Cbx^4+\frac{1}{3}Dax^3+\frac{1}{3}Bbx^3+\frac{1}{2}Cax^2+\frac{1}{2}Abx^2+Bax+Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A
*b*x^2 + B*a*x + A*a*log(abs(x))
```

maple [A] time = 0.00, size = 53, normalized size = 0.95

$$\frac{Db\,x^5}{5}+\frac{Cb\,x^4}{4}+\frac{Bb\,x^3}{3}+\frac{Da\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Aa\ln(x)+Bax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/5*b*D*x^5+1/4*b*C*x^4+1/3*B*b*x^3+1/3*D*x^3*a+1/2*A*x^2*b+1/2*C*x^2*a+a*B
*x+a*A*ln(x)
```

maxima [A] time = 1.35, size = 48, normalized size = 0.86

$$\frac{1}{5} Dbx^5 + \frac{1}{4} Cbx^4 + \frac{1}{3} (Da + Bb)x^3 + Bax + \frac{1}{2} (Ca + Ab)x^2 + Aa \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

mupad [B] time = 1.17, size = 52, normalized size = 0.93

$$\frac{a\, x^3\, D}{3} + \frac{b\, x^5\, D}{5} + B\, a\, x + \frac{A\, b\, x^2}{2} + \frac{C\, a\, x^2}{2} + \frac{B\, b\, x^3}{3} + \frac{C\, b\, x^4}{4} + A\, a\, \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/3
 + (C*b*x^4)/4 + A*a*log(x)
```

sympy [A] time = 0.32, size = 54, normalized size = 0.96

$$Aa \log{(x)} + Bax + \frac{Cbx^4}{4} + \frac{Dbx^5}{5} + x^3 \left(\frac{Bb}{3} + \frac{Da}{3}\right) + x^2 \left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2*
(A*b/2 + C*a/2)
```

3.63 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=54

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+(bB+aD)x+bCx^2+bDx^3\right)dx\\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2, x]
```

fricas [A] time = 0.81, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log(x)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*a
 + A*b)*x^2 - 12*A*a)/x
```

giac [A] time = 0.39, size = 50, normalized size = 0.93

$$\frac{1}{4}\,Dbx^4+\frac{1}{3}\,Cbx^3+\frac{1}{2}\,Dax^2+\frac{1}{2}\,Bbx^2+Cax+Abx+Ba\log(|x|)-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a
*log(abs(x)) - A*a/x
```

maple [A] time = 0.01, size = 50, normalized size = 0.93

$$\frac{Db\,x^4}{4}+\frac{Cb\,x^3}{3}+\frac{Bb\,x^2}{2}+\frac{Da\,x^2}{2}+Abx+Ba\ln(x)+Cax-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/4*b*D*x^4+1/3*b*C*x^3+1/2*B*x^2*b+1/2*D*x^2*a+A*b*x+a*C*x-a*A/x+a*B*ln(x)
```

maxima [A] time = 1.34, size = 48, normalized size = 0.89

$$\frac{1}{4}\,Dbx^{4} + \frac{1}{3}\,Cbx^{3} + \frac{1}{2}\,(Da + Bb)x^{2} + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)*
x - A*a/x
```

mupad [B] time = 1.14, size = 49, normalized size = 0.91

$$\frac{a\,x^2\,D}{2} + \frac{b\,x^4\,D}{4} + A\,b\,x + C\,a\,x - \frac{A\,a}{x} + \frac{B\,b\,x^2}{2} + \frac{C\,b\,x^3}{3} + B\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x^
3)/3 + B*a*log(x)
```

sympy [A] time = 0.28, size = 49, normalized size = 0.91

$$-\frac{Aa}{x} + Ba\log{(x)} + \frac{Cbx^{3}}{3} + \frac{Dbx^{4}}{4} + x^{2}\left(\frac{Bb}{2} + \frac{Da}{2}\right) + x\,(Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(A
*b + C*a)
```

3.64 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal. Leaf size=54

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \int \left(bB\left(1+\frac{aD}{bB}\right)+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+bCx+bDx^2\right)dx\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.94

$$\log(x)(aC+Ab)-\frac{a\left(A+2Bx-2Dx^3\right)}{2x^2}+\frac{1}{6}bx\left(6B+3Cx+2Dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
+ a*C)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3, x]
```

fricas [A] time = 0.62, size = 55, normalized size = 1.02

$$\frac{2\,Dbx^5+3\,Cbx^4+6\,(Da+Bb)x^3+6\,(Ca+Ab)x^2\log(x)-6\,Bax-3\,Aa}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x) -
 6*B*a*x - 3*A*a)/x^2
```

giac [A] time = 0.38, size = 48, normalized size = 0.89

$$\frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+Dax+Bbx+(Ca+Ab)\log(|x|)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(
2*B*a*x + A*a)/x^2
```

maple [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{Db\,x^3}{3}+\frac{Cb\,x^2}{2}+Ab\ln(x)+Bbx+Ca\ln(x)+Dax-\frac{Ba}{x}-\frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/3*b*D*x^3+1/2*b*C*x^2+B*b*x+a*D*x-1/2*a*A/x^2-a*B/x+A*ln(x)*b+C*ln(x)*a
```

maxima [A] time = 1.32, size = 48, normalized size = 0.89

$$\frac{1}{3}Dbx^3 + \frac{1}{2}Cbx^2 + (Da + Bb)x + (Ca + Ab)\log(x) - \frac{2Bax + Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*a
*x + A*a)/x^2
```

mupad [B] time = 1.14, size = 47, normalized size = 0.87

$$\frac{b\,x^3\,D}{3} + B\,b\,x - \frac{A\,a}{2\,x^2} - \frac{B\,a}{x} + \frac{C\,b\,x^2}{2} + A\,b\,\ln(x) + C\,a\,\ln(x) + a\,x\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
C*a*log(x) + a*x*D
```

sympy [A] time = 0.51, size = 51, normalized size = 0.94

$$\frac{Cbx^2}{2} + \frac{Dbx^3}{3} + x\,(Bb + Da) + (Ab + Ca)\log\,(x) + \frac{-Aa - 2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B*
a*x)/(2*x**2)
```

3.65 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1802}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= \int \left(bC+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB+aD}{x}+bDx\right)dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.02

$$\frac{-aC-Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 + (
b*B + a*D)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4, x]
```

fricas [A] time = 0.75, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log(x)-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
A*b)*x^2 - 2*A*a)/x^3
```

giac [A] time = 0.43, size = 50, normalized size = 0.93

$$\frac{1}{2}\,Dbx^2+Cbx+(Da+Bb)\log(|x|)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b
)*x^2 + 2*A*a)/x^3
```

maple [A] time = 0.00, size = 51, normalized size = 0.94

$$\frac{Db\,x^2}{2}+Bb\ln(x)+Cbx+Da\ln(x)-\frac{Ab}{x}-\frac{Ca}{x}-\frac{Ba}{2x^2}-\frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/2*b*D*x^2+b*C*x-1/3*A*a/x^3-1/2*B*a/x^2-1/x*A*b-1/x*a*C+B*b*ln(x)+D*ln(x)
*a
```

maxima [A] time = 1.33, size = 49, normalized size = 0.91

$$\frac{1}{2}\,Dbx^2+Cbx+(Da+Bb)\log(x)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2
 + 2*A*a)/x^3
```

mupad [B] time = 1.15, size = 50, normalized size = 0.93

$$\frac{b\,x^2\,D}{2}+a\,\ln(x)\,D+C\,b\,x-\frac{A\,a}{3\,x^3}-\frac{A\,b}{x}-\frac{B\,a}{2\,x^2}-\frac{C\,a}{x}+B\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
- (C*a)/x + B*b*log(x)
```

sympy [A] time = 1.01, size = 54, normalized size = 1.00

$$Cbx+\frac{Dbx^2}{2}+(Bb+Da)\log{(x)}+\frac{-2Aa-3Bax+x^2\,(-6Ab-6Ca)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
- 6*C*a))/(6*x**3)
```

3.66 $\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=109

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(a^2Ax^3+a^2Bx^4+a(2Ab+aC)x^5+a(2bB+aD)x^6+b(Ab+2a\right.$$

$$=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.90

$$a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))+\frac{b^2x^8\left(495A+4x\left(110B+99Cx+90Dx^2\right)\right)}{3960}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4*
x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*C
+ 8*D*x))))/252
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^2\right)^2 \left(A + Bx + Cx^2 + Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.35, size = 105, normalized size = 0.96

$$\frac{1}{11} Db^2x^{11} + \frac{1}{10} Cb^2x^{10} + \frac{2}{9} Dabx^9 + \frac{1}{9} Bb^2x^9 + \frac{1}{4} Cabx^8 + \frac{1}{8} Ab^2x^8 + \frac{1}{7} Da^2x^7 + \frac{2}{7} Babx^7 + \frac{1}{6} Ca^2x^6 + \frac{1}{3} Aabx^6 + \frac{1}{5} Ba^2x^5 + \frac{1}{4} Aa^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*a
*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 + 1/
3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

maple [A] time = 0.00, size = 102, normalized size = 0.94

$$\frac{Db^2x^{11}}{11} + \frac{C\,b^2x^{10}}{10} + \frac{\left(b^2B + 2abD\right)x^9}{9} + \frac{B\,a^2x^5}{5} + \frac{\left(b^2A + 2abC\right)x^8}{8} + \frac{A\,a^2x^4}{4} + \frac{\left(2abB + a^2D\right)x^7}{7} + \frac{\left(2Aab + a^2C\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b)
*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^2*
A*x^4
```

maxima [A] time = 1.36, size = 101, normalized size = 0.93

$$\frac{1}{11}\,Db^2x^{11} + \frac{1}{10}\,Cb^2x^{10} + \frac{1}{9}\left(2\,Dab + Bb^2\right)x^9 + \frac{1}{8}\left(2\,Cab + Ab^2\right)x^8 + \frac{1}{5}\,Ba^2x^5 + \frac{1}{7}\left(Da^2 + 2\,Bab\right)x^7 + \frac{1}{4}\,Aa^2x^4 + \frac{1}{6}\left(Ca^2 + 2\,Aab\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

mupad [B] time = 1.13, size = 108, normalized size = 0.99

$$\frac{a^2\,x^7\,D}{7} + \frac{b^2\,x^{11}\,D}{11} + \frac{A\,x^4\,\left(6\,a^2 + 8\,a\,b\,x^2 + 3\,b^2\,x^4\right)}{24} + \frac{B\,x^5\,\left(63\,a^2 + 90\,a\,b\,x^2 + 35\,b^2\,x^4\right)}{315} + \frac{C\,x^6\,\left(10\,a^2 + 15\,a\,b\,x^2 + 6\,b^2\,x^4\right)}{60} + \frac{2\,a\,b\,x^9\,D}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/2
4 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b^2
*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

sympy [A] time = 0.14, size = 110, normalized size = 1.01

$$\frac{Aa^2x^4}{4} + \frac{Ba^2x^5}{5} + \frac{Cb^2x^{10}}{10} + \frac{Db^2x^{11}}{11} + x^9\left(\frac{Bb^2}{9} + \frac{2Dab}{9}\right) + x^8\left(\frac{Ab^2}{8} + \frac{Cab}{4}\right) + x^7\left(\frac{2Bab}{7} + \frac{Da^2}{7}\right) + x^6\left(\frac{Aab}{3} + \frac{Ca^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(B
*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a**2
/7) + x**6*(A*a*b/3 + C*a**2/6)
```

3.67 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=109

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+a(2bB+aD)x^5+b(Ab+\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(A\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 0.84

$$\frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7x(45B+4x(10C+9Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x*
(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*
x))))/2520
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.37, size = 105, normalized size = 0.96

$$\frac{1}{10} Db^2x^{10} + \frac{1}{9} Cb^2x^9 + \frac{1}{4} Dabx^8 + \frac{1}{8} Bb^2x^8 + \frac{2}{7} Cabx^7 + \frac{1}{7} Ab^2x^7 + \frac{1}{6} Da^2x^6 + \frac{1}{3} Babx^6 + \frac{1}{5} Ca^2x^5 + \frac{2}{5} Aabx^5 + \frac{1}{4} Ba^2x^4 + \frac{1}{3} Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*b
*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*
A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

maple [A] time = 0.00, size = 102, normalized size = 0.94

$$\frac{Db^2x^{10}}{10} + \frac{C\,b^2x^9}{9} + \frac{\left(b^2B+2abD\right)x^8}{8} + \frac{B\,a^2x^4}{4} + \frac{\left(b^2A+2abC\right)x^7}{7} + \frac{A\,a^2x^3}{3} + \frac{\left(2abB+a^2D\right)x^6}{6} + \frac{\left(2Aab+a^2C\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*x
^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*A*a^2*
x^3
```

maxima [A] time = 1.30, size = 101, normalized size = 0.93

$$\frac{1}{10}Db^2x^{10} + \frac{1}{9}Cb^2x^9 + \frac{1}{8}\left(2\,Dab + Bb^2\right)x^8 + \frac{1}{7}\left(2\,Cab + Ab^2\right)x^7 + \frac{1}{4}Ba^2x^4 + \frac{1}{6}\left(Da^2 + 2\,Bab\right)x^6 + \frac{1}{3}Aa^2x^3 + \frac{1}{5}\left(Ca^2 + 2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

mupad [B] time = 1.11, size = 108, normalized size = 0.99

$$\frac{a^2\,x^6\,D}{6} + \frac{b^2\,x^{10}\,D}{10} + \frac{A\,x^3\,\left(35\,a^2 + 42\,a\,b\,x^2 + 15\,b^2\,x^4\right)}{105} + \frac{B\,x^4\,\left(6\,a^2 + 8\,a\,b\,x^2 + 3\,b^2\,x^4\right)}{24} + \frac{C\,x^5\,\left(63\,a^2 + 90\,a\,b\,x^2 + 35\,b^2\,x^4\right)}{315} + \frac{a\,b\,x^8\,D}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2)
)/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*b^
2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

sympy [A] time = 0.14, size = 110, normalized size = 1.01

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Cb^2x^9}{9} + \frac{Db^2x^{10}}{10} + x^8\left(\frac{Bb^2}{8} + \frac{Dab}{4}\right) + x^7\left(\frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^6\left(\frac{Bab}{3} + \frac{Da^2}{6}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*b
**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6)
+ x**5*(2*A*a*b/5 + C*a**2/5)
```

3.68 $\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a+bx^2\right)^2\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)\,dx$$
$$=\frac{A\left(a+bx^2\right)^3}{6b}+\int\left(a^2Bx^2+a^2Cx^3+a(2bB+aD)x^4+2abCx^5+b\right.$$
$$=\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5+\frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.88

$$\frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+x(72B+7x(9C+8Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*x
*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x
))))/2520
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 105, normalized size = 1.01

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{2}{7}Dabx^7+\frac{1}{7}Bb^2x^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{5}Da^2x^5+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*x
^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*
a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

maple [A] time = 0.00, size = 102, normalized size = 0.98

$$\frac{Db^2x^9}{9}+\frac{C\,b^2x^8}{8}+\frac{\left(b^2B+2abD\right)x^7}{7}+\frac{B\,a^2x^3}{3}+\frac{\left(b^2A+2abC\right)x^6}{6}+\frac{A\,a^2x^2}{2}+\frac{\left(2abB+a^2D\right)x^5}{5}+\frac{\left(2Aab+a^2C\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^6
+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

maxima [A] time = 1.39, size = 101, normalized size = 0.97

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{1}{7}\left(2\,Dab+Bb^2\right)x^7+\frac{1}{6}\left(2\,Cab+Ab^2\right)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{5}\left(Da^2+2\,Bab\right)x^5+\frac{1}{2}Aa^2x^2+\frac{1}{4}\left(Ca^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

mupad [B] time = 1.11, size = 107, normalized size = 1.03

$$\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{B\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{C\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{2\,a\,b\,x^7\,D}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (
B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4 +
 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

sympy [A] time = 0.09, size = 110, normalized size = 1.06

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^5\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b**
2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5)
+ x**4*(A*a*b/2 + C*a**2/4)
```

3.69 $\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=99

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{B\left(a+bx^2\right)^3}{6b} + \int \left(a+bx^2\right)^2 \left(A+Cx^2+Dx^3\right)\,dx$$
$$= \frac{B\left(a+bx^2\right)^3}{6b} + \int \left(a^2A + a(2Ab+aC)x^2 + a^2Dx^3 + b(Ab+2aC)x^4$$
$$= a^2Ax + \frac{1}{3}a(2Ab+aC)x^3 + \frac{1}{4}a^2Dx^4 + \frac{1}{5}b(Ab+2aC)x^5 + \frac{1}{3}abDx^6 +$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.89

$$\frac{1}{840}\left(70a^2x(12A+x(6B+x(4C+3Dx)))+28abx^3(20A+x(15B+2x(6C+5Dx)))+b^2x^5(168A+5x(28B+3x(8C+7Dx)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B +
 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/84
0
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.38, size = 102, normalized size = 1.03

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{3}Dabx^6 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{4}Da^2x^4 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x
^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*
a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

maple [A] time = 0.00, size = 99, normalized size = 1.00

$$\frac{Db^2x^8}{8}+\frac{C\,b^2x^7}{7}+\frac{\left(b^2B+2abD\right)x^6}{6}+\frac{B\,a^2x^2}{2}+\frac{\left(b^2A+2abC\right)x^5}{5}+A\,a^2x+\frac{\left(2abB+a^2D\right)x^4}{4}+\frac{\left(2Aab+a^2C\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^5
+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

maxima [A] time = 1.35, size = 98, normalized size = 0.99

$$\frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{6}\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}Ba^2x^2+\frac{1}{4}\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

mupad [B] time = 1.11, size = 105, normalized size = 1.06

$$\frac{A\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{C\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{a\,b\,x^6\,D}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
+ (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 + 4
2*a*b*x^2))/105 + (a*b*x^6*D)/3
```

sympy [A] time = 0.09, size = 107, normalized size = 1.08

$$Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8}+x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6 +
 D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**3*
(2*A*a*b/3 + C*a**2/3)
```

3.70 $\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} \, dx$

Optimal. Leaf size=92

$$a^2 A \log(x)+a^2 Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$a^2 A \log(x)+a^2 Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx = \frac{C\left(a+bx^2\right)^3}{6b} + \int \frac{\left(a+bx^2\right)^2\left(A+Bx+Dx^3\right)}{x}\,dx$$
$$= \frac{C\left(a+bx^2\right)^3}{6b} + \int \left(a^2B + \frac{a^2A}{x} + 2aAbx + a(2bB+aD)x^2 + Ab^2x^3 \right.$$
$$= a^2Bx + aAbx^2 + \frac{1}{3}a(2bB+aD)x^3 + \frac{1}{4}Ab^2x^4 + \frac{1}{5}b(bB+2aD)x^5 + \frac{1}{7}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.96

$$\frac{1}{420}x\left(70a^2(6B+x(3C+2Dx))+14abx(30A+x(20B+3x(5C+4Dx)))+b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right)+a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A*L
og[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x, x]
```

fricas [A] time = 0.80, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

giac [A] time = 0.46, size = 100, normalized size = 1.09

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{2}{5}Dabx^5+\frac{1}{5}Bb^2x^5+\frac{1}{2}Cabx^4+\frac{1}{4}Ab^2x^4+\frac{1}{3}Da^2x^3+\frac{2}{3}Babx^3+\frac{1}{2}Ca^2x^2+Aabx^2+Ba^2x+Aa^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*x
^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*
x^2 + B*a^2*x + A*a^2*log(abs(x))
```

maple [A] time = 0.00, size = 100, normalized size = 1.09

$$\frac{Db^2x^7}{7}+\frac{C\,b^2x^6}{6}+\frac{B\,b^2x^5}{5}+\frac{2Dab\,x^5}{5}+\frac{A\,b^2x^4}{4}+\frac{Cab\,x^4}{2}+\frac{2Bab\,x^3}{3}+\frac{Da^2x^3}{3}+Aab\,x^2+\frac{C\,a^2x^2}{2}+A\,a^2\ln(x)+B\,a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/7*b^2*D*x^7+1/6*C*b^2*x^6+1/5*B*b^2*x^5+2/5*D*x^5*a*b+1/4*A*b^2*x^4+1/2*C
*x^4*a*b+2/3*B*x^3*a*b+1/3*D*x^3*a^2+A*a*b*x^2+1/2*C*x^2*a^2+B*a^2*x+a^2*A*
ln(x)
```

maxima [A] time = 1.38, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2\,Dab+Bb^2\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2\,Bab\right)x^3+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

mupad [B] time = 1.11, size = 103, normalized size = 1.12

$$\frac{A\,\left(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+\frac{B\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^3\,D}{3}+\frac{b^2\,x^7\,D}{7}+\frac{C\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{2\,a\,b\,x^5\,D}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10*
a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 + 3*
a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

sympy [A] time = 0.32, size = 104, normalized size = 1.13

$$Aa^2\log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5\left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3\left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5 +
 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x**
2*(A*a*b + C*a**2/2)
```

3.71 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1628}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{D(a+bx^2)^3}{6b} + \int \frac{(a+bx^2)^2 (A+Bx+Cx^2)}{x^2}\,dx$$

$$= \frac{D(a+bx^2)^3}{6b} + \int \left(a(2Ab+aC) + \frac{a^2A}{x^2} + \frac{a^2B}{x} + 2abBx + b(Ab+2$$

$$= -\frac{a^2A}{x} + a(2Ab+aC)x + abBx^2 + \frac{1}{3}b(Ab+2aC)x^3 + \frac{1}{4}b^2Bx^4 + \frac{1}{5}b^2$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.98

$$a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^2B\log(x)+\frac{1}{6}abx(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))
/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2, x]
```

fricas [A] time = 0.51, size = 103, normalized size = 1.14

$$\frac{10\,Db^2x^7+12\,Cb^2x^6+15\left(2\,Dab+Bb^2\right)x^5+20\left(2\,Cab+Ab^2\right)x^4+60\,Ba^2x\log(x)+30\left(Da^2+2\,Bab\right)x^3-60\,Aa^2+60\left(Ca^2+2\,Aab\right)x^2}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
+ A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 + 60
*(C*a^2 + 2*A*a*b)*x^2)/x
```

giac [A] time = 0.38, size = 98, normalized size = 1.09

$$\frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{2}\,Dabx^4 + \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Cabx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\log\left(|x|\right) - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*x
^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^
2*log(abs(x)) - A*a^2/x
```

maple [A] time = 0.01, size = 98, normalized size = 1.09

$$\frac{Db^2x^6}{6} + \frac{C\,b^2x^5}{5} + \frac{B\,b^2x^4}{4} + \frac{Dab\,x^4}{2} + \frac{A\,b^2x^3}{3} + \frac{2Cab\,x^3}{3} + Bab\,x^2 + \frac{Da^2x^2}{2} + 2Aabx + B\,a^2\ln(x) + C\,a^2x - \frac{A\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/6*D*b^2*x^6+1/5*b^2*C*x^5+1/4*B*b^2*x^4+1/2*D*x^4*a*b+1/3*A*x^3*b^2+2/3*C
*x^3*a*b+B*a*b*x^2+1/2*D*x^2*a^2+2*A*a*b*x+a^2*C*x-A*a^2/x+a^2*B*ln(x)
```

maxima [A] time = 1.32, size = 96, normalized size = 1.07

$$\frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{4}\left(2\,Dab + Bb^2\right)x^4 + \frac{1}{3}\left(2\,Cab + Ab^2\right)x^3 + Ba^2\log(x) + \frac{1}{2}\left(Da^2 + 2\,Bab\right)x^2 - \frac{Aa^2}{x} + \left(Ca^2 + 2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2 +
2*A*a*b)*x
```

mupad [B] time = 1.11, size = 92, normalized size = 1.02

$$\frac{B\,\left(4\,a^2\,\ln(x) + b^2\,x^4 + 4\,a\,b\,x^2\right)}{4} + \frac{{\left(b\,x^2 + a\right)}^3\,D}{6\,b} + \frac{C\,x\,\left(15\,a^2 + 10\,a\,b\,x^2 + 3\,b^2\,x^4\right)}{15} + \frac{A\,\left(-3\,a^2 + 6\,a\,b\,x^2 + b^2\,x^4\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*x
*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/
(3*x)
```

sympy [A] time = 0.35, size = 99, normalized size = 1.10

$$-\frac{Aa^2}{x} + Ba^2\log(x) + \frac{Cb^2x^5}{5} + \frac{Db^2x^6}{6} + x^4\left(\frac{Bb^2}{4} + \frac{Dab}{2}\right) + x^3\left(\frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x^2\left(Bab + \frac{Da^2}{2}\right) + x\left(2Aab + Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
+ D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2*A
*a*b + C*a**2)
```

3.72 $\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} \, dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} \, dx = \int \left(a(2bB+aD)+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+b(b$$

$$= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3$$

Mathematica [A] time = 0.04, size = 87, normalized size = 0.89

$$-\frac{a^2(A+2Bx-2Dx^3)}{2x^2}+a\log(x)(aC+2Ab)+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
(b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 (A + Bx + Cx^2 + Dx^3)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3, x]
```

fricas [A] time = 0.66, size = 103, normalized size = 1.05

$$\frac{12\,Db^2x^7 + 15\,Cb^2x^6 + 20\left(2\,Dab + Bb^2\right)x^5 + 30\left(2\,Cab + Ab^2\right)x^4 - 60\,Ba^2x + 60\left(Da^2 + 2\,Bab\right)x^3 + 60\left(Ca^2 + 2\,Aab\right)x^2\log(x) - 30\,Aa^2}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
+ A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*b)
*x^2*log(x) - 30*A*a^2)/x^2
```

giac [A] time = 0.34, size = 97, normalized size = 0.99

$$\frac{1}{5}\,Db^2x^5 + \frac{1}{4}\,Cb^2x^4 + \frac{2}{3}\,Dabx^3 + \frac{1}{3}\,Bb^2x^3 + Cabx^2 + \frac{1}{2}\,Ab^2x^2 + Da^2x + 2\,Babx + \left(Ca^2 + 2\,Aab\right)\log\left(|x|\right) - \frac{2\,Ba^2x + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2 +
 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*
(2*B*a^2*x + A*a^2)/x^2
```

maple [A] time = 0.01, size = 97, normalized size = 0.99

$$\frac{Db^2x^5}{5} + \frac{C\,b^2x^4}{4} + \frac{B\,b^2x^3}{3} + \frac{2Dab\,x^3}{3} + \frac{A\,b^2x^2}{2} + Cab\,x^2 + 2Aab\ln(x) + 2Babx + C\,a^2\ln(x) + Da^2x - \frac{B\,a^2}{x} - \frac{A\,a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x)

[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*B*x^3*b^2+2/3*D*x^3*a*b+1/2*A*b^2*x^2+C*x^2*a*b+2*B*x*a*b+a^2*D*x-1/2*a^2*A/x^2-a^2*B/x+2*A*ln(x)*a*b+C*ln(x)*a^2

maxima [A] time = 1.32, size = 96, normalized size = 0.98

$$\frac{1}{5}Db^2x^5+\frac{1}{4}Cb^2x^4+\frac{1}{3}\left(2\,Dab+Bb^2\right)x^3+\frac{1}{2}\left(2\,Cab+Ab^2\right)x^2+\left(Da^2+2\,Bab\right)x+\left(Ca^2+2\,Aab\right)\log(x)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")

[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b + A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*x + A*a^2)/x^2

mupad [B] time = 1.11, size = 103, normalized size = 1.05

$$\frac{C\,\left(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+a^2\,x\,D+\frac{b^2\,x^5\,D}{5}+\frac{A\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x)\right)}{2\,x^2}+\frac{B\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}+\frac{2\,a\,b\,x^3\,D}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)

[Out] (C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/(3*x) + (2*a*b*x^3*D)/3

sympy [A] time = 0.58, size = 100, normalized size = 1.02

$$\frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a\left(2Ab+Ca\right)\log\left(x\right)+x^3\left(\frac{Bb^2}{3}+\frac{2Dab}{3}\right)+x^2\left(\frac{Ab^2}{2}+Cab\right)+x\left(2Bab+Da^2\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)

[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 + 2*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2*B*a**2*x)/(2*x**2)

3.73 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4} dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \int \left(b(Ab+2aC)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{a(2bB+aD)}{x}+b(\right.$$
$$= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.85

$$-\frac{a^2(2A+3x(B+2Cx))}{6x^3}-\frac{2aAb}{x}+a\log(x)(aD+2bB)+abx(2C+Dx)+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
(b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4, x]
```

fricas [A] time = 0.88, size = 103, normalized size = 1.05

$$\frac{3\,Db^2x^7+4\,Cb^2x^6+6\left(2\,Dab+Bb^2\right)x^5+12\left(2\,Cab+Ab^2\right)x^4+12\left(Da^2+2\,Bab\right)x^3\log(x)-6\,Ba^2x-4\,Aa^2-12\left(Ca^2+2\,Aab\right)x^2}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b + A
*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

giac [A] time = 0.43, size = 97, normalized size = 0.99

$$\frac{1}{4}\,Db^2x^4+\frac{1}{3}\,Cb^2x^3+Dabx^2+\frac{1}{2}\,Bb^2x^2+2\,Cabx+Ab^2x+\left(Da^2+2\,Bab\right)\log\left(|x|\right)-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aab\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*b
^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

maple [A] time = 0.01, size = 97, normalized size = 0.99

$$\frac{Db^2x^4}{4}+\frac{C\,b^2x^3}{3}+\frac{B\,b^2x^2}{2}+Dab\,x^2+A\,b^2x+2Bab\ln(x)+2Cabx+Da^2\ln(x)-\frac{2Aab}{x}-\frac{C\,a^2}{x}-\frac{B\,a^2}{2x^2}-\frac{A\,a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/4*b^2*D*x^4+1/3*b^2*C*x^3+1/2*B*b^2*x^2+D*x^2*a*b+A*x*b^2+2*a*b*C*x-1/3*a
^2*A/x^3-1/2*B*a^2/x^2-2*a/x*A*b-a^2/x*C+2*B*a*b*ln(x)+D*ln(x)*a^2
```

maxima [A] time = 1.37, size = 97, normalized size = 0.99

$$\frac{1}{4}\,Db^2x^4+\frac{1}{3}\,Cb^2x^3+\frac{1}{2}\left(2\,Dab+Bb^2\right)x^2+\left(2\,Cab+Ab^2\right)x+\left(Da^2+2\,Bab\right)\log(x)-\frac{3\,Ba^2x+2\,Aa^2+6\left(Ca^2+2\,Aab\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b^
2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A
*a*b)*x^2)/x^3
```

mupad [B] time = 1.28, size = 106, normalized size = 1.08

$$\frac{b^2\,x^4\,D}{4}+\frac{a^2\,\ln\left(x^2\right)\,D}{2}-\frac{A\,\left(a^2+6\,a\,b\,x^2-3\,b^2\,x^4\right)}{3\,x^3}+\frac{B\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x)\right)}{2\,x^2}+\frac{C\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}+a\,b\,x^2\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*x
^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^2
+ 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

sympy [A] time = 1.46, size = 100, normalized size = 1.02

$$\frac{Cb^2x^3}{3}+\frac{Db^2x^4}{4}+a\,(2Bb+Da)\log(x)+x^2\left(\frac{Bb^2}{2}+Dab\right)+x\left(Ab^2+2Cab\right)+\frac{-2Aa^2-3Ba^2x+x^2\left(-12Aab-6Ca^2\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 + D
*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b -
6*C*a**2))/(6*x**3)
```

3.74 $\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+$$

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{5}a^3Bx^5+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(a^3Ax^3+a^3Bx^4+a^2(3Ab+aC)x^5+a^2(3bB+aD)x^6+3ab(Ab\right.\\ &= \frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab\end{aligned}$$

Mathematica [A] time = 0.03, size = 149, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.46, size = 153, normalized size = 1.03

$$\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{3}{11}\,Dab^2x^{11}+\frac{1}{11}\,Bb^3x^{11}+\frac{3}{10}\,Cab^2x^{10}+\frac{1}{10}\,Ab^3x^{10}+\frac{1}{3}\,Da^2bx^9+\frac{1}{3}\,Bab^2x^9+\frac{3}{8}\,Ca^2bx^8+\frac{3}{8}\,Aab^2x^8+\frac{1}{7}\,Da^3x^7+\frac{3}{7}\,Ba^2bx^7+\frac{1}{6}\,Ca^3x^6+\frac{1}{2}\,Aa^2bx^6+\frac{1}{5}\,Ba^3x^5+\frac{1}{4}\,Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 + 3
/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 + 3/
8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a
^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

maple [A] time = 0.00, size = 150, normalized size = 1.01

$$\frac{Db^3x^{13}}{13}+\frac{C\,b^3x^{12}}{12}+\frac{\left(b^3B+3a\,b^2D\right)x^{11}}{11}+\frac{\left(A\,b^3+3a\,b^2C\right)x^{10}}{10}+\frac{B\,a^3x^5}{5}+\frac{\left(3a\,b^2B+3a^2bD\right)x^9}{9}+\frac{A\,a^3x^4}{4}+\frac{\left(3a\,b^2A+3a^2bC\right)x^8}{8}+\frac{\left(3a^2bB+a^3D\right)x^7}{7}+\frac{\left(3A\,a^2b+a^3C\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+1/11*(B*b^3+3*D*a*b^2)*x^11+1/10*(A*b^3+3*C
*a*b^2)*x^10+1/9*(3*B*a*b^2+3*D*a^2*b)*x^9+1/8*(3*A*a*b^2+3*C*a^2*b)*x^8+1/
```

```
7*(3*B*a^2*b+D*a^3)*x^7+1/6*(3*A*a^2*b+C*a^3)*x^6+1/5*a^3*B*x^5+1/4*a^3*A*x
^4
```

maxima [A] time = 1.33, size = 145, normalized size = 0.97

$$\frac{1}{13} Db^3x^{13} + \frac{1}{12} Cb^3x^{12} + \frac{1}{11}\left(3\,Dab^2 + Bb^3\right)x^{11} + \frac{1}{10}\left(3\,Cab^2 + Ab^3\right)x^{10} + \frac{1}{3}\left(Da^2b + Bab^2\right)x^9 + \frac{1}{5} Ba^3x^5 + \frac{3}{8}\left(Ca^2b + Aab^2\right)x^8 + \frac{1}{4} Aa^3x^4 + \frac{1}{7}\left(Da^3 + 3\,Ba^2b\right)x^7 + \frac{1}{6}\left(Ca^3 + 3\,Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

mupad [B] time = 1.30, size = 153, normalized size = 1.03

$$\frac{A\,a^3\,x^4}{4} + \frac{B\,a^3\,x^5}{5} + \frac{A\,b^3\,x^{10}}{10} + \frac{C\,a^3\,x^6}{6} + \frac{B\,b^3\,x^{11}}{11} + \frac{C\,b^3\,x^{12}}{12} + \frac{a^3\,x^7\,D}{7} + \frac{b^3\,x^{13}\,D}{13} + \frac{a^2\,b\,x^9\,D}{3} + \frac{3\,a\,b^2\,x^{11}\,D}{11} + \frac{A\,a^2\,b\,x^6}{2} + \frac{3\,A\,a\,b^2\,x^8}{8} + \frac{3\,B\,a^2\,b\,x^7}{7} + \frac{B\,a\,b^2\,x^9}{3} + \frac{3\,C\,a^2\,b\,x^8}{8} + \frac{3\,C\,a\,b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x^
11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D)/
3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2*b*
x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

sympy [A] time = 0.17, size = 163, normalized size = 1.09

$$\frac{Aa^3x^4}{4} + \frac{Ba^3x^5}{5} + \frac{Cb^3x^{12}}{12} + \frac{Db^3x^{13}}{13} + x^{11}\left(\frac{Bb^3}{11} + \frac{3Dab^2}{11}\right) + x^{10}\left(\frac{Ab^3}{10} + \frac{3Cab^2}{10}\right) + x^9\left(\frac{Bab^2}{3} + \frac{Da^2b}{3}\right) + x^8\left(\frac{3Aab^2}{8} + \frac{3Ca^2b}{8}\right) + x^7\left(\frac{3Ba^2b}{7} + \frac{Da^3}{7}\right) + x^6\left(\frac{Aa^2b}{2} + \frac{Ca^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*(
B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*a*
b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a**2*
b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

3.75 $\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+$$

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{3}a^3Ax^3+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{4}a^3Bx^4+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx &= \int \left(a^3Ax^2+a^3Bx^3+a^2(3Ab+aC)x^4+a^2(3bB+aD)x^5+3ab(A\right.\\ &= \frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}a\end{aligned}$$

Mathematica [A] time = 0.06, size = 125, normalized size = 0.84

$$\frac{462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A+5x(28B+3x(8C+7Dx)))+33ab^2x^7(360A+7x(45B+4x(10C+9Dx)))+14b^3x^9\left(220A+3x\left(66B+60Cx+55Dx^2\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*C
+ 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x)))))/27720
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.46, size = 153, normalized size = 1.03

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{3}{10}Dab^2x^{10}+\frac{1}{10}Bb^3x^{10}+\frac{1}{3}Cab^2x^9+\frac{1}{9}Ab^3x^9+\frac{3}{8}Da^2bx^8+\frac{3}{8}Bab^2x^8+\frac{3}{7}Ca^2bx^7+\frac{3}{7}Aab^2x^7+\frac{1}{6}Da^3x^6+\frac{1}{2}Ba^2bx^6+\frac{1}{5}Ca^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{4}Ba^3x^4+\frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 + 1
/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*C*
a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x
^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

maple [A] time = 0.00, size = 150, normalized size = 1.01

$$\frac{Db^3x^{12}}{12}+\frac{C\,b^3x^{11}}{11}+\frac{\left(b^3B+3a\,b^2D\right)x^{10}}{10}+\frac{\left(A\,b^3+3a\,b^2C\right)x^9}{9}+\frac{B\,a^3x^4}{4}+\frac{\left(3a\,b^2B+3a^2bD\right)x^8}{8}+\frac{A\,a^3x^3}{3}+\frac{\left(3a\,b^2A+3a^2bC\right)x^7}{7}+\frac{\left(3a^2bB+a^3D\right)x^6}{6}+\frac{\left(3A\,a^2b+a^3C\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+1/10*(B*b^3+3*D*a*b^2)*x^10+1/9*(A*b^3+3*C*
a*b^2)*x^9+1/8*(3*B*a*b^2+3*D*a^2*b)*x^8+1/7*(3*A*a*b^2+3*C*a^2*b)*x^7+1/6*
(3*B*a^2*b+D*a^3)*x^6+1/5*(3*A*a^2*b+C*a^3)*x^5+1/4*a^3*B*x^4+1/3*a^3*A*x^3
```

maxima [A] time = 1.36, size = 145, normalized size = 0.97

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}\left(3\,Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3\,Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}\left(Da^3+3\,Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3\,Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

mupad [B] time = 1.28, size = 153, normalized size = 1.03

$$\frac{A\,a^3\,x^3}{3}+\frac{B\,a^3\,x^4}{4}+\frac{A\,b^3\,x^9}{9}+\frac{C\,a^3\,x^5}{5}+\frac{B\,b^3\,x^{10}}{10}+\frac{C\,b^3\,x^{11}}{11}+\frac{a^3\,x^6\,D}{6}+\frac{b^3\,x^{12}\,D}{12}+\frac{3\,a^2\,b\,x^8\,D}{8}+\frac{3\,a\,b^2\,x^{10}\,D}{10}+\frac{3\,A\,a^2\,b\,x^5}{5}+\frac{3\,A\,a\,b^2\,x^7}{7}+\frac{B\,a^2\,b\,x^6}{2}+\frac{3\,B\,a\,b^2\,x^8}{8}+\frac{3\,C\,a^2\,b\,x^7}{7}+\frac{C\,a\,b^2\,x^9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^10
)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D)/
8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2*b*
x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

sympy [A] time = 0.14, size = 165, normalized size = 1.11

$$\frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\left(\frac{3Aab^2}{7}+\frac{3Ca^2b}{7}\right)+x^6\left(\frac{Ba^2b}{2}+\frac{Da^3}{6}\right)+x^5\left(\frac{3Aa^2b}{5}+\frac{Ca^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*(
B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b**
2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*b/2
 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

3.76 $\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=138

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b$$

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1582, 1810}

$$\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx=\frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a+bx^2\right)^3\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)dx$$
$$=\frac{A\left(a+bx^2\right)^4}{8b}+\int\left(a^3Bx^2+a^3Cx^3+a^2(3bB+aD)x^4+3a^2bCx^5+\right.$$
$$=\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD$$

Mathematica [A] time = 0.06, size = 124, normalized size = 0.90

$$\frac{462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105A+2x(42B+5x(7C+6Dx)))+165ab^2x^6(84A+x(72B+7x(9C+8Dx)))+7b^3x^8\left(495A+4x\left(110B+99Cx+90Dx^2\right)\right)}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/27720
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
[Out] IntegrateAlgebraic[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.36, size = 153, normalized size = 1.11

$$\frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{3}Dab^2x^9+\frac{1}{9}Bb^3x^9+\frac{3}{8}Cab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{7}Da^2bx^7+\frac{3}{7}Bab^2x^7+\frac{1}{2}Ca^2bx^6+\frac{1}{2}Aab^2x^6+\frac{1}{5}Da^3x^5+\frac{3}{5}Ba^2bx^5+\frac{1}{4}Ca^3x^4+\frac{3}{4}Aa^2bx^4+\frac{1}{3}Ba^3x^3+\frac{1}{2}Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*C
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^2*
b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^4 +
 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

maple [A] time = 0.00, size = 150, normalized size = 1.09

$$\frac{Db^3x^{11}}{11}+\frac{C\,b^3x^{10}}{10}+\frac{\left(b^3B+3a\,b^2D\right)x^9}{9}+\frac{\left(A\,b^3+3a\,b^2C\right)x^8}{8}+\frac{B\,a^3x^3}{3}+\frac{\left(3a\,b^2B+3a^2bD\right)x^7}{7}+\frac{A\,a^3x^2}{2}+\frac{\left(3a\,b^2A+3a^2bC\right)x^6}{6}+\frac{\left(3a^2bB+a^3D\right)x^5}{5}+\frac{\left(3A\,a^2b+a^3C\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+1/9*(B*b^3+3*D*a*b^2)*x^9+1/8*(A*b^3+3*C*a*
b^2)*x^8+1/7*(3*B*a*b^2+3*D*a^2*b)*x^7+1/6*(3*A*a*b^2+3*C*a^2*b)*x^6+1/5*(3
*B*a^2*b+D*a^3)*x^5+1/4*(3*A*a^2*b+C*a^3)*x^4+1/3*a^3*B*x^3+1/2*a^3*A*x^2
```

maxima [A] time = 1.34, size = 145, normalized size = 1.05

$$\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\left(3\,Dab^2+Bb^3\right)x^9+\frac{1}{8}\left(3\,Cab^2+Ab^3\right)x^8+\frac{3}{7}\left(Da^2b+Bab^2\right)x^7+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\left(Ca^2b+Aab^2\right)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\left(Da^3+3\,Ba^2b\right)x^5+\frac{1}{4}\left(Ca^3+3\,Aa^2b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

mupad [B] time = 1.28, size = 153, normalized size = 1.11

$$\frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4}+\frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5}+\frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3}+\frac{3\,A\,a^2\,b\,x^4}{4}+\frac{A\,a\,b^2\,x^6}{2}+\frac{3\,B\,a^2\,b\,x^5}{5}+\frac{3\,B\,a\,b^2\,x^7}{7}+\frac{C\,a^2\,b\,x^6}{2}+\frac{3\,C\,a\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9)
/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/7
+ (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5)/5
 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

sympy [A] time = 0.14, size = 163, normalized size = 1.18

$$\frac{Aa^3x^2}{2} + \frac{Ba^3x^3}{3} + \frac{Cb^3x^{10}}{10} + \frac{Db^3x^{11}}{11} + x^9\left(\frac{Bb^3}{9} + \frac{Dab^2}{3}\right) + x^8\left(\frac{Ab^3}{8} + \frac{3Cab^2}{8}\right) + x^7\left(\frac{3Bab^2}{7} + \frac{3Da^2b}{7}\right) + x^6\left(\frac{Aab^2}{2} + \frac{Ca^2b}{2}\right) + x^5\left(\frac{3Ba^2b}{5} + \frac{Da^3}{5}\right) + x^4\left(\frac{3Aa^2b}{4} + \frac{Ca^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(B
*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2/7
 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 + D*
a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

3.77 $\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=133

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3C$$

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aC+3Ab)+a^3Ax+\frac{1}{2}a^2bDx^6+\frac{1}{4}a^3Dx^4+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{B\left(a+bx^2\right)^4}{8b} + \int \left(a+bx^2\right)^3 \left(A+Cx^2+Dx^3\right) dx$$
$$= \frac{B\left(a+bx^2\right)^4}{8b} + \int \left(a^3A + a^2(3Ab+aC)x^2 + a^3Dx^3 + 3ab(Ab+aC)\right.$$
$$= a^3Ax + \frac{1}{3}a^2(3Ab+aC)x^3 + \frac{1}{4}a^3Dx^4 + \frac{3}{5}ab(Ab+aC)x^5 + \frac{1}{2}a^2bDx^6$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.91

$$\frac{210a^3x(12A+x(6B+x(4C+3Dx))) + 126a^2bx^3(20A+x(15B+2x(6C+5Dx))) + 9ab^2x^5(168A+5x(28B+3x(8C+7Dx))) + b^3x^7(360A+7x(45B+4x(10C+9Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*
x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> keys do not match self's parent
```

giac [A] time = 0.40, size = 149, normalized size = 1.12

$$\frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Da^3x^4 + \frac{3}{4}Ba^2bx^4 + \frac{1}{3}Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*a
*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*b*
x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3 + A
*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

maple [A] time = 0.00, size = 147, normalized size = 1.11

$$\frac{Db^3x^{10}}{10}+\frac{C\,b^3x^9}{9}+\frac{\left(b^3B+3a\,b^2D\right)x^8}{8}+\frac{\left(A\,b^3+3a\,b^2C\right)x^7}{7}+\frac{B\,a^3x^2}{2}+\frac{\left(3a\,b^2B+3a^2bD\right)x^6}{6}+A\,a^3x+\frac{\left(3a\,b^2A+3a^2bC\right)x^5}{5}+\frac{\left(3a^2bB+a^3D\right)x^4}{4}+\frac{\left(3A\,a^2b+a^3C\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+1/8*(B*b^3+3*D*a*b^2)*x^8+1/7*(A*b^3+3*C*a*b^
2)*x^7+1/6*(3*B*a*b^2+3*D*a^2*b)*x^6+1/5*(3*A*a*b^2+3*C*a^2*b)*x^5+1/4*(3*B
*a^2*b+D*a^3)*x^4+1/3*(3*A*a^2*b+C*a^3)*x^3+1/2*a^3*B*x^2+a^3*A*x
```

maxima [A] time = 1.34, size = 142, normalized size = 1.07

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\left(3\,Dab^2+Bb^3\right)x^8+\frac{1}{7}\left(3\,Cab^2+Ab^3\right)x^7+\frac{1}{2}\left(Da^2b+Bab^2\right)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\left(Ca^2b+Aab^2\right)x^5+Aa^3x+\frac{1}{4}\left(Da^3+3\,Ba^2b\right)x^4+\frac{1}{3}\left(Ca^3+3\,Aa^2b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

mupad [B] time = 1.26, size = 149, normalized size = 1.12

$$\frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9)
/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b^2
*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b^2*
x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

sympy [A] time = 0.13, size = 158, normalized size = 1.19

$$Aa^3x + \frac{Ba^3x^2}{2} + \frac{Cb^3x^9}{9} + \frac{Db^3x^{10}}{10} + x^8\left(\frac{Bb^3}{8} + \frac{3Dab^2}{8}\right) + x^7\left(\frac{Ab^3}{7} + \frac{3Cab^2}{7}\right) + x^6\left(\frac{Bab^2}{2} + \frac{Da^2b}{2}\right) + x^5\left(\frac{3Aab^2}{5} + \frac{3Ca^2b}{5}\right) + x^4\left(\frac{3Ba^2b}{4} + \frac{Da^3}{4}\right) + x^3\left(Aa^2b + \frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/8
 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*a*
*2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a**3/
4) + x**3*(A*a**2*b + C*a**3/3)
```

3.78 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=129

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C\left(a+bx^2\right)^4}{8b}+\frac{1}{6}A$$

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1802}

$$\frac{3}{2}a^2Abx^2+a^3A\log(x)+\frac{1}{3}a^2x^3(aD+3bB)+a^3Bx+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C\left(a+bx^2\right)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{C(a+bx^2)^4}{8b} + \int \frac{(a+bx^2)^3 (A+Bx+Dx^3)}{x}\,dx$$
$$= \frac{C(a+bx^2)^4}{8b} + \int \left(a^3B + \frac{a^3A}{x} + 3a^2Abx + a^2(3bB+aD)x^2 + 3aAb\right.$$
$$= a^3Bx + \frac{3}{2}a^2Abx^2 + \frac{1}{3}a^2(3bB+aD)x^3 + \frac{3}{4}aAb^2x^4 + \frac{3}{5}ab(bB+aD)x$$

Mathematica [A] time = 0.07, size = 121, normalized size = 0.94

$$a^3A\log(x) + \frac{x\left(420a^3(6B+x(3C+2Dx)) + 126a^2bx(30A+x(20B+3x(5C+4Dx))) + 18ab^2x^3(105A+2x(42B+5x(7C+6Dx))) + 5b^3x^5(84A+x(72B+7x(9C+8Dx)))\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5*
C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^
3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x, x]
```

fricas [A] time = 0.74, size = 140, normalized size = 1.09

$$\frac{1}{9}Db^3x^9 + \frac{1}{8}Cb^3x^8 + \frac{1}{7}\left(3Dab^2 + Bb^3\right)x^7 + \frac{1}{6}\left(3Cab^2 + Ab^3\right)x^6 + \frac{3}{5}\left(Da^2b + Bab^2\right)x^5 + Ba^3x + \frac{3}{4}\left(Ca^2b + Aab^2\right)x^4 + Aa^3\log(x) + \frac{1}{3}\left(Da^3 + 3Ba^2b\right)x^3 + \frac{1}{2}\left(Ca^3 + 3Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

giac [A] time = 0.33, size = 148, normalized size = 1.15

$$\frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{3}{7}Dab^2x^7+\frac{1}{7}Bb^3x^7+\frac{1}{2}Cab^2x^6+\frac{1}{6}Ab^3x^6+\frac{3}{5}Da^2bx^5+\frac{3}{5}Bab^2x^5+\frac{3}{4}Ca^2bx^4+\frac{3}{4}Aab^2x^4+\frac{1}{3}Da^3x^3+Ba^2bx^3+\frac{1}{2}Ca^3x^2+\frac{3}{2}Aa^2bx^2+Ba^3x+Aa^3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*b
^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*x^
4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a
^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

maple [A] time = 0.00, size = 148, normalized size = 1.15

$$\frac{Db^3x^9}{9}+\frac{Cb^3x^8}{8}+\frac{Bb^3x^7}{7}+\frac{3Dab^2x^7}{7}+\frac{Ab^3x^6}{6}+\frac{Cab^2x^6}{2}+\frac{3Bab^2x^5}{5}+\frac{3Da^2bx^5}{5}+\frac{3Aab^2x^4}{4}+\frac{3Ca^2bx^4}{4}+Ba^2bx^3+\frac{Da^3x^3}{3}+\frac{3Aa^2bx^2}{2}+\frac{Ca^3x^2}{2}+Aa^3\ln(x)+Ba^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x)
```

```
[Out] 1/9*b^3*D*x^9+1/8*C*b^3*x^8+1/7*B*x^7*b^3+3/7*D*x^7*a*b^2+1/6*A*b^3*x^6+1/2
*C*x^6*a*b^2+3/5*B*x^5*a*b^2+3/5*D*x^5*a^2*b+3/4*a*A*b^2*x^4+3/4*C*x^4*a^2*
b+B*x^3*a^2*b+1/3*D*x^3*a^3+3/2*a^2*A*b*x^2+1/2*C*x^2*a^3+a^3*B*x+a^3*A*ln(
x)
```

maxima [A] time = 1.32, size = 140, normalized size = 1.09

$$\frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{1}{7}\left(3Dab^2+Bb^3\right)x^7+\frac{1}{6}\left(3Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5+Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4+Aa^3\log(x)+\frac{1}{3}\left(Da^3+3Ba^2b\right)x^3+\frac{1}{2}\left(Ca^3+3Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

mupad [B] time = 1.26, size = 147, normalized size = 1.14

$$\frac{Ab^3x^6}{6}+\frac{Ca^3x^2}{2}+\frac{Bb^3x^7}{7}+\frac{Cb^3x^8}{8}+Aa^3\ln(x)+\frac{a^3x^3D}{3}+\frac{b^3x^9D}{9}+Ba^3x+\frac{3a^2bx^5D}{5}+\frac{3ab^2x^7D}{7}+\frac{3Aa^2bx^2}{2}+\frac{3Aab^2x^4}{4}+Ba^2bx^3+\frac{3Bab^2x^5}{5}+\frac{3Ca^2bx^4}{4}+\frac{Cab^2x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(x
) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^2*
x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*b^2
*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

sympy [A] time = 0.40, size = 158, normalized size = 1.22

$$Aa^3\log(x) + Ba^3x + \frac{Cb^3x^8}{8} + \frac{Db^3x^9}{9} + x^7\left(\frac{Bb^3}{7} + \frac{3Dab^2}{7}\right) + x^6\left(\frac{Ab^3}{6} + \frac{Cab^2}{2}\right) + x^5\left(\frac{3Bab^2}{5} + \frac{3Da^2b}{5}\right) + x^4\left(\frac{3Aab^2}{4} + \frac{3Ca^2b}{4}\right) + x^3\left(Ba^2b + \frac{Da^3}{3}\right) + x^2\left(\frac{3Aa^2b}{2} + \frac{Ca^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7 +
 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*a*
*2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3) +
 x**2*(3*A*a**2*b/2 + C*a**3/2)
```

3.79 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3Bx^6$$

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1583, 1628}

$$a^2x(aC+3Ab)-\frac{a^3A}{x}+\frac{3}{2}a^2bBx^2+a^3B\log(x)+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rule 1583

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(Coe
ff[Px, x, n - m - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx = \frac{D\left(a+bx^2\right)^4}{8b} + \int \frac{\left(a+bx^2\right)^3 \left(A+Bx+Cx^2\right)}{x^2}\,dx$$
$$= \frac{D\left(a+bx^2\right)^4}{8b} + \int \left(a^2(3Ab+aC) + \frac{a^3A}{x^2} + \frac{a^3B}{x} + 3a^2bBx + 3ab(Al\right.$$
$$= -\frac{a^3A}{x} + a^2(3Ab+aC)x + \frac{3}{2}a^2bBx^2 + ab(Ab+aC)x^3 + \frac{3}{4}ab^2Bx^4 +$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.99

$$a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+a^3B\log(x)+\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))+\frac{1}{840}b^3x^5(168A+5x(28B+3x(8C+7Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))
))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(168
*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2, x]
```

fricas [A] time = 0.81, size = 147, normalized size = 1.19

$$\frac{105\,Db^3x^9+120\,Cb^3x^8+140\left(3\,Dab^2+Bb^3\right)x^7+168\left(3\,Cab^2+Ab^3\right)x^6+630\left(Da^2b+Bab^2\right)x^5+840\,Ba^3x\log(x)+840\left(Ca^2b+Aab^2\right)x^4-840\,Aa^3+420\left(Da^3+3\,Ba^2b\right)x^3+840\left(Ca^3+3\,Aa^2b\right)x^2}{840\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(3
*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x) +
840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 840
*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

giac [A] time = 0.41, size = 145, normalized size = 1.17

$$\frac{1}{8}Db^3x^8+\frac{1}{7}Cb^3x^7+\frac{1}{2}Dab^2x^6+\frac{1}{6}Bb^3x^6+\frac{3}{5}Cab^2x^5+\frac{1}{5}Ab^3x^5+\frac{3}{4}Da^2bx^4+\frac{3}{4}Bab^2x^4+Ca^2bx^3+Aab^2x^3+\frac{1}{2}Da^3x^2+\frac{3}{2}Ba^2bx^2+Ca^3x+3\,Aa^2bx+Ba^3\log\left(|x|\right)-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*b
^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3 +
A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x + B*a
^3*log(abs(x)) - A*a^3/x
```

maple [A] time = 0.01, size = 145, normalized size = 1.17

$$\frac{Db^3x^8}{8}+\frac{C\,b^3x^7}{7}+\frac{B\,b^3x^6}{6}+\frac{Da\,b^2x^6}{2}+\frac{A\,b^3x^5}{5}+\frac{3Ca\,b^2x^5}{5}+\frac{3Ba\,b^2x^4}{4}+\frac{3Da^2b\,x^4}{4}+Aa\,b^2x^3+C\,a^2b\,x^3+\frac{3B\,a^2b\,x^2}{2}+\frac{Da^3x^2}{2}+3A\,a^2bx+B\,a^3\ln(x)+C\,a^3x-\frac{A\,a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

```
[Out] 1/8*D*b^3*x^8+1/7*b^3*C*x^7+1/6*B*b^3*x^6+1/2*D*x^6*a*b^2+1/5*A*x^5*b^3+3/5
*C*x^5*a*b^2+3/4*B*a*b^2*x^4+3/4*D*x^4*a^2*b+A*x^3*a*b^2+C*x^3*a^2*b+3/2*B*
a^2*b*x^2+1/2*D*x^2*a^3+3*A*a^2*b*x+a^3*C*x-a^3*A/x+a^3*B*ln(x)
```

maxima [A] time = 1.35, size = 139, normalized size = 1.12

$$\frac{1}{8}Db^3x^8+\frac{1}{7}Cb^3x^7+\frac{1}{6}\left(3\,Dab^2+Bb^3\right)x^6+\frac{1}{5}\left(3\,Cab^2+Ab^3\right)x^5+\frac{3}{4}\left(Da^2b+Bab^2\right)x^4+Ba^3\log(x)+\left(Ca^2b+Aab^2\right)x^3-\frac{Aa^3}{x}+\frac{1}{2}\left(Da^3+3\,Ba^2b\right)x^2+\left(Ca^3+3\,Aa^2b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b^
2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b + A*
a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

mupad [B] time = 1.18, size = 121, normalized size = 0.98

$$\frac{{\left(b\,x^2+a\right)}^4\,D}{8\,b}-\frac{A\,a^3}{x}+\frac{A\,b^3\,x^5}{5}+\frac{B\,b^3\,x^6}{6}+\frac{C\,b^3\,x^7}{7}+B\,a^3\,\ln(x)+C\,a^3\,x+3\,A\,a^2\,b\,x+A\,a\,b^2\,x^3+\frac{3\,B\,a^2\,b\,x^2}{2}+\frac{3\,B\,a\,b^2\,x^4}{4}+C\,a^2\,b\,x^3+\frac{3\,C\,a\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] ((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b^
3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*b*
x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

sympy [A] time = 0.46, size = 150, normalized size = 1.21

$$-\frac{Aa^3}{x} + Ba^3\log(x) + \frac{Cb^3x^7}{7} + \frac{Db^3x^8}{8} + x^6\left(\frac{Bb^3}{6} + \frac{Dab^2}{2}\right) + x^5\left(\frac{Ab^3}{5} + \frac{3Cab^2}{5}\right) + x^4\left(\frac{3Bab^2}{4} + \frac{3Da^2b}{4}\right) + x^3\left(Aab^2 + Ca^2b\right) + x^2\left(\frac{3Ba^2b}{2} + \frac{Da^3}{2}\right) + x\left(3Aa^2b + Ca^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
+ D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D*a
**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) + x*
(3*A*a**2*b + C*a**3)
```

3.80 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD$$

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$a^2\log(x)(aC+3Ab)-\frac{a^3A}{2x^2}+a^2x(aD+3bB)-\frac{a^3B}{x}+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \int \left(a^2(3bB+aD)+\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\right.$$
$$= -\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3+$$

Mathematica [A] time = 0.06, size = 124, normalized size = 0.92

$$-\frac{a^3(A+2Bx-2Dx^3)}{2x^2}+a^2\log(x)(aC+3Ab)+\frac{1}{2}a^2bx(6B+x(3C+2Dx))+\frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx)))+\frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
+ (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A +
2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3, x]
```

fricas [A] time = 0.84, size = 147, normalized size = 1.09

$$\frac{60\,Db^3x^9+70\,Cb^3x^8+84\,\left(3\,Dab^2+Bb^3\right)x^7+105\,\left(3\,Cab^2+Ab^3\right)x^6+420\,\left(Da^2b+Bab^2\right)x^5-420\,Ba^3x+630\,\left(Ca^2b+Aab^2\right)x^4-210\,Aa^3+420\,\left(Da^3+3\,Ba^2b\right)x^3+420\,\left(Ca^3+3\,Aa^2b\right)x^2\log(x)}{420\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C*
a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a^2
*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3 +
3*A*a^2*b)*x^2*log(x))/x^2
```

giac [A] time = 0.41, size = 144, normalized size = 1.07

$$\frac{1}{7}\,Db^3x^7+\frac{1}{6}\,Cb^3x^6+\frac{3}{5}\,Dab^2x^5+\frac{1}{5}\,Bb^3x^5+\frac{3}{4}\,Cab^2x^4+\frac{1}{4}\,Ab^3x^4+Da^2bx^3+Bab^2x^3+\frac{3}{2}\,Ca^2bx^2+\frac{3}{2}\,Aab^2x^2+Da^3x+3\,Ba^2bx+\left(Ca^3+3\,Aa^2b\right)\log\left(|x|\right)-\frac{2\,Ba^3x+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*b
^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/2*
A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) - 1/2
*(2*B*a^3*x + A*a^3)/x^2
```

maple [A] time = 0.01, size = 144, normalized size = 1.07

$$\frac{Db^3x^7}{7}+\frac{C\,b^3x^6}{6}+\frac{B\,b^3x^5}{5}+\frac{3Da\,b^2x^5}{5}+\frac{A\,b^3x^4}{4}+\frac{3Ca\,b^2x^4}{4}+Ba\,b^2x^3+Da^2b\,x^3+\frac{3Aa\,b^2x^2}{2}+\frac{3C\,a^2b\,x^2}{2}+3A\,a^2b\ln(x)+3B\,a^2bx+C\,a^3\ln(x)+Da^3x-\frac{B\,a^3}{x}-\frac{A\,a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*B*x^5*b^3+3/5*D*x^5*a*b^2+1/4*A*b^3*x^4+3/4
*C*x^4*a*b^2+B*a*b^2*x^3+D*x^3*a^2*b+3/2*A*a*b^2*x^2+3/2*C*x^2*a^2*b+3*B*a^
2*b*x+a^3*D*x-1/2*a^3*A/x^2-a^3*B/x+3*A*ln(x)*a^2*b+C*ln(x)*a^3
```

maxima [A] time = 1.36, size = 139, normalized size = 1.03

$$\frac{1}{7}Db^3x^7 + \frac{1}{6}Cb^3x^6 + \frac{1}{5}\left(3\,Dab^2 + Bb^3\right)x^5 + \frac{1}{4}\left(3\,Cab^2 + Ab^3\right)x^4 + \left(Da^2b + Bab^2\right)x^3 + \frac{3}{2}\left(Ca^2b + Aab^2\right)x^2 + \left(Da^3 + 3\,Ba^2b\right)x + \left(Ca^3 + 3\,Aa^2b\right)\log(x) - \frac{2\,Ba^3x + Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b^
2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 + (D
*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^3)/
x^2
```

mupad [B] time = 1.26, size = 143, normalized size = 1.06

$$\frac{A\,b^3\,x^4}{4} - \frac{B\,a^3}{x} - \frac{A\,a^3}{2\,x^2} + \frac{B\,b^3\,x^5}{5} + \frac{C\,b^3\,x^6}{6} + C\,a^3\,\ln(x) + a^3\,x\,D + \frac{b^3\,x^7\,D}{7} + a^2\,b\,x^3\,D + \frac{3\,a\,b^2\,x^5\,D}{5} + 3\,B\,a^2\,b\,x + \frac{3\,A\,a\,b^2\,x^2}{2} + B\,a\,b^2\,x^3 + \frac{3\,C\,a^2\,b\,x^2}{2} + \frac{3\,C\,a\,b^2\,x^4}{4} + 3\,A\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/6
 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)/5
 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (3*C
*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

sympy [A] time = 0.67, size = 151, normalized size = 1.12

$$\frac{Cb^3x^6}{6} + \frac{Db^3x^7}{7} + a^2\left(3Ab + Ca\right)\log\left(x\right) + x^5\left(\frac{Bb^3}{5} + \frac{3Dab^2}{5}\right) + x^4\left(\frac{Ab^3}{4} + \frac{3Cab^2}{4}\right) + x^3\left(Bab^2 + Da^2b\right) + x^2\left(\frac{3Aab^2}{2} + \frac{3Ca^2b}{2}\right) + x\left(3Ba^2b + Da^3\right) + \frac{-Aa^3 - 2Ba^3x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
+ 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**2*
b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*a**
3 - 2*B*a**3*x)/(2*x**2)
```

3.81 $\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx$

Optimal. Leaf size=139

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(a$$

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1802}

$$-\frac{a^2(aC+3Ab)}{x}-\frac{a^3A}{3x^3}+a^2\log(x)(aD+3bB)-\frac{a^3B}{2x^2}+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -(a^3*A)/(3*x^3) - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx = \int \left(3ab(Ab+aC)+\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(3Ab+aC)}{x^2}+\frac{a^2(3bB+aD)}{x}+\right.$$

$$= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2$$

Mathematica [A] time = 0.05, size = 124, normalized size = 0.89

$$-\frac{a^3(2A+3x(B+2Cx))}{6x^3}+\frac{3a^2b\left(x^2(2C+Dx)-2A\right)}{2x}+a^2\log(x)(aD+3bB)+\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x)))
/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4, x]
```

fricas [A] time = 0.77, size = 147, normalized size = 1.06

$$\frac{10\,Db^3x^9 + 12\,Cb^3x^8 + 15\left(3\,Dab^2 + Bb^3\right)x^7 + 20\left(3\,Cab^2 + Ab^3\right)x^6 + 90\left(Da^2b + Bab^2\right)x^5 - 30\,Ba^3x + 180\left(Ca^2b + Aab^2\right)x^4 + 60\left(Da^3 + 3\,Ba^2b\right)x^3\log(x) - 20\,Aa^3 - 60\left(Ca^3 + 3\,Aa^2b\right)x^2}{60\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a*
b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b +
 A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3 +
3*A*a^2*b)*x^2)/x^3
```

giac [A] time = 0.40, size = 146, normalized size = 1.05

$$\frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{3}{4}\,Dab^2x^4 + \frac{1}{4}\,Bb^3x^4 + Cab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Da^2bx^2 + \frac{3}{2}\,Bab^2x^2 + 3\,Ca^2bx + 3\,Aab^2x + \left(Da^3 + 3\,Ba^2b\right)\log\left(|x|\right) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\left(Ca^3 + 3\,Aa^2b\right)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*x
^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*A*
a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C
*a^3 + 3*A*a^2*b)*x^2)/x^3
```

maple [A] time = 0.01, size = 146, normalized size = 1.05

$$\frac{Db^3x^6}{6} + \frac{C\,b^3x^5}{5} + \frac{B\,b^3x^4}{4} + \frac{3Da\,b^2x^4}{4} + \frac{A\,b^3x^3}{3} + Ca\,b^2x^3 + \frac{3Ba\,b^2x^2}{2} + \frac{3Da^2b\,x^2}{2} + 3Aa\,b^2x + 3B\,a^2b\ln(x) + 3C\,a^2bx + Da^3\ln(x) - \frac{3A\,a^2b}{x} - \frac{C\,a^3}{x} - \frac{B\,a^3}{2x^2} - \frac{A\,a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*x^4*b^3+3/4*D*x^4*a*b^2+1/3*A*b^3*x^3+C*x
^3*a*b^2+3/2*B*x^2*a*b^2+3/2*D*x^2*a^2*b+3*A*a*b^2*x+3*a^2*b*C*x-1/3*a^3*A/
x^3-1/2*a^3*B/x^2-3*a^2/x*A*b-a^3/x*C+3*B*ln(x)*a^2*b+D*ln(x)*a^3
```

maxima [A] time = 1.32, size = 142, normalized size = 1.02

$$\frac{1}{6}Db^3x^6+\frac{1}{5}Cb^3x^5+\frac{1}{4}\left(3Dab^2+Bb^3\right)x^4+\frac{1}{3}\left(3Cab^2+Ab^3\right)x^3+\frac{3}{2}\left(Da^2b+Bab^2\right)x^2+3\left(Ca^2b+Aab^2\right)x+\left(Da^3+3Ba^2b\right)\log(x)-\frac{3Ba^3x+2Aa^3+6\left(Ca^3+3Aa^2b\right)x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b^
2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x + (D
*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)
*x^2)/x^3
```

mupad [B] time = 1.37, size = 148, normalized size = 1.06

$$\frac{B\,b^3\,x^4}{4}-\frac{C\,a^3}{x}-\frac{B\,a^3}{2\,x^2}+\frac{C\,b^3\,x^5}{5}+\frac{b^3\,x^6\,D}{6}-\frac{A\,\left(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6\right)}{3\,x^3}+\frac{a^3\,\ln\left(x^2\right)\,D}{2}+\frac{3\,a^2\,b\,x^2\,D}{2}+3\,C\,a^2\,b\,x+\frac{3\,a\,b^2\,x^4\,D}{4}+\frac{3\,B\,a\,b^2\,x^2}{2}+C\,a\,b^2\,x^3+3\,B\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/6
 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2)*
D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*x^2
)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

sympy [A] time = 1.09, size = 155, normalized size = 1.12

$$\frac{Cb^3x^5}{5}+\frac{Db^3x^6}{6}+a^2\left(3Bb+Da\right)\log(x)+x^4\left(\frac{Bb^3}{4}+\frac{3Dab^2}{4}\right)+x^3\left(\frac{Ab^3}{3}+Cab^2\right)+x^2\left(\frac{3Bab^2}{2}+\frac{3Da^2b}{2}\right)+x\left(3Aab^2+3Ca^2b\right)+\frac{-2Aa^3-3Ba^3x+x^2\left(-18Aa^2b-6Ca^3\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
+ 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a**
2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-18*
A*a**2*b - 6*C*a**3))/(6*x**3)
```

3.82 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2} \, dx$

Optimal. Leaf size=151

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}$$

Rubi [A] time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax(Ab-aC)}{b^3}+\frac{x^4(bB-aD)}{4b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int\left(-\frac{a(Ab-aC)}{b^3}-\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{b^2}+\frac{(bB-aD)x^3}{b^2}+\frac{Cx^4}{b}+\cdots\right.$$

$$= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx\cdots}{6b\cdots}$$

$$= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx\cdots}{6b\cdots}$$

$$= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx\cdots}{6b\cdots}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 0.86

$$\frac{-60a^{3/2}\sqrt{b}\,(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+bx\left(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5Dx)))\right)-30a^2(aD-bB)\log\left(a+bx^2\right)}{60b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*x
^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) + a*
C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(60*
b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2), x]
```

fricas [A] time = 0.96, size = 332, normalized size = 2.20

$$\left[\frac{10\,Db^3x^6+12\,Cb^3x^5-15\left(Dab^2-Bb^3\right)x^4-20\left(Cab^2-Ab^3\right)x^3+30\left(Da^2b-Bab^2\right)x^2-30\left(Ca^2b-Aab^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+60\left(Ca^2b-Aab^2\right)x-30\left(Da^3-Ba^2b\right)\log\left(bx^2+a\right)}{60\,b^4},\ \frac{10\,Db^3x^6+12\,Cb^3x^5-15\left(Dab^2-Bb^3\right)x^4-20\left(Cab^2-Ab^3\right)x^3+30\left(Da^2b-Bab^2\right)x^2-60\left(Ca^2b-Aab^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+60\left(Ca^2b-Aab^2\right)x-30\left(Da^3-Ba^2b\right)\log\left(bx^2+a\right)}{60\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2
 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*b - A*a*b^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(C*a^2*b - A*a*b^2
)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12*C*b
^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b
- B*a*b^2)*x^2 - 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4]
```

giac [A] time = 0.43, size = 161, normalized size = 1.07

$$-\frac{\left(Ca^3 - Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\log\left(bx^2 + a\right)}{2\,b^4} + \frac{10\,Db^5x^6 + 12\,Cb^5x^5 - 15\,Dab^4x^4 + 15\,Bb^5x^4 - 20\,Cab^4x^3 + 20\,Ab^5x^3 + 30\,Da^2b^3x^2 - 30\,Bab^4x^2 + 60\,Ca^2b^3x - 60\,Aab^4x}{60\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*a
^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4*x
^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 30*B
*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

maple [A] time = 0.01, size = 176, normalized size = 1.17

$$\frac{Dx^6}{6b} + \frac{C\,x^5}{5b} + \frac{B\,x^4}{4b} - \frac{Da\,x^4}{4b^2} + \frac{A\,x^3}{3b} - \frac{Ca\,x^3}{3b^2} + \frac{A\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{Ba\,x^2}{2b^2} - \frac{C\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} + \frac{Da^2x^2}{2b^3} - \frac{Aax}{b^2} + \frac{B\,a^2\ln\left(b\,x^2 + a\right)}{2b^3} + \frac{C\,a^2x}{b^3} - \frac{Da^3\ln\left(b\,x^2 + a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/6*D*x^6/b+1/5*C*x^5/b+1/4/b*B*x^4-1/4/b^2*D*x^4*a+1/3/b*A*x^3-1/3/b^2*C*x
^3*a-1/2/b^2*B*x^2*a+1/2/b^3*D*x^2*a^2-1/b^2*A*a*x+1/b^3*a^2*C*x+1/2*a^2/b^
3*ln(b*x^2+a)*B-1/2*a^3/b^4*ln(b*x^2+a)*D+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)*A-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.96, size = 145, normalized size = 0.96

$$-\frac{\left(Ca^3 - Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{10\,Db^2x^6 + 12\,Cb^2x^5 - 15\left(Dab - Bb^2\right)x^4 - 20\left(Cab - Ab^2\right)x^3 + 30\left(Da^2 - Bab\right)x^2 + 60\left(Ca^2 - Aab\right)x}{60\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(10*D*b^2*x
^6 + 12*C*b^2*x^5 - 15*(D*a*b - B*b^2)*x^4 - 20*(C*a*b - A*b^2)*x^3 + 30*(D
```

```
*a^2 - B*a*b)*x^2 + 60*(C*a^2 - A*a*b)*x)/b^3 - 1/2*(D*a^3 - B*a^2*b)*log(b
*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \left(A + B\,x + C\,x^2 + x^3\,D\right)}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.43, size = 316, normalized size = 2.09

$$\frac{Cx^5}{5b} + \frac{Dx^6}{6b} + x^4\left(\frac{B}{4b} - \frac{Da}{4b^2}\right) + x^3\left(\frac{A}{3b} - \frac{Ca}{3b^2}\right) + x^2\left(-\frac{Ba}{2b^2} + \frac{Da^2}{2b^3}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ca^2}{b^3}\right) + \left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}\,(-Ab+Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right) + \left(-\frac{a^2(-Bb+Da)}{2b^4} + \frac{\sqrt{-a^3b^9}\,(-Ab+Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} + \frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + x**4*(B/(4*b) - D*a/(4*b**2)) + x**3*(A/(3*b)
 - C*a/(3*b**2)) + x**2*(-B*a/(2*b**2) + D*a**2/(2*b**3)) + x*(-A*a/b**2 +
C*a**2/b**3) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*a)
/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4)
 - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) + (-a**
2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (
B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*
(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b))
```

3.83 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$

Optimal. Leaf size=130

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^2(Ab-aC)}{2b^2}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^3(bB-aD)}{3b^2}-\frac{ax(bB-aD)}{b^3}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int\left(-\frac{a(bB-aD)}{b^3}+\frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{b^2}+\frac{Cx^3}{b}+\frac{Dx^4}{b}+\frac{a^2(bB-aD}{b^3}\right.$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{\int \frac{a^2(bB-aD)-a}{a+b}}{b^3}$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}-\frac{(a(Ab-aC)}{b^2}$$

$$= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD}{b}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 0.88

$$\frac{x\left(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx)))\right)+30a(aC-Ab)\log\left(a+bx^2\right)}{60b^3}-\frac{a^{3/2}(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a^
2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C + 4
*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2), x]
```

fricas [A] time = 0.94, size = 270, normalized size = 2.08

$$\left[\frac{12\,Db^2x^5+15\,Cb^2x^4-20\left(Dab-Bb^2\right)x^3-30\left(Cab-Ab^2\right)x^2+30\left(Da^2-Bab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+60\left(Da^2-Bab\right)x+30\left(Ca^2-Aab\right)\log\left(bx^2+a\right)}{60\,b^3},\ \frac{12\,Db^2x^5+15\,Cb^2x^4-20\left(Dab-Bb^2\right)x^3-30\left(Cab-Ab^2\right)x^2-60\left(Da^2-Bab\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+60\left(Da^2-Bab\right)x+30\left(Ca^2-Aab\right)\log\left(bx^2+a\right)}{60\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b - A
*b^2)*x^2 + 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/
b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b
 - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 60*(
D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

giac [A] time = 0.35, size = 137, normalized size = 1.05

$$\frac{\left(Ca^2 - Aab\right)\log\left(bx^2 + a\right)}{2\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{12\,Db^4x^5 + 15\,Cb^4x^4 - 20\,Dab^3x^3 + 20\,Bb^4x^3 - 30\,Cab^3x^2 + 30\,Ab^4x^2 + 60\,Da^2b^2x - 60\,Bab^3x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^3
+ 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a*b^
3*x)/b^5
```

maple [A] time = 0.01, size = 152, normalized size = 1.17

$$\frac{Dx^5}{5b} + \frac{C\,x^4}{4b} + \frac{B\,x^3}{3b} - \frac{Da\,x^3}{3b^2} + \frac{A\,x^2}{2b} + \frac{B\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{Ca\,x^2}{2b^2} - \frac{Da^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} - \frac{Aa\ln\left(b\,x^2 + a\right)}{2b^2} - \frac{Bax}{b^2} + \frac{C\,a^2\ln\left(b\,x^2 + a\right)}{2b^3} + \frac{Da^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/5*D*x^5/b+1/4*C*x^4/b+1/3*B/b*x^3-1/3/b^2*D*x^3*a+1/2/b*A*x^2-1/2/b^2*C*x
^2*a-B*a/b^2*x+1/b^3*a^2*D*x-1/2*a/b^2*ln(b*x^2+a)*A+1/2*a^2/b^3*ln(b*x^2+a
)*C+1/(a*b)^(1/2)*B*a^2/b^2*arctan(1/(a*b)^(1/2)*b*x)-a^3/b^3/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*D
```

maxima [A] time = 2.93, size = 127, normalized size = 0.98

$$\frac{\left(Ca^2 - Aab\right)\log\left(bx^2 + a\right)}{2\,b^3} - \frac{\left(Da^3 - Ba^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{12\,Db^2x^5 + 15\,Cb^2x^4 - 20\,\left(Dab - Bb^2\right)x^3 - 30\,\left(Cab - Ab^2\right)x^2 + 60\,\left(Da^2 - Bab\right)x}{60\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b
^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(A + B\,x + C\,x^2 + x^3\,D\right)}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.35, size = 274, normalized size = 2.11

$$\frac{Cx^4}{4b} + \frac{Dx^5}{5b} + x^3\left(\frac{B}{3b} - \frac{Da}{3b^2}\right) + x^2\left(\frac{A}{2b} - \frac{Ca}{2b^2}\right) + x\left(-\frac{Ba}{b^2} + \frac{Da^2}{b^3}\right) + \left(\frac{a\left(-Ab + Ca\right)}{2b^3} - \frac{\sqrt{-a^3b^7}\,\left(-Bb + Da\right)}{2b^7}\right)\log\left(x + \frac{-Aab + Ca^2 - 2b^3\left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}\,(-Bb+Da)}{2b^7}\right)}{-Bab + Da^2}\right) + \left(\frac{a\left(-Ab + Ca\right)}{2b^3} + \frac{\sqrt{-a^3b^7}\,\left(-Bb + Da\right)}{2b^7}\right)\log\left(x + \frac{-Aab + Ca^2 - 2b^3\left(\frac{a(-Ab+Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}\,(-Bb+Da)}{2b^7}\right)}{-Bab + Da^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b)
 - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3) -
 sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*
(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b
 + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b
**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sqrt(-a*
*3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

3.84 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{x(Ab-aC)}{b^2}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^2(bB-aD)}{2b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx &= \int \left(\frac{Ab-aC}{b^2}+\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}-\frac{a(Ab-aC)+a(bB-aD)x}{b^2\left(a+bx^2\right)}\right) dx\\
&= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\int \frac{a(Ab-aC)+a(bB-aD)x}{a+bx^2}\,dx}{b^2}\\
&= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{(a(Ab-aC))\int \frac{1}{a+bx^2}\,dx}{b^2}-\frac{(a(b
\end{aligned}
$$

$$
= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(
$$

Mathematica [A] time = 0.06, size = 95, normalized size = 0.86

$$
\frac{bx\left(-6a(2C+Dx)+12Ab+bx\left(6B+4Cx+3Dx^2\right)\right)+12\sqrt{a}\,\sqrt{b}\,(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+6a(aD-bB)\log\left(a+bx^2\right)}{12b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*Log
[a + b*x^2])/(12*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2), x]
```

fricas [A] time = 0.92, size = 238, normalized size = 2.14

$$
\left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\left(Dab-Bb^2\right)x^2-6\left(Cab-Ab^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\left(Cab-Ab^2\right)x+6\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{12\,b^3},\ \frac{3\,Db^2x^4+4\,Cb^2x^3-6\left(Dab-Bb^2\right)x^2+12\left(Cab-Ab^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-12\left(Cab-Ab^2\right)x+6\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{12\,b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^2
)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b -
A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*C*b
^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3]
```

giac [**A**] time = 0.40, size = 112, normalized size = 1.01

$$\frac{\left(Ca^{2}-Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{\left(Da^{2}-Bab\right)\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{3\,Db^{3}x^{4}+4\,Cb^{3}x^{3}-6\,Dab^{2}x^{2}+6\,Bb^{3}x^{2}-12\,Cab^{2}x+12\,Ab^{3}x}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b)
*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6*B
*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

maple [**A**] time = 0.00, size = 128, normalized size = 1.15

$$\frac{Dx^{4}}{4b}+\frac{C\,x^{3}}{3b}-\frac{Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{B\,x^{2}}{2b}+\frac{C\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}-\frac{Da\,x^{2}}{2b^{2}}+\frac{Ax}{b}-\frac{Ba\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{Cax}{b^{2}}+\frac{Da^{2}\ln\left(b\,x^{2}+a\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/4*D*x^4/b+1/3*C*x^3/b+1/2*B/b*x^2-1/2/b^2*D*x^2*a+A/b*x-1/b^2*a*C*x-1/2*B
*a/b^2*ln(b*x^2+a)+1/2*a^2/b^3*ln(b*x^2+a)*D-1/(a*b)^(1/2)*A*a/b*arctan(1/(
a*b)^(1/2)*b*x)+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [**A**] time = 3.00, size = 98, normalized size = 0.88

$$\frac{\left(Ca^{2}-Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{3\,Dbx^{4}+4\,Cbx^{3}-6\,(Da-Bb)x^{2}-12\,(Ca-Ab)x}{12\,b^{2}}+\frac{\left(Da^{2}-Bab\right)\log\left(bx^{2}+a\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 + 4
*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b)*
log(b*x^2 + a)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(A + B\,x + C\,x^2 + x^3\,D\right)}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 1.65, size = 245, normalized size = 2.21

$$\frac{Cx^3}{3b} + \frac{Dx^4}{4b} + x^2\left(\frac{B}{2b} - \frac{Da}{2b^2}\right) + x\left(\frac{A}{b} - \frac{Ca}{b^2}\right) + \left(\frac{a\left(-Bb + Da\right)}{2b^3} - \frac{\sqrt{-ab^7}\,\left(-Ab + Ca\right)}{2b^6}\right)\log\left(x + \frac{Bab - Da^2 + 2b^3\left(\frac{a(-Bb+Da)}{2b^3} - \frac{\sqrt{-ab^7}\,(-Ab+Ca)}{2b^6}\right)}{-Ab^2 + Cab}\right) + \left(\frac{a\left(-Bb + Da\right)}{2b^3} + \frac{\sqrt{-ab^7}\,\left(-Ab + Ca\right)}{2b^6}\right)\log\left(x + \frac{Bab - Da^2 + 2b^3\left(\frac{a(-Bb+Da)}{2b^3} + \frac{\sqrt{-ab^7}\,(-Ab+Ca)}{2b^6}\right)}{-Ab^2 + Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a/
b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*log
(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqrt(-a
*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D
*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b))
```

3.85 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

Optimal. Leaf size=92

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}+\frac{x(bB-aD)}{b^2}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \int \left(\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}-\frac{a(bB-aD)-b(Ab-aC)x}{b^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{b}-\frac{(a(bB-aD))\int \frac{1}{a+bx^2}\,dx}{b^2}$$

$$= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log\left(a+\right.}{2b^2}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 0.88

$$\frac{3(Ab-aC)\log\left(a+bx^2\right)+x(-6aD+6bB+bx(3C+2Dx))}{6b^2}+\frac{\sqrt{a}\,(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2), x]
```

fricas [A] time = 0.98, size = 180, normalized size = 1.96

$$\left[\frac{2\,Dbx^3+3\,Cbx^2+3\,(Da-Bb)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2},\ \frac{2\,Dbx^3+3\,Cbx^2+6\,(Da-Bb)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*D*b*x^3 + 3*C*b*x^2 + 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a
))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

giac [A] time = 0.42, size = 88, normalized size = 0.96

$$-\frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,b^2} + \frac{\left(Da^2 - Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{2\,Db^2x^3 + 3\,Cb^2x^2 - 6\,Dabx + 6\,Bb^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x)/
b^3
```

maple [A] time = 0.01, size = 106, normalized size = 1.15

$$\frac{Dx^3}{3b} - \frac{Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{C\,x^2}{2b} + \frac{Da^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} + \frac{A\ln\left(b\,x^2 + a\right)}{2b} + \frac{Bx}{b} - \frac{Ca\ln\left(b\,x^2 + a\right)}{2b^2} - \frac{Dax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/3*D*x^3/b+1/2*C*x^2/b+B/b*x-1/b^2*a*D*x+1/2*A/b*ln(b*x^2+a)-1/2/b^2*ln(b*
x^2+a)*a*C-1/(a*b)^(1/2)*B*a/b*arctan(1/(a*b)^(1/2)*b*x)+1/b^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*a^2*D
```

maxima [A] time = 3.01, size = 82, normalized size = 0.89

$$-\frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,b^2} + \frac{\left(Da^2 - Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{2\,Dbx^3 + 3\,Cbx^2 - 6\,(Da - Bb)x}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

sympy [B] time = 0.99, size = 211, normalized size = 2.29

$$\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+x\left(\frac{B}{b}-\frac{Da}{b^2}\right)+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)+\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
- sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b
 + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) + (-
(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b
+ C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5
)))/(-B*b + D*a))
```

3.86 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} \, dx$

Optimal. Leaf size=73

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1810, 635, 205, 260}

$$\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx &= \int \left(\frac{C}{b}+\frac{Dx}{b}+\frac{Ab-aC+(bB-aD)x}{b\left(a+bx^2\right)}\right)dx \\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{\int \frac{Ab-aC+(bB-aD)x}{a+bx^2}\,dx}{b} \\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\int \frac{1}{a+bx^2}\,dx}{b}+\frac{(bB-aD)\int \frac{x}{a+bx^2}\,dx}{b} \\ &= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.93

$$\frac{\frac{2\sqrt{b}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)+bx(2C+Dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2), x]
```

fricas [A] time = 0.76, size = 157, normalized size = 2.15

$$\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{Dabx^2+2\,Cabx-2\,(Ca-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-\left(Da^2-Bab\right)\log\left(bx^2+a\right)}{2\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D*a
*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^2
 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

giac [A] time = 0.46, size = 66, normalized size = 0.90

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}+\frac{Dbx^2+2\,Cbx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x^
2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

maple [A] time = 0.00, size = 83, normalized size = 1.14

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Ca\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{Dx^2}{2b}+\frac{B\ln\left(b\,x^2+a\right)}{2b}+\frac{Cx}{b}-\frac{Da\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] 1/2*D*x^2/b+C*x/b+1/2*B/b*ln(b*x^2+a)-1/2/b^2*ln(b*x^2+a)*a*D+1/(a*b)^(1/2)
*A*arctan(1/(a*b)^(1/2)*b*x)-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*C
```

maxima [A] time = 2.93, size = 64, normalized size = 0.88

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{Dx^2+2\,Cx}{2\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

mupad [B] time = 1.42, size = 79, normalized size = 1.08

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

```
[Out] (B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/b
 + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(1/
2)*x)/a^(1/2)))/b^(3/2)
```

sympy [B] time = 0.88, size = 219, normalized size = 3.00

$$\frac{Cx}{b} + \frac{Dx^2}{2b} + \left(-\frac{-Bb + Da}{2b^2} - \frac{\sqrt{-ab^5}\,(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right) + \left(-\frac{-Bb + Da}{2b^2} + \frac{\sqrt{-ab^5}\,(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} + \frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)
/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) - s
qrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*a)/
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 -
 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))
/(-A*b**2 + C*a*b))
```

3.87 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx &= \int \left(\frac{D}{b}+\frac{A}{ax}+\frac{a(bB-aD)-b(Ab-aC)x}{ab\left(a+bx^2\right)}\right)dx\\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}+\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{ab}\\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{a}+\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{b}\\
&= \frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.01

$$\frac{(aC-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}-\frac{(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
(A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.92, size = 158, normalized size = 2.19

$$\left[\frac{2\,Dabx+2\,Ab^2\log(x)-(Da-Bb)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{2\,Dabx+2\,Ab^2\log(x)-2\,(Da-Bb)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*D*a*b*x + 2*A*b^2*log(x) - (D*a - B*b)*sqrt(-a*b)*log((b*x^2 + 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1/2
*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

giac [A] time = 0.45, size = 66, normalized size = 0.92

$$\frac{Dx}{b}+\frac{A\log\left(|x|\right)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) +
 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Da\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,\,b}+\frac{A\ln(x)}{a}-\frac{A\ln\left(b\,x^2+a\right)}{2a}+\frac{C\ln\left(b\,x^2+a\right)}{2b}+\frac{Dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x)
```

```
[Out] D*x/b-1/2*A/a*ln(b*x^2+a)+1/2/b*ln(b*x^2+a)*C+1/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*B-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A/a*ln(x)
```

maxima [A] time = 2.93, size = 65, normalized size = 0.90

$$\frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*
(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.88 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)} \, dx$

Optimal. Leaf size=76

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx = \int \left(\frac{A}{ax^2}+\frac{B}{ax}+\frac{-Ab+aC-(bB-aD)x}{a\left(a+bx^2\right)}\right)dx$$
$$= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{\int \frac{-Ab+aC-(bB-aD)x}{a+bx^2}\,dx}{a}$$
$$= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{(-Ab+aC)\int \frac{1}{a+bx^2}\,dx}{a}+\frac{(-bB+aD)\int \frac{x}{a+bx^2}\,dx}{a}$$
$$= -\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.99

$$\frac{(aC-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}+\frac{(aD-bB)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
 + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.77, size = 165, normalized size = 2.17

$$\left[\frac{2\,Babx\log(x)+(Ca-Ab)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,Aab+\left(Da^2-Bab\right)x\log\left(bx^2+a\right)}{2\,a^2bx},\ \frac{2\,Babx\log(x)+2\,(Ca-Ab)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\,Aab+\left(Da^2-Bab\right)x\log\left(bx^2+a\right)}{2\,a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x
), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a)
- 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

giac [A] time = 0.38, size = 68, normalized size = 0.89

$$\frac{B\log\left(|x|\right)}{a}+\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*
a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

maple [A] time = 0.01, size = 83, normalized size = 1.09

$$-\frac{Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{B\ln(x)}{a}-\frac{B\ln\left(b\,x^2+a\right)}{2a}+\frac{D\ln\left(b\,x^2+a\right)}{2b}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x)
```

```
[Out] -1/2*B/a*ln(b*x^2+a)+1/2/b*ln(b*x^2+a)*D-1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*A*b+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a/x+B/a*ln(x)
```

maxima [A] time = 3.00, size = 67, normalized size = 0.88

$$\frac{B\log(x)}{a}+\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a - B
*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

mupad [B] time = 1.21, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^2+a\right)\,D}{2\,b}-\frac{A}{a\,x}-\frac{B\,\left(\ln\left(b\,x^2+a\right)-2\,\ln(x)\right)}{2\,a}-\frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
- (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(1/
2)))/(a^(1/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.89 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)} \, dx$

Optimal. Leaf size=92

$$-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1802, 635, 205, 260}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)
*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx=\int \left(\frac{A}{ax^3}+\frac{B}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{-a(bB-aD)+b(Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\int \frac{-a(bB-aD)+b(Ab-aC)x}{a+bx^2}\,dx}{a^2}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(b(Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(bB-aD)\int \frac{1}{a+bx^2}\,\text{d}}{a}$$

$$=-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+\right.}{2a^2}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.91

$$\frac{(Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-Ab)-\frac{aA}{x^2}+\frac{2\sqrt{a}\,(aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 1.04, size = 205, normalized size = 2.23

$$\left[-\frac{(Da-Bb)\sqrt{-ab}\,x^2\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,Babx+\left(Cab-Ab^2\right)x^2\log\left(bx^2+a\right)-2\left(Cab-Ab^2\right)x^2\log(x)+Aab}{2\,a^2bx^2},\ \frac{2\,(Da-Bb)\sqrt{ab}\,x^2\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\,Babx-\left(Cab-Ab^2\right)x^2\log\left(bx^2+a\right)+2\left(Cab-Ab^2\right)x^2\log(x)-Aab}{2\,a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 2*B*a*b*x + (C*a*b - A*b^2)*x^2*log(b*x^2 + a) - 2*(C*a*b - A*b^2)*x
^2*log(x) + A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(sqr
t(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b - A
*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

giac [A] time = 0.39, size = 80, normalized size = 0.87

$$\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ca-Ab)\log\left(|x|\right)}{a^2}-\frac{2\,Bax+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 102, normalized size = 1.11

$$-\frac{Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(b\,x^2+a\right)}{2a^2}+\frac{C\ln(x)}{a}-\frac{C\ln\left(b\,x^2+a\right)}{2a}-\frac{B}{ax}-\frac{A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x)
```

```
[Out] 1/2*A/a^2*b*ln(b*x^2+a)-1/2/a*ln(b*x^2+a)*C-1/(a*b)^(1/2)*B/a*b*arctan(1/(a
*b)^(1/2)*b*x)+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-1/2*A/a/x^2-B/a/x-
A/a^2*b*ln(x)+1/a*ln(x)*C
```

maxima [A] time = 3.05, size = 76, normalized size = 0.83

$$\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ca-Ab)\log(x)}{a^2}-\frac{2\,Bx+A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

mupad [B] time = 1.30, size = 97, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)D}{\sqrt{a}\,\sqrt{b}}-\frac{B}{a\,x}-\frac{C\,\left(\ln\left(b\,x^2+a\right)-2\,\ln(x)\right)}{2\,a}-\frac{A}{2\,a\,x^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{A\,b\,\ln(x)}{a^2}-\frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*x
^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*b*
log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.90 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{a}\,(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3Ab-5aC)}{2b^3}-\frac{x^3(3Ab-5aC)}{6ab^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{a(2bB-3aD)\,\mathrm{lo}}{2b^4}$$

Rubi [A] time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^3(3Ab-5aC)}{6ab^2}+\frac{x(3Ab-5aC)}{2b^3}-\frac{\sqrt{a}\,(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x^2(2bB-3aD)}{2b^3}-\frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4}+\frac{Dx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(3Ab-5aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(-\frac{a(3Ab-5aC)}{b^2}-\frac{2a(2bB-3aD)x}{b^2}+\frac{(3Ab-5aC)x^2}{b}\right.}{2ab}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)\right.}{2ab\left(a\right.}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)\right.}{2ab\left(a\right.}$$

$$= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)\right.}{2ab\left(a\right.}$$

Mathematica [A] time = 0.13, size = 139, normalized size = 0.79

$$\frac{\frac{6a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{a+bx^2}+12bx(Ab-2aC)+6\sqrt{a}\sqrt{b}(5aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6bx^2(bB-2aD)+6a(3aD-2bB)\log\left(a+bx^2\right)+4b^2Cx^3+3b^2Dx^4}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4 +
 (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*(-
3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a + b
*x^2])/(12*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(A + Bx + Cx^2 + Dx^3\right)}{\left(a + bx^2\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.58, size = 468, normalized size = 2.66

$$\left[\frac{3 \, D b^{3} x^{6} + 4 \, C b^{3} x^{5} - 3 \, {\left(3 \, D a b^{2} - 2 \, B b^{3}\right)} x^{4} + 6 \, D a^{3} - 6 \, B a^{2} b - 4 \, {\left(5 \, C a b^{2} - 3 \, A b^{3}\right)} x^{3} - 6 \, {\left(2 \, D a^{2} b - B a b^{2}\right)} x^{2} - 3 \, {\left(5 \, C a^{2} b - 3 \, A a b^{2} + {\left(5 \, C a b^{2} - 3 \, A b^{3}\right)} x^{2}\right)} \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2} - 2 \, b x \sqrt{-\frac{a}{b}} - a}{b x^{2} + a}\right) - 6 \, {\left(5 \, C a^{2} b - 3 \, A a b^{2}\right)} x + 6 \, {\left(3 \, D a^{3} - 2 \, B a^{2} b + {\left(3 \, D a^{2} b - 2 \, B a b^{2}\right)} x^{2}\right)} \log\left(b x^{2} + a\right)}{12 \, {\left(b^{5} x^{2} + a b^{4}\right)}}, \frac{3 \, D b^{3} x^{6} + 4 \, C b^{3} x^{5} - 3 \, {\left(3 \, D a b^{2} - 2 \, B b^{3}\right)} x^{4} + 6 \, D a^{3} - 6 \, B a^{2} b - 4 \, {\left(5 \, C a b^{2} - 3 \, A b^{3}\right)} x^{3} - 6 \, {\left(2 \, D a^{2} b - B a b^{2}\right)} x^{2} + 6 \, {\left(5 \, C a^{2} b - 3 \, A a b^{2} + {\left(5 \, C a b^{2} - 3 \, A b^{3}\right)} x^{2}\right)} \sqrt{\frac{a}{b}} \arctan\left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right) - 6 \, {\left(5 \, C a^{2} b - 3 \, A a b^{2}\right)} x + 6 \, {\left(3 \, D a^{3} - 2 \, B a^{2} b + {\left(3 \, D a^{2} b - 2 \, B a b^{2}\right)} x^{2}\right)} \log\left(b x^{2} + a\right)}{12 \, {\left(b^{5} x^{2} + a b^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3*(
5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a
^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*
b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a
^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2
+ 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^2*b + (3*
D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.43, size = 159, normalized size = 0.90

$$\frac{{\left(5 \, C a^{2} - 3 \, A a b\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} b^{3}} + \frac{{\left(3 \, D a^{2} - 2 \, B a b\right)} \log\left(b x^{2} + a\right)}{2 \, b^{4}} + \frac{D a^{3} - B a^{2} b - {\left(C a^{2} b - A a b^{2}\right)} x}{2 \, {\left(b x^{2} + a\right)} b^{4}} + \frac{3 \, D b^{6} x^{4} + 4 \, C b^{6} x^{3} - 12 \, D a b^{5} x^{2} + 6 \, B b^{6} x^{2} - 24 \, C a b^{5} x + 12 \, A b^{6} x}{12 \, b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a^
2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2
```

```
)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^2 +
 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

maple [A] time = 0.01, size = 201, normalized size = 1.14

$$\frac{Dx^4}{4b^2}+\frac{Cx^3}{3b^2}+\frac{Aax}{2\left(bx^2+a\right)b^2}-\frac{3Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^2}+\frac{Bx^2}{2b^2}-\frac{Ca^2x}{2\left(bx^2+a\right)b^3}+\frac{5Ca^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3}-\frac{Dax^2}{b^3}+\frac{Ax}{b^2}-\frac{Ba^2}{2\left(bx^2+a\right)b^3}-\frac{Ba\ln\left(bx^2+a\right)}{b^3}-\frac{2Cax}{b^3}+\frac{Da^3}{2\left(bx^2+a\right)b^4}+\frac{3Da^2\ln\left(bx^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*D*x^4/b^2+1/3/b^2*C*x^3+1/2*B/b^2*x^2-1/b^3*D*x^2*a+1/b^2*A*x-2/b^3*a*C
*x+1/2*a/b^2/(b*x^2+a)*A*x-1/2*a^2/b^3/(b*x^2+a)*C*x-1/2/(b*x^2+a)*B*a^2/b^
3+1/2*a^3/b^4/(b*x^2+a)*D-B*a/b^3*ln(b*x^2+a)+3/2*a^2/b^4*ln(b*x^2+a)*D-3/2
*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+5/2*a^2/b^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.83, size = 150, normalized size = 0.85

$$\frac{Da^3-Ba^2b-\left(Ca^2b-Aab^2\right)x}{2\left(b^5x^2+ab^4\right)}+\frac{\left(5\,Ca^2-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-Ab)x}{12\,b^3}+\frac{\left(3\,Da^2-2\,Bab\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C*
a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 + 4*
C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 - 2*
B*a*b)*log(b*x^2 + a)/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 4.77, size = 335, normalized size = 1.90

$$\frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}+x^2\left(\frac{B}{2b^2}-\frac{Da}{b^3}\right)+x\left(\frac{A}{b^2}-\frac{2Ca}{b^3}\right)+\left(\frac{a\left(-2Bb+3Da\right)}{2b^4}-\frac{\sqrt{-ab^9}\,\left(-3Ab+5Ca\right)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}-\frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)+\left(\frac{a\left(-2Bb+3Da\right)}{2b^4}+\frac{\sqrt{-ab^9}\,\left(-3Ab+5Ca\right)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}+\frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)+\frac{-Ba^2b+Da^3+x\left(Aab^2-Ca^2b\right)}{2ab^4+2b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b**
2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*
C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*
b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) +
(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log
(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b*
*9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b + D*a**
3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

3.91 $\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=154

$$\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(Ab-2aC)}{2ab^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3bB}{2}$$

Rubi [A] time = 0.24, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x^2(Ab-2aC)}{2ab^2}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{x(3bB-5aD)}{2b^3}-\frac{\sqrt{a}\,(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)+2(Ab-2aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \left(-\frac{a(3bB-5aD)}{b^2}+\frac{2(Ab-2aC)x}{b}-\frac{2aDx^2}{b}+\frac{a^2(3bB \cdots}{\cdots}\right.}{2ab}$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{a \cdots}{\cdots}}{\cdots}$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} + \frac{(Ab \cdots}{\cdots}$$

$$= \frac{(3bB-5aD)x}{2b^3} - \frac{(Ab-2aC)x^2}{2ab^2} + \frac{Dx^3}{3b^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\sqrt{a} \cdots}{\cdots}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 0.83

$$\frac{a(-a(C+Dx)+Ab+bBx)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}+\frac{\sqrt{a}\,(5aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(bB-2aD)}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*x
 - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(A + Bx + Cx^2 + Dx^3\right)}{\left(a + bx^2\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.86, size = 372, normalized size = 2.42

$$\left[\frac{4\,Db^2x^5 + 6\,Cb^2x^4 + 6\,Cabx^2 - 4\left(5\,Dab - 3\,Bb^2\right)x^3 - 6\,Ca^2 + 6\,Aab + 3\left(5\,Da^2 - 3\,Bab + \left(5\,Dab - 3\,Bb^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 6\left(5\,Da^2 - 3\,Bab\right)x - 6\left(2\,Ca^2 - Aab + \left(2\,Cab - Ab^2\right)x^2\right)\log\left(bx^2 + a\right)}{12\left(b^4x^2 + ab^3\right)}, \frac{2\,Db^2x^5 + 3\,Cb^2x^4 + 3\,Cabx^2 - 2\left(5\,Dab - 3\,Bb^2\right)x^3 - 3\,Ca^2 + 3\,Aab + 3\left(5\,Da^2 - 3\,Bab + \left(5\,Dab - 3\,Bb^2\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 3\left(5\,Da^2 - 3\,Bab\right)x - 3\left(2\,Ca^2 - Aab + \left(2\,Cab - Ab^2\right)x^2\right)\log\left(bx^2 + a\right)}{6\left(b^4x^2 + ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
- 6*C*a^2 + 6*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B*a*
b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2
 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b - 3*B*
b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x
^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*(2*C*a^
2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.38, size = 131, normalized size = 0.85

$$-\frac{(2\,Ca - Ab)\log\left(bx^2 + a\right)}{2\,b^3} + \frac{\left(5\,Da^2 - 3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} - \frac{Ca^2 - Aab + \left(Da^2 - Bab\right)x}{2\left(bx^2 + a\right)b^3} + \frac{2\,Db^4x^3 + 3\,Cb^4x^2 - 12\,Dab^3x + 6\,Bb^4x}{6\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)/b^
6
```

maple [A] time = 0.01, size = 177, normalized size = 1.15

$$\frac{Dx^3}{3b^2}+\frac{Bax}{2\left(b\,x^2+a\right)b^2}-\frac{3Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{C\,x^2}{2b^2}-\frac{Da^2x}{2\left(b\,x^2+a\right)b^3}+\frac{5Da^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{Aa}{2\left(b\,x^2+a\right)b^2}+\frac{A\ln\left(b\,x^2+a\right)}{2b^2}+\frac{Bx}{b^2}-\frac{C\,a^2}{2\left(b\,x^2+a\right)b^3}-\frac{Ca\ln\left(b\,x^2+a\right)}{b^3}-\frac{2Dax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*D*x^3/b^2+1/2/b^2*C*x^2+1/b^2*B*x-2/b^3*a*D*x+1/2/b^2/(b*x^2+a)*B*x*a-1
/2/b^3/(b*x^2+a)*a^2*D*x+1/2/(b*x^2+a)*A*a/b^2-1/2/b^3/(b*x^2+a)*a^2*C+1/2*
A/b^2*ln(b*x^2+a)-1/b^3*ln(b*x^2+a)*a*C-3/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*a*B+5/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*D
```

maxima [A] time = 3.00, size = 127, normalized size = 0.82

$$-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\left(b^4x^2+ab^3\right)}-\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3}+\frac{\left(5\,Da^2-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{2\,Dbx^3+3\,Cbx^2-6\,(2\,Da-Bb)x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a - A
*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 3.87, size = 289, normalized size = 1.88

$$\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+x\left(\frac{B}{b^2}-\frac{2Da}{b^3}\right)+\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\frac{Aab-Ca^2+x\left(Bab-Da^2\right)}{2ab^3+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*C
*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4
*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4
*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*
B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2
*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a)) + (A*a
*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

3.92 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} dx$

Optimal. Leaf size=134

$$\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 1802, 635, 205, 260}

$$-\frac{x(Ab-3aC)}{2ab^2}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 3*a*C)*x)/(2*a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) - (A
*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)+(Ab-3aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)} - \frac{\int\left(A-\frac{3aC}{b}-\frac{2aDx}{b}-\frac{a(Ab-3aC)+2a(bB-2aD)x}{b\left(a+bx^2\right)}\right.}{2ab}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int\frac{a(Ab-3aC)+2a(bB-2}{a+bx^2}}{2ab^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\int\frac{1}{a+bx}}{2b^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\tan^{-1}(}{2\sqrt{a}\,b^{5/2}}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 0.75

$$\frac{\frac{a^2(-D)+ab(B+Cx)-Ab^2x}{a+bx^2}+\frac{\sqrt{b}\,(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log\left(a+bx^2\right)+2bCx+bDx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (Sq
rt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*Lo
g[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.88, size = 357, normalized size = 2.66

$$\left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+\left(3\,Ca^2-Aab+\left(3\,Cab-Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Ca^2b-Aab^2\right)x-2\left(2\,Da^3-Ba^2b+\left(2\,Da^2b-Bab^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(ab^4x^2+a^2b^3\right)},\ \frac{Dab^2x^4+2\,Cab^2x^3+Da^2bx^2-Da^3+Ba^2b-\left(3\,Ca^2-Aab+\left(3\,Cab-Ab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Ca^2b-Aab^2\right)x-\left(2\,Da^3-Ba^2b+\left(2\,Da^2b-Bab^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(ab^4x^2+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b +
 (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^2*b
 + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/2*(D
*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2 - A*a
*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^2*b -
A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a)
)/(a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.39, size = 111, normalized size = 0.83

$$-\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}-\frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}+\frac{Db^2x^2+2\,Cb^2x}{2\,b^4}-\frac{Da^2-Bab-\left(Cab-Ab^2\right)x}{2\left(bx^2+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*b
)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a*b
 - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

maple [A] time = 0.01, size = 154, normalized size = 1.15

$$-\frac{Ax}{2\left(b\,x^{2}+a\right)b}+\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}+\frac{Cax}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3Ca\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^{2}}+\frac{Dx^{2}}{2b^{2}}+\frac{Ba}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}+\frac{Cx}{b^{2}}-\frac{Da^{2}}{2\left(b\,x^{2}+a\right)b^{3}}-\frac{Da\ln\left(b\,x^{2}+a\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*D*x^2/b^2+1/b^2*C*x-1/2/b/(b*x^2+a)*A*x+1/2/b^2/(b*x^2+a)*a*C*x+1/2/b^2
/(b*x^2+a)*B*a-1/2/b^3/(b*x^2+a)*a^2*D+1/2/b^2*ln(b*x^2+a)*B-1/b^3*ln(b*x^2
+a)*a*D+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-3/2/b^2/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*a*C
```

maxima [A] time = 2.96, size = 108, normalized size = 0.81

$$-\frac{Da^{2}-Bab-\left(Cab-Ab^{2}\right)x}{2\left(b^{4}x^{2}+ab^{3}\right)}-\frac{\left(3\,Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{2}}+\frac{Dx^{2}+2\,Cx}{2\,b^{2}}-\frac{\left(2\,Da-Bb\right)\log\left(bx^{2}+a\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a - A
*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2*(
2*D*a - B*b)*log(b*x^2 + a)/b^3
```

mupad [B] time = 1.29, size = 152, normalized size = 1.13

$$\frac{B\,\ln\left(b\,x^{2}+a\right)}{2\,b^{2}}+\frac{x^{2}\,D}{2\,b^{2}}+\frac{C\,x}{b^{2}}-\frac{a^{2}\,D}{2\,b^{3}\,\left(b\,x^{2}+a\right)}+\frac{B\,a}{2\,b^{2}\,\left(b\,x^{2}+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^{2}+a\right)}+\frac{C\,a\,x}{2\,\left(b^{3}\,x^{2}+a\,b^{2}\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{3\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}-\frac{a\,\ln\left(b\,x^{2}+a\right)\,D}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*(
a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*x)
/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
- (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^2)*D
)/b^3
```

sympy [B] time = 4.61, size = 284, normalized size = 2.12

$$\frac{Cx}{b^{2}}+\frac{Dx^{2}}{2b^{2}}+\left(-\frac{-Bb+2Da}{2b^{3}}-\frac{\sqrt{-ab^{7}}\,\left(-Ab+3Ca\right)}{4ab^{6}}\right)\log\left(x+\frac{2Bab-4Da^{2}-4ab^{3}\left(-\frac{-Bb+2Da}{2b^{3}}-\frac{\sqrt{-ab^{7}}\,(-Ab+3Ca)}{4ab^{6}}\right)}{-Ab^{2}+3Cab}\right)+\left(-\frac{-Bb+2Da}{2b^{3}}+\frac{\sqrt{-ab^{7}}\,\left(-Ab+3Ca\right)}{4ab^{6}}\right)\log\left(x+\frac{2Bab-4Da^{2}-4ab^{3}\left(-\frac{-Bb+2Da}{2b^{3}}+\frac{\sqrt{-ab^{7}}\,(-Ab+3Ca)}{4ab^{6}}\right)}{-Ab^{2}+3Cab}\right)+\frac{Bab-Da^{2}+x\left(-Ab^{2}+Cab\right)}{2ab^{3}+2b^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D
*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b
)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*l
og(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b
**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x
*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

3.93 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1804, 1810, 635, 205, 260}

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2aCx-2aDx^2}{a+bx^2}\,dx}{2ab}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2aD}{b}-\frac{a(bB-3aD)+2abCx}{b\left(a+bx^2\right)}\right)dx}{2ab}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(bB-3aD)+2abCx}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{C\int \frac{x}{a+bx^2}\,dx}{b}+\frac{(bB-3aD)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx\right.}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.91

$$\frac{aC+aDx-Ab-bBx}{2b^2\left(a+bx^2\right)}-\frac{(3aD-bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B) +
 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x^2
])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.81, size = 287, normalized size = 2.84

$$\left[\frac{4\,Dab^2x^3+2\,Ca^2b-2\,Aab^2-\left(3\,Da^2-Bab+\left(3\,Dab-Bb^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Da^2b-Bab^2\right)x+2\left(Cab^2x^2+Ca^2b\right)\log\left(bx^2+a\right)}{4\left(ab^4x^2+a^2b^3\right)},\ \frac{2\,Dab^2x^3+Ca^2b-Aab^2-\left(3\,Da^2-Bab+\left(3\,Dab-Bb^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Da^2b-Bab^2\right)x+\left(Cab^2x^2+Ca^2b\right)\log\left(bx^2+a\right)}{2\left(ab^4x^2+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 - (3*D*a^2 - B*a*b + (3*D*a*b -
 B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x
^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*b +
(3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b - B*a*b
^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 0.43, size = 81, normalized size = 0.80

$$\frac{Dx}{b^2}+\frac{C\log\left(bx^2+a\right)}{2\,b^2}-\frac{\left(3\,Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{Ca-Ab+\left(Da-Bb\right)x}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 127, normalized size = 1.26

$$-\frac{Bx}{2\left(b\,x^2+a\right)b}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}+\frac{Dax}{2\left(b\,x^2+a\right)b^2}-\frac{3Da\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^2}-\frac{A}{2\left(b\,x^2+a\right)b}+\frac{Ca}{2\left(b\,x^2+a\right)b^2}+\frac{C\ln\left(b\,x^2+a\right)}{2b^2}+\frac{Dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] D*x/b^2-1/2/b/(b*x^2+a)*B*x+1/2/b^2/(b*x^2+a)*a*D*x-1/2/b/(b*x^2+a)*A+1/2/b
^2/(b*x^2+a)*a*C+1/2*C*ln(b*x^2+a)/b^2+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*B-3/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*D
```

maxima [A] time = 3.02, size = 84, normalized size = 0.83

$$\frac{Ca - Ab + (Da - Bb)x}{2\left(b^3x^2 + ab^2\right)} + \frac{Dx}{b^2} + \frac{C\log\left(bx^2 + a\right)}{2\,b^2} - \frac{(3\,Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*x
^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A + B\,x + C\,x^2 + x^3\,D\right)}{\left(b\,x^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

sympy [B] time = 5.76, size = 212, normalized size = 2.10

$$\frac{Dx}{b^2} + \left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}\,(-Bb + 3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}\,(-Bb+3Da)}{4ab^5}\right)}{-Bb + 3Da}\right) + \left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}\,(-Bb + 3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}\,(-Bb+3Da)}{4ab^5}\right)}{-Bb + 3Da}\right) + \frac{-Ab + Ca + x\,(-Bb + Da)}{2ab^2 + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x + (
2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-
B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(
x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)
))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

3.94 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=93

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aC)-a\left(B-\frac{aD}{b}\right)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1814, 635, 205, 260}

$$\frac{(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-\frac{Ab+aC}{b}-\frac{2aDx}{b}}{a+bx^2}\,dx}{2a}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aC)\int \frac{1}{a+bx^2}\,dx}{2ab}+\frac{D\int \frac{x}{a+bx^2}\,dx}{b}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\,(aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{a^2D-ab(B+Cx)+Ab^2x}{a\left(a+bx^2\right)}+D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.85, size = 257, normalized size = 2.76

$$\left[\frac{2\,Da^{3}-2\,Ba^{2}b-\left(Ca^{2}+Aab+\left(Cab+Ab^{2}\right)x^{2}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\,\sqrt{-ab}x-a}{bx^{2}+a}\right)-2\,\left(Ca^{2}b-Aab^{2}\right)x+2\,\left(Da^{2}bx^{2}+Da^{3}\right)\log\left(bx^{2}+a\right)}{4\,\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)},\ \frac{Da^{3}-Ba^{2}b+\left(Ca^{2}+Aab+\left(Cab+Ab^{2}\right)x^{2}\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(Ca^{2}b-Aab^{2}\right)x+\left(Da^{2}bx^{2}+Da^{3}\right)\log\left(bx^{2}+a\right)}{2\,\left(a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x +
 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D*a^
3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a
^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.38, size = 88, normalized size = 0.95

$$\frac{D\log\left(bx^{2}+a\right)}{2\,b^{2}}+\frac{\left(Ca+Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{\left(Ca-Ab\right)x-\frac{Da^{2}-Bab}{b}}{2\,\left(bx^{2}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 97, normalized size = 1.04

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}+\frac{D\ln\left(b\,x^{2}+a\right)}{2b^{2}}+\frac{\frac{(Ab-aC)x}{2ab}-\frac{bB-aD}{2b^{2}}}{b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2*D*ln(b*x^2+a)/b^2+1/2
/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)*C
```

maxima [A] time = 2.94, size = 89, normalized size = 0.96

$$\frac{Da^{2}-Bab-\left(Cab-Ab^{2}\right)x}{2\,\left(ab^{3}x^{2}+a^{2}b^{2}\right)}+\frac{D\log\left(bx^{2}+a\right)}{2\,b^{2}}+\frac{\left(Ca+Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(b
*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 1.32, size = 110, normalized size = 1.18

$$\frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2}-\frac{B}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{C\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

```
[Out] ((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)/
(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))
/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

sympy [B] time = 3.09, size = 233, normalized size = 2.51

$$\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)+\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right)+\frac{-Bab+Da^2+x\left(Ab^2-Cab\right)}{2a^2b^2+2ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**
2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/
(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4
))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 - C*a*
b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

3.95 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=95

$$\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 801, 635, 205, 260}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-\frac{(bB+aD)x}{b}}{x\left(a+bx^2\right)}\,dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax}+\frac{-abB-a^2D+2Ab^2x}{ab\left(a+bx^2\right)}\right)dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{\int \frac{-abB-a^2D+2Ab^2x}{a+bx^2}\,dx}{2a^2b}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^2}+\frac{(bB+aD)\int \frac{1}{a+bx^2}\,dx}{2ab}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 85, normalized size = 0.89

$$
\frac{\frac{a(-a(C+Dx)+Ab+bBx)}{b\left(a+bx^2\right)}-A\log\left(a+bx^2\right)+\frac{\sqrt{a}\,(aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x\left(a + bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 0.87, size = 296, normalized size = 3.12

$$\left[-\frac{2\,Ca^2b - 2\,Aab^2 + \left(Da^2 + Bab + \left(Dab + Bb^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 2\left(Da^2b - Bab^2\right)x + 2\left(Ab^3x^2 + Aab^2\right)\log\left(bx^2 + a\right) - 4\left(Ab^3x^2 + Aab^2\right)\log(x)}{4\left(a^2b^3x^2 + a^3b^2\right)}, -\frac{Ca^2b - Aab^2 - \left(Da^2 + Bab + \left(Dab + Bb^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + \left(Da^2b - Bab^2\right)x + \left(Ab^3x^2 + Aab^2\right)\log\left(bx^2 + a\right) - 2\left(Ab^3x^2 + Aab^2\right)\log(x)}{2\left(a^2b^3x^2 + a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(-
a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2)*
x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log(x)
)/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (D*a*
b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*x + (
A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))/(a^2*
b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.35, size = 93, normalized size = 0.98

$$-\frac{A\log\left(bx^2 + a\right)}{2\,a^2} + \frac{A\log\left(|x|\right)}{a^2} + \frac{(Da + Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab} - \frac{Ca^2 - Aab + \left(Da^2 - Bab\right)x}{2\left(bx^2 + a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*a^2*b)
```

maple [A] time = 0.01, size = 125, normalized size = 1.32

$$\frac{Bx}{2\left(b\,x^2 + a\right)a} + \frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a} - \frac{Dx}{2\left(b\,x^2 + a\right)b} + \frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b} + \frac{A}{2\left(b\,x^2 + a\right)a} + \frac{A\ln(x)}{a^2} - \frac{A\ln\left(b\,x^2 + a\right)}{2a^2} - \frac{C}{2\left(b\,x^2 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)*B*x-1/2/(b*x^2+a)/b*x*D+1/2/a/(b*x^2+a)*A-1/2/(b*x^2+a)/b*C
-1/2*A*ln(b*x^2+a)/a^2+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B+1/2/b/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A*ln(x)/a^2
```

maxima [A] time = 2.91, size = 87, normalized size = 0.92

$$-\frac{Ca-Ab+(Da-Bb)x}{2\left(ab^2x^2+a^2b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log(x)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)
/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.96 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{\int \frac{-2A-2Bx+\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{2a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{\int \left(-\frac{2A}{ax^2}-\frac{2B}{ax}+\frac{3Ab-aC+2bBx}{a\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} + \frac{B\log(x)}{a^2} - \frac{\int \frac{3Ab-aC+2bBx}{a+bx^2}\,dx}{2a^2}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} + \frac{B\log(x)}{a^2} - \frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^2} - \frac{(3Ab-aC)\int \frac{1}{a+b}}{2a^2}$$

$$= -\frac{A}{a^2x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)} - \frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log\left(a\right.}{2a}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 1.00

$$\frac{(aC-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{a^2(-D)+abB+abCx-Ab^2x}{2a^2b\left(a+bx^2\right)} - \frac{A}{a^2x} - \frac{B\log\left(a+bx^2\right)}{2a^2} + \frac{B\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x
])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.88, size = 336, normalized size = 3.05

$$\left[-\frac{4\,Aa^2b-2\left(Ca^2b-3\,Aab^2\right)x^2-\left(\left(Cab-3\,Ab^2\right)x^3+\left(Ca^2-3\,Aab\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+2\left(Da^3-Ba^2b\right)x+2\left(Bab^2x^3+Ba^2bx\right)\log\left(bx^2+a\right)-4\left(Bab^2x^3+Ba^2bx\right)\log\left(x\right)}{4\left(a^3b^2x^3+a^4bx\right)},\ -\frac{2\,Aa^2b-\left(Ca^2b-3\,Aab^2\right)x^2-\left(\left(Cab-3\,Ab^2\right)x^3+\left(Ca^2-3\,Aab\right)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(Da^3-Ba^2b\right)x+\left(Bab^2x^3+Ba^2bx\right)\log\left(bx^2+a\right)-2\left(Bab^2x^3+Ba^2bx\right)\log\left(x\right)}{2\left(a^3b^2x^3+a^4bx\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (
C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) - 4*
(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b
- (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*b)*x)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3 + B*a
^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 +
 a^4*b*x)]
```

giac [A] time = 0.34, size = 103, normalized size = 0.94

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log\left(|x|\right)}{a^2}+\frac{\left(Ca-3\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}+\frac{Cabx^2-3\,Ab^2x^2-Da^2x+Babx-2\,Aab}{2\left(bx^3+ax\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B*a
*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

maple [A] time = 0.01, size = 136, normalized size = 1.24

$$-\frac{Abx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{Cx}{2\left(b\,x^{2}+a\right)a}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{B}{2\left(b\,x^{2}+a\right)a}+\frac{B\ln(x)}{a^{2}}-\frac{B\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{D}{2\left(b\,x^{2}+a\right)b}-\frac{A}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/(b*x^2+a)*A*b*x+1/2/a/(b*x^2+a)*C*x+1/2/a/(b*x^2+a)*B-1/2/(b*x^2+a
)/b*D-1/2*B/a^2*ln(b*x^2+a)-3/2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A
*b+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a^2/x+B/a^2*ln(x)
```

maxima [A] time = 2.98, size = 105, normalized size = 0.95

$$-\frac{2\,Aab-\left(Cab-3\,Ab^{2}\right)x^{2}+\left(Da^{2}-Bab\right)x}{2\left(a^{2}b^{2}x^{3}+a^{3}bx\right)}-\frac{B\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{B\log(x)}{a^{2}}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 + a
^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.41, size = 133, normalized size = 1.21

$$\frac{B}{2\,a\,\left(b\,x^{2}+a\right)}-\frac{\frac{A}{a}+\frac{3\,A\,b\,x^{2}}{2\,a^{2}}}{b\,x^{3}+a\,x}-\frac{B\,\ln\left(b\,x^{2}+a\right)}{2\,a^{2}}+\frac{B\,\ln(x)}{a^{2}}-\frac{D}{2\,b\,\left(b\,x^{2}+a\right)}+\frac{C\,x}{2\,a\,\left(b\,x^{2}+a\right)}-\frac{3\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
+ b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a +
b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan((b^
(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.97 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*x^
2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - ((2
*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2A-2Bx+2\left(\frac{Ab}{a}-C\right)x^2+\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2A}{ax^3}-\frac{2B}{ax^2}-\frac{2(-2Ab+aC)}{a^2x}+\frac{a(3bB-aD)-2b(2Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}-\frac{\int \frac{a(3bB-aD)-2b(2Ab-aC}{a+bx^2}}{2a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(b(2Ab-aC))\int \frac{x}{a+b}}{a^3}$$

$$= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{(2Ab-aC)\log}{a^3}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.83

$$\frac{\frac{a(a(C+Dx)-Ab-bBx)}{a+bx^2}+(2Ab-aC)\log\left(a+bx^2\right)+2\log(x)(aC-2Ab)-\frac{aA}{x^2}+\frac{\sqrt{a}\,(aD-3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{2aB}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
+ (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*b
+ a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 \left(a + bx^2\right)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2), x]
```

fricas [A] time = 1.02, size = 441, normalized size = 3.27

$$\left[-\frac{4 Ba^2bx + 2 Aa^2b - 2 \left(Da^2b - 3 Bab^2\right)x^3 - 2 \left(Ca^2b - 2 Aab^2\right)x^2 + \left(\left(Dab - 3 Bb^2\right)x^4 + \left(Da^2 - 3 Bab\right)x^2\right)\sqrt{-ab} \log\left(\frac{bx^2 - 2 \sqrt{-ab}x - a}{bx^2 + a}\right) + 2 \left(\left(Cab^2 - 2 Ab^3\right)x^4 + \left(Ca^2b - 2 Aab^2\right)x^2\right) \log\left(bx^2 + a\right) - 4 \left(\left(Cab^2 - 2 Ab^3\right)x^4 + \left(Ca^2b - 2 Aab^2\right)x^2\right) \log(x)}{4 \left(a^3b^2x^4 + a^4bx^2\right)}, -\frac{2 Ba^2bx + Aa^2b - \left(Da^2b - 3 Bab^2\right)x^3 - \left(Ca^2b - 2 Aab^2\right)x^2 - \left(\left(Dab - 3 Bb^2\right)x^4 + \left(Da^2 - 3 Bab\right)x^2\right)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right) + \left(\left(Cab^2 - 2 Ab^3\right)x^4 + \left(Ca^2b - 2 Aab^2\right)x^2\right) \log\left(bx^2 + a\right) - 2 \left(\left(Cab^2 - 2 Ab^3\right)x^4 + \left(Ca^2b - 2 Aab^2\right)x^2\right) \log(x)}{2 \left(a^3b^2x^4 + a^4bx^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b -
 2*A*a*b^2)*x^2 + ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)*x^
4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*x^4
+ (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(2*B*a
^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*x^2 -
((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a)
 - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2
*x^4 + a^4*b*x^2)]
```

giac [A] time = 0.42, size = 126, normalized size = 0.93

$$\frac{(Da - 3 Bb) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} a^2} - \frac{(Ca - 2 Ab) \log\left(bx^2 + a\right)}{2 a^3} + \frac{(Ca - 2 Ab) \log(|x|)}{a^3} - \frac{2 Ba^2x - \left(Da^2 - 3 Bab\right)x^3 + Aa^2 - \left(Ca^2 - 2 Aab\right)x^2}{2 \left(bx^2 + a\right)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b)
*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D*a
^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

maple [A] time = 0.02, size = 169, normalized size = 1.25

$$-\frac{Bbx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{Dx}{2\left(b\,x^{2}+a\right)a}+\frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{Ab}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{2Ab\ln(x)}{a^{3}}+\frac{Ab\ln\left(b\,x^{2}+a\right)}{a^{3}}+\frac{C}{2\left(b\,x^{2}+a\right)a}+\frac{C\ln(x)}{a^{2}}-\frac{C\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{B}{a^{2}x}-\frac{A}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/(b*x^2+a)*B/a^2*b*x+1/2/a/(b*x^2+a)*D*x-1/2/a^2/(b*x^2+a)*A*b+1/2/a/(b
*x^2+a)*C+1/a^3*b*ln(b*x^2+a)*A-1/2/a^2*ln(b*x^2+a)*C-3/2/(a*b)^(1/2)*B/a^2
*b*arctan(1/(a*b)^(1/2)*b*x)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-
1/2*A/a^2/x^2-B/a^2/x-2/a^3*ln(x)*A*b+1/a^2*ln(x)*C
```

maxima [A] time = 2.94, size = 117, normalized size = 0.87

$$\frac{(Da-3\,Bb)x^{3}-2\,Bax+(Ca-2\,Ab)x^{2}-Aa}{2\left(a^{2}bx^{4}+a^{3}x^{2}\right)}+\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}-\frac{(Ca-2\,Ab)\log\left(bx^{2}+a\right)}{2\,a^{3}}+\frac{(Ca-2\,Ab)\log(x)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^
3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a
 - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

mupad [B] time = 1.35, size = 158, normalized size = 1.17

$$\frac{C}{2\,a\,\left(b\,x^{2}+a\right)}-\frac{\frac{A}{2\,a}+\frac{A\,b\,x^{2}}{a^{2}}}{b\,x^{4}+a\,x^{2}}-\frac{\frac{B}{a}+\frac{3\,B\,b\,x^{2}}{2\,a^{2}}}{b\,x^{3}+a\,x}-\frac{C\,\ln\left(b\,x^{2}+a\right)}{2\,a^{2}}+\frac{C\,\ln(x)}{a^{2}}+\frac{A\,b\,\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{2\,A\,b\,\ln(x)}{a^{3}}+\frac{x\,D\,{}_{2}\mathrm{F}_{1}\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{b\,x^{2}}{a}\right)}{a^{2}}-\frac{3\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (3
*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x))/
a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/2,
2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)

[Out] Timed out
```

3.98 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=185

$$\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{(bB-$$

Rubi [A] time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 801, 635, 205, 260}

$$\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(bB-3aD)}{2ab^3}+\frac{(bB-3aD)\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(Ab-5aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{x^2(-3a\ldots}{\ldots}}{\ldots}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int\left(-\frac{3a(\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-\ldots}{\ldots}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-\ldots}{\ldots}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-\ldots}{\ldots}$$

Mathematica [A] time = 0.12, size = 139, normalized size = 0.75

$$\frac{\frac{2a\left(a^2D-ab(B+Cx)+Ab^2x\right)}{\left(a+bx^2\right)^2}+\frac{-12a^2D+8abB+9abCx-5Ab^2x}{a+bx^2}+\frac{3\sqrt{b}\,(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)\log\left(a+bx^2\right)+8bCx+4bDx^2}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b*
x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]*(
A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[a +
 b*x^2])/(8*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.84, size = 574, normalized size = 3.10

$$\left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\left(5\,Ca^2b^2-Aab^3\right)x^3-16\left(Da^3b-Ba^2b^2\right)x^2+3\left(\left(5\,Cab^2-Ab^3\right)x^4+5\,Ca^3-Aa^2b+2\left(5\,Ca^2b-Aab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(5\,Ca^3b-Aa^2b^2\right)x-8\left(3\,Da^4-Ba^3b+\left(3\,Da^2b^2-Bab^3\right)x^4+2\left(3\,Da^3b-Ba^2b^2\right)x^2\right)\log\left(bx^2+a\right)}{16\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)},\ \frac{4\,Dab^3x^6+8\,Cab^3x^5+8\,Da^2b^2x^4-10\,Da^4+6\,Ba^3b+5\left(5\,Ca^2b^2-Aab^3\right)x^3-8\left(Da^3b-Ba^2b^2\right)x^2-3\left(\left(5\,Cab^2-Ab^3\right)x^4+5\,Ca^3-Aa^2b+2\left(5\,Ca^2b-Aab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(5\,Ca^3b-Aa^2b^2\right)x-4\left(3\,Da^4-Ba^3b+\left(3\,Da^2b^2-Bab^3\right)x^4+2\left(3\,Da^3b-Ba^2b^2\right)x^2\right)\log\left(bx^2+a\right)}{8\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B*
a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3*(
(5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*
sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b - A
*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a
^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4
), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 + 6*B*a^
3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 - 3*((5*C
*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*D*a^4 - B*
a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*
x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.41, size = 157, normalized size = 0.85

$$-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Db^3x^2+2\,Cb^3x}{2\,b^6}-\frac{10\,Da^3-6\,Ba^2b-\left(9\,Cab^2-5\,Ab^3\right)x^3+4\left(3\,Da^2b-2\,Bab^2\right)x^2-\left(7\,Ca^2b-3\,Aab^2\right)x}{8\left(bx^2+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*b
)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 - 6*
B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7*C*
a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

maple [A] time = 0.01, size = 235, normalized size = 1.27

$$-\frac{5A\,x^3}{8\left(b\,x^2+a\right)^2b}+\frac{9Ca\,x^3}{8\left(b\,x^2+a\right)^2b^2}+\frac{Ba\,x^2}{\left(b\,x^2+a\right)^2b^2}-\frac{3Da^2x^2}{2\left(b\,x^2+a\right)^2b^3}-\frac{3Aax}{8\left(b\,x^2+a\right)^2b^2}+\frac{7C\,a^2x}{8\left(b\,x^2+a\right)^2b^3}+\frac{3A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}+\frac{3B\,a^2}{4\left(b\,x^2+a\right)^2b^3}-\frac{15Ca\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}-\frac{5Da^3}{4\left(b\,x^2+a\right)^2b^4}+\frac{Dx^2}{2b^3}+\frac{B\ln\left(b\,x^2+a\right)}{2b^3}+\frac{Cx}{b^3}-\frac{3Da\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2/b^3*D*x^2+1/b^3*C*x-5/8/(b*x^2+a)^2*A/b*x^3+9/8/b^2/(b*x^2+a)^2*C*x^3*a
+1/b^2/(b*x^2+a)^2*B*x^2*a-3/2/b^3/(b*x^2+a)^2*D*x^2*a^2-3/8/(b*x^2+a)^2*A*
a/b^2*x+7/8/b^3/(b*x^2+a)^2*a^2*C*x+3/4/(b*x^2+a)^2*B*a^2/b^3-5/4/b^4/(b*x^
2+a)^2*a^3*D+1/2*B/b^3*ln(b*x^2+a)-3/2/b^4*ln(b*x^2+a)*a*D+3/8/(a*b)^(1/2)*
A/b^2*arctan(1/(a*b)^(1/2)*b*x)-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*a*C
```

maxima [A] time = 3.07, size = 165, normalized size = 0.89

$$-\frac{10\,Da^3-6\,Ba^2b-\left(9\,Cab^2-5\,Ab^3\right)x^3+4\left(3\,Da^2b-2\,Bab^2\right)x^2-\left(7\,Ca^2b-3\,Aab^2\right)x}{8\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{Dx^2+2\,Cx}{2\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B
*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
- 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 + 2*
C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

mupad [B] time = 1.56, size = 232, normalized size = 1.25

$$\frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}-\frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{D\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\left(b\,x^2+a\right)^2}\right)}{2\,b^4}+\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3}+\frac{C\,x}{b^3}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}-\frac{15\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*A
*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)/(
4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*x^2
) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*log(
a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2
)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

sympy [B] time = 29.59, size = 357, normalized size = 1.93

$$\frac{Cx}{b^3} + \frac{Dx^2}{2b^3} + \left(-\frac{-Bb + 3Da}{2b^4} - \frac{3\sqrt{-ab^9}\,(-Ab + 5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \left(-\frac{-Bb + 3Da}{2b^4} + \frac{3\sqrt{-ab^9}\,(-Ab + 5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \frac{6Ba^2b - 10Da^3 + x^3\left(-5Ab^3 + 9Cab^2\right) + x^2\left(8Bab^2 - 12Da^2b\right) + x\left(-3Aab^2 + 7Ca^2b\right)}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-
A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b
+ 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2
 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/
(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b
**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b))
 + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B*a*b*
*2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16*a*b**5*
x**2 + 8*b**6*x**4)
```

3.99 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=155

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(bB-5aD)}{8ab^3}+\frac{C\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(4aC-x(3bB-}{8ab^2\left(a+bx\right.}$$

Rubi [A] time = 0.23, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1804, 774, 635, 205, 260}

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}-\frac{3x(bB-5aD)}{8ab^3}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)-4aCx-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{x\left(8a^2C-3a(bB-5\right.}{a+bx^2}}{8a^2b^2}$$

$$= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+$$

$$= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+$$

$$= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.81

$$\frac{a(-a(C+Dx)+Ab+bBx)}{4b^3\left(a+bx^2\right)^2}+\frac{8aC+9aDx-4Ab-5bBx}{8b^3\left(a+bx^2\right)}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}+\frac{Dx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*(
A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.93, size = 480, normalized size = 3.10

$$\left[\frac{16\,Dab^3x^5+12\,Ca^3b-4\,Aa^2b^2+10\left(5\,Da^2b^2-Bab^3\right)x^3+8\left(2\,Ca^2b^2-Aab^3\right)x^2-3\left(\left(5\,Dab^2-Bb^3\right)x^4+5\,Da^3-Ba^2b+2\left(5\,Da^2b-Bab^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(5\,Da^3b-Ba^2b^2\right)x+8\left(Cab^3x^4+2\,Ca^2b^2x^2+Ca^3b\right)\log\left(bx^2+a\right)}{16\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)},\ \frac{8\,Dab^3x^5+6\,Ca^3b-2\,Aa^2b^2+5\left(5\,Da^2b^2-Bab^3\right)x^3+4\left(2\,Ca^2b^2-Aab^3\right)x^2-3\left(\left(5\,Dab^2-Bb^3\right)x^4+5\,Da^3-Ba^2b+2\left(5\,Da^2b-Bab^2\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+3\left(5\,Da^3b-Ba^2b^2\right)x+4\left(Cab^3x^4+2\,Ca^2b^2x^2+Ca^3b\right)\log\left(bx^2+a\right)}{8\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D*a
^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^4 +
2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3
*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2 - B*a*
b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^2 + C*a^3
*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.38, size = 122, normalized size = 0.79

$$\frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\left(5\,Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+\left(7\,Da^2-3\,Bab\right)x}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2
*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

maple [A] time = 0.01, size = 206, normalized size = 1.33

$$-\frac{5Bx^3}{8\left(bx^2+a\right)^2 b}+\frac{9Dax^3}{8\left(bx^2+a\right)^2 b^2}-\frac{Ax^2}{2\left(bx^2+a\right)^2 b}+\frac{Cax^2}{\left(bx^2+a\right)^2 b^2}-\frac{3Bax}{8\left(bx^2+a\right)^2 b^2}+\frac{7Da^2x}{8\left(bx^2+a\right)^2 b^3}-\frac{Aa}{4\left(bx^2+a\right)^2 b^2}+\frac{3B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}+\frac{3Ca^2}{4\left(bx^2+a\right)^2 b^3}-\frac{15Da\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}+\frac{C\ln\left(bx^2+a\right)}{2b^3}+\frac{Dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] D/b^3*x-5/8/b/(b*x^2+a)^2*B*x^3+9/8/b^2/(b*x^2+a)^2*D*x^3*a-1/2/b/(b*x^2+a)
^2*A*x^2+1/b^2/(b*x^2+a)^2*C*x^2*a-3/8/b^2/(b*x^2+a)^2*B*x*a+7/8/b^3/(b*x^2
+a)^2*a^2*D*x-1/4/(b*x^2+a)^2*A*a/b^2+3/4/b^3/(b*x^2+a)^2*a^2*C+1/2*C*ln(b*
x^2+a)/b^3+3/8/(a*b)^(1/2)*B/b^2*arctan(1/(a*b)^(1/2)*b*x)-15/8/b^3/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*D
```

maxima [A] time = 2.98, size = 136, normalized size = 0.88

$$\frac{\left(9\,Dab-5\,Bb^2\right)x^3+6\,Ca^2-2\,Aab+4\left(2\,Cab-Ab^2\right)x^2+\left(7\,Da^2-3\,Bab\right)x}{8\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\left(5\,Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2
+ (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/2*
C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

sympy [B] time = 29.73, size = 282, normalized size = 1.82

$$\frac{Dx}{b^3}+\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}-\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x+\frac{8Ca-16ab^3\left(\frac{C}{2b^3}+\frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)+\frac{-2Aab+6Ca^2+x^3\left(-5Bb^2+9Dab\right)+x^2\left(-4Ab^2+8Cab\right)+x\left(-3Bab+7Da^2\right)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
+ (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*
a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5
*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*
b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*
a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

3.100 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=136

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log(a+bx^2)}{2b^3}$$

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1804, 635, 205, 260}

$$\frac{(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2(a+bx^2)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}+\frac{D\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (x*(A*b +
3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)-(Ab+3aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{a(Ab+3a}{a+}}{8a}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3a}{8}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3a}{8}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.90

$$\frac{\frac{\sqrt{b}(3aC+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{-2a^2D+2ab(B+Cx)-2Ab^2x}{\left(a+bx^2\right)^2}+\frac{8a^2D-ab(4B+5Cx)+Ab^2x}{a\left(a+bx^2\right)}+4D\log\left(a+bx^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2*
x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.86, size = 447, normalized size = 3.29

$$\left[\frac{12\,Da^4 - 4\,Ba^3b - 2\left(5\,Ca^2b^2 - Aab^3\right)x^3 + 8\left(2\,Da^3b - Ba^2b^2\right)x^2 - \left(\left(3\,Cab^2 + Ab^3\right)x^4 + 3\,Ca^3 + Aa^2b + 2\left(3\,Ca^2b + Aab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 2\left(3\,Ca^3b + Aa^2b^2\right)x + 8\left(Da^2b^2x^4 + 2\,Da^3bx^2 + Da^4\right)\log\left(bx^2 + a\right)}{16\left(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3\right)}, \frac{6\,Da^4 - 2\,Ba^3b - \left(5\,Ca^2b^2 - Aab^3\right)x^3 + 4\left(2\,Da^3b - Ba^2b^2\right)x^2 + \left(\left(3\,Cab^2 + Ab^3\right)x^4 + 3\,Ca^3 + Aa^2b + 2\left(3\,Ca^2b + Aab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - \left(3\,Ca^3b + Aa^2b^2\right)x + 4\left(Da^2b^2x^4 + 2\,Da^3bx^2 + Da^4\right)\log\left(bx^2 + a\right)}{8\left(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
- B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^
2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)
*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4 - 2*
B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2 + ((3
*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^2*b^2*x^
4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

giac [A] time = 0.42, size = 128, normalized size = 0.94

$$\frac{D\log\left(bx^2 + a\right)}{2\,b^3} + \frac{\left(3\,Ca + Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2} - \frac{\left(5\,Cab - Ab^2\right)x^3 - 4\left(2\,Da^2 - Bab\right)x^2 + \left(3\,Ca^2 + Aab\right)x - \frac{2\left(3\,Da^3 - Ba^2b\right)}{b}}{8\left(bx^2 + a\right)^2 ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a^2
 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

maple [A] time = 0.01, size = 133, normalized size = 0.98

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab} + \frac{3C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2} + \frac{D\ln\left(b\,x^2 + a\right)}{2b^3} + \frac{\frac{(Ab-5aC)x^3}{8ab} - \frac{(bB-2aD)x^2}{2b^2} - \frac{(Ab+3aC)x}{8b^2} - \frac{(bB-3aD)a}{4b^3}}{\left(b\,x^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4*
a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/2*D*ln(b*x^2+a)/b^3+1/8/(a*b)^(1/2)*A/a/b*
arctan(1/(a*b)^(1/2)*b*x)+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 2.98, size = 146, normalized size = 1.07

$$\frac{6\,Da^3 - 2\,Ba^2b - \left(5\,Cab^2 - Ab^3\right)x^3 + 4\left(2\,Da^2b - Bab^2\right)x^2 - \left(3\,Ca^2b + Aab^2\right)x}{8\left(ab^5x^4 + 2\,a^2b^4x^2 + a^3b^3\right)} + \frac{D\log\left(bx^2 + a\right)}{2\,b^3} + \frac{(3\,Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^2
)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) + 1/
2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^2)
```

mupad [B] time = 1.39, size = 195, normalized size = 1.43

$$\frac{\frac{A\,x^3}{8\,a}-\frac{A\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{B\,x^2}{2\,b}+\frac{B\,a}{4\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{5\,C\,x^3}{8\,b}+\frac{3\,C\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{D\left(\ln\left(b\,x^2+a\right)+\frac{2\,a}{b\,x^2+a}-\frac{a^2}{2\left(b\,x^2+a\right)^2}\right)}{2\,b^3}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
+ (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*x)
/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a + b*x
^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^
(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

sympy [B] time = 20.01, size = 304, normalized size = 2.24

$$\left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)+\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)+\frac{-2Ba^2b+6Da^3+x^3\left(Ab^3-5Cab^2\right)+x^2\left(-4Bab^2+8Da^2b\right)+x\left(-Aab^2-3Ca^2b\right)}{8a^3b^3+16a^2b^4x^2+8ab^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*
a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b
**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(
16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b
**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b + 6*
D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b) + x*(
-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**4)
```

3.101 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=119

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1804, 1814, 12, 205}

$$\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) - (2*(A*b + a*
C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2(Ab+aC)x-4aDx^2}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{a\left(B+\frac{3aD}{b}\right)}{a+bx^2}\,dx}{8a^2b}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{(bB+3aD)\int}{8ab^2}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{(bB+3aD)\,\mathrm{ta}}{8a^{3/2}b}$$

Mathematica [A] time = 0.13, size = 99, normalized size = 0.83

$$\frac{\frac{(3aD+bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}\left(-a^2(2C+3Dx)-ab\left(2A+x\left(B+4Cx+5Dx^2\right)\right)+b^2Bx^3\right)}{a\left(a+bx^2\right)^2}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x^
2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/
2))/(8*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.77, size = 357, normalized size = 3.00

$$\left[-\frac{8\,Ca^2b^2x^2+4\,Ca^3b+4\,Aa^2b^2+2\left(5\,Da^2b^2-Bab^3\right)x^3+\left(\left(3\,Dab^2+Bb^3\right)x^4+3\,Da^3+Ba^2b+2\left(3\,Da^2b+Bab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Da^3b+Ba^2b^2\right)x}{16\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)},-\frac{4\,Ca^2b^2x^2+2\,Ca^3b+2\,Aa^2b^2+\left(5\,Da^2b^2-Bab^3\right)x^3-\left(\left(3\,Dab^2+Bb^3\right)x^4+3\,Da^3+Ba^2b+2\left(3\,Da^2b+Bab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Da^3b+Ba^2b^2\right)x}{8\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*D
*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4*C*a
^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - ((3*D*
a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^
3*b^4*x^2 + a^4*b^3)]
```

giac [A] time = 0.39, size = 97, normalized size = 0.82

$$\frac{\left(3\,Da+Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}-\frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\left(bx^2+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x^
3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((b*
x^2 + a)^2*a*b^2)
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{3D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2}+\frac{-\frac{C\,x^2}{2b}+\frac{(bB-5aD)x^3}{8ab}-\frac{(bB+3aD)x}{8b^2}-\frac{Ab+aC}{4b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C/b*x^2-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b^
2)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B+3/8/b^2/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D
```

maxima [A] time = 2.99, size = 111, normalized size = 0.93

$$-\frac{4\,Cabx^2+\left(5\,Dab-Bb^2\right)x^3+2\,Ca^2+2\,Aab+\left(3\,Da^2+Bab\right)x}{8\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)}+\frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

sympy [A] time = 16.37, size = 178, normalized size = 1.50

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{-2Aab-2Ca^2-4Cabx^2+x^3\left(Bb^2-5Dab\right)+x\left(-Bab-3Da^2\right)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b) +
 x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

3.102 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=116

$$\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{x(Ab-aC)-a\left(B-\frac{aD}{b}\right)}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1814, 639, 205}

$$-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}+\frac{(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -(a*(B - (a*D)/b) - (A*b - a*C)*x)/(4*a*b*(a + b*x^2)^2) - (4*a^2*D - b*(3*
A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-3A-\frac{aC}{b}-\frac{4aDx}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)} + \frac{(3Ab+aC)\int \frac{1}{a+bx^2}\,dx}{8a^2b}$$

$$= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)} + \frac{(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.11, size = 104, normalized size = 0.90

$$\frac{\frac{\sqrt{a}\left(-2a^3D-a^2b(2B+x(C+4Dx))+ab^2x\left(5A+Cx^2\right)+3Ab^3x^3\right)}{\left(a+bx^2\right)^2}+\sqrt{b}\,(aC+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x*
(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3, x]
```

fricas [A] time = 0.88, size = 346, normalized size = 2.98

$$\left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\left(Ca^2b^2+3\,Aab^3\right)x^3+\left(\left(Cab^2+3\,Ab^3\right)x^4+Ca^3+3\,Aa^2b+2\left(Ca^2b+3\,Aab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(Ca^3b-5\,Aa^2b^2\right)x}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)},-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-\left(Ca^2b^2+3\,Aab^3\right)x^3-\left(\left(Cab^2+3\,Ab^3\right)x^4+Ca^3+3\,Aa^2b+2\left(Ca^2b+3\,Aab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(Ca^3b-5\,Aa^2b^2\right)x}{8\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^3
 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x
^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*b -
 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3*b*x
^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3*A*b^
3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 +
a^5*b^2)]
```

giac [A] time = 0.39, size = 106, normalized size = 0.91

$$\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\left(bx^2+a\right)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x^
3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B*a
^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

maple [A] time = 0.01, size = 111, normalized size = 0.96

$$\frac{3A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^2}+\frac{C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{-\frac{Dx^2}{2b}+\frac{(3Ab+aC)x^3}{8a^2}+\frac{(5Ab-aC)x}{8ab}-\frac{bB+aD}{4b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D/b*x^2+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b^
2)/(b*x^2+a)^2+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+1/8/a/b/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C
```

maxima [A] time = 3.00, size = 122, normalized size = 1.05

$$-\frac{4\,Da^{2}bx^{2}+2\,Da^{3}+2\,Ba^{2}b-\left(Cab^{2}+3\,Ab^{3}\right)x^{3}+\left(Ca^{2}b-5\,Aab^{2}\right)x}{8\left(a^{2}b^{4}x^{4}+2\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}+\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a^
2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a + 3*
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 1.33, size = 163, normalized size = 1.41

$$\frac{\frac{C\,x^{3}}{8\,a}-\frac{C\,x}{8\,b}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}+\frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^{3}}{8\,a^{2}}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}-\frac{B}{4\,b\,\left(a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}\right)}-\frac{\left(2\,b\,x^{2}+a\right)\,D}{4\,b^{2}\,\left(b\,x^{2}+a\right)^{2}}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

```
[Out] ((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
+ (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(1/2
)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(3/
2)*b^(3/2))
```

sympy [A] time = 11.27, size = 184, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\,(3Ab+Ca)\log\left(-a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{5}b^{3}}}\,(3Ab+Ca)\log\left(a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x\right)}{16}+\frac{-2Ba^{2}b-2Da^{3}-4Da^{2}bx^{2}+x^{3}\left(3Ab^{3}+Cab^{2}\right)+x\left(5Aab^{2}-Ca^{2}b\right)}{8a^{4}b^{2}+16a^{3}b^{3}x^{2}+8a^{2}b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b**2
) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b*
*4*x**4)
```

3.103 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1805, 823, 801, 635, 205, 260}

$$\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-\frac{(3bB+aD)x}{b}}{x\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \frac{8aAb+a(3bB+aD)x}{x\left(a+bx^2\right)}\,dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{\int \left(\frac{8Ab}{x}+\frac{3abB+a^2D-8Ab^2x}{a+bx^2}\right)dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} + \frac{\int \frac{3abB+a^2D-8Ab^2x}{a+bx^2}\,dx}{8a^3b}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{A\log(x)}{a^3} - \frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^3} + \frac{(3b\cdots}{\cdots}$$

$$= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2} + \frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)} + \frac{(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} + \frac{A\log\cdots}{a^3}$$

Mathematica [A] time = 0.11, size = 117, normalized size = 0.90

$$\frac{\frac{2a^2(-a(C+Dx)+Ab+bBx)}{b\left(a+bx^2\right)^2}+\frac{a(aDx+4Ab+3bBx)}{b\left(a+bx^2\right)}-4A\log\left(a+bx^2\right)+\frac{\sqrt{a}\,(aD+3bB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(C
 + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3), x]
```

fricas [B] time = 0.96, size = 488, normalized size = 3.75

$$\left[\frac{8\,Aab^3x^2 - 4\,Ca^3b + 12\,Aa^2b^2 + 2\left(Da^2b^2 + 3\,Bab^3\right)x^3 - \left(\left(Dab^2 + 3\,Bb^3\right)x^4 + Da^3 + 3\,Ba^2b + 2\left(Da^2b + 3\,Bab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 2\left(Da^3b - 5\,Ba^2b^2\right)x - 8\left(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2\right)\log\left(bx^2 + a\right) + 16\left(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2\right)\log(x)}{16\left(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2\right)}, \frac{4\,Aab^3x^2 - 2\,Ca^3b + 6\,Aa^2b^2 + \left(Da^2b^2 + 3\,Bab^3\right)x^3 + \left(\left(Dab^2 + 3\,Bb^3\right)x^4 + Da^3 + 3\,Ba^2b + 2\left(Da^2b + 3\,Bab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - \left(Da^3b - 5\,Ba^2b^2\right)x - 4\left(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2\right)\log\left(bx^2 + a\right) + 8\left(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2\right)\log(x)}{8\left(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3)
*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*b^
2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*a^3
*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 + (D*a^
2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(
D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*b - 5*B*
a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 + a) + 8*(
A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

giac [A] time = 0.48, size = 128, normalized size = 0.98

$$-\frac{A\log\left(bx^2 + a\right)}{2\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{(Da + 3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b} + \frac{4\,Aab^2x^2 - 2\,Ca^3 + 6\,Aa^2b + \left(Da^2b + 3\,Bab^2\right)x^3 - \left(Da^3 - 5\,Ba^2b\right)x}{8\left(bx^2 + a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b +
 (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b)
```

maple [A] time = 0.02, size = 184, normalized size = 1.42

$$\frac{3Bb\,x^3}{8\left(b\,x^2 + a\right)^2a^2} + \frac{Dx^3}{8\left(b\,x^2 + a\right)^2a} + \frac{Ab\,x^2}{2\left(b\,x^2 + a\right)^2a^2} + \frac{5Bx}{8\left(b\,x^2 + a\right)^2a} - \frac{Dx}{8\left(b\,x^2 + a\right)^2b} + \frac{3A}{4\left(b\,x^2 + a\right)^2a} + \frac{3B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2} - \frac{C}{4\left(b\,x^2 + a\right)^2b} + \frac{D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab} + \frac{A\ln(x)}{a^3} - \frac{A\ln\left(b\,x^2 + a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x)
```

```
[Out] 3/8/(b*x^2+a)^2*B/a^2*b*x^3+1/8/a/(b*x^2+a)^2*D*x^3+1/2/a^2/(b*x^2+a)^2*A*x
^2*b+5/8/(b*x^2+a)^2*B/a*x-1/8/(b*x^2+a)^2/b*x*D+3/4/(b*x^2+a)^2*A/a-1/4/(b
*x^2+a)^2/b*C-1/2*A/a^3*ln(b*x^2+a)+3/8/(a*b)^(1/2)*B/a^2*arctan(1/(a*b)^(1
/2)*b*x)+1/8/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D+A/a^3*ln(x)
```

maxima [A] time = 2.90, size = 133, normalized size = 1.02

$$\frac{4\,Ab^2x^2+\left(Dab+3\,Bb^2\right)x^3-2\,Ca^2+6\,Aab-\left(Da^2-5\,Bab\right)x}{8\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log(x)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*B
*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3 +
 A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-4A-4Bx+3\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}+\frac{\int \frac{8A+8Bx-\left(\frac{7Ab}{a}-3C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}+\frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab+3aC-8bBx}{a\left(a+bx^2\right)}\right)dx}{8a^2}$$

$$= -\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}+\frac{B\log(x)}{a^3}+\frac{\int \frac{-15Ab+3aC-8}{a+bx^2}}{8a^3}$$

$$= -\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}+\frac{B\log(x)}{a^3}-\frac{(bB)\int \frac{x}{a+bx^2}\,d}{a^3}$$

$$= -\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

Mathematica [A]   time = 0.10, size = 141, normalized size = 0.98

$$\frac{3(aC-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{4aB+3aCx-7Abx}{8a^3\left(a+bx^2\right)}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{a^2(-D)+abB+abCx-Ab^2x}{4a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
+ (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b
*x^2])/(2*a^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3), x]
```

fricas [B]   time = 0.90, size = 524, normalized size = 3.64

$$\left[\frac{8Ba^2b^2x^3-16Aa^3b+6\left(Ca^2b^2-5Aab^3\right)x^4+10\left(Ca^3b-5Aa^2b^2\right)x^2+3\left(\left(Cab^2-5Ab^3\right)x^5+2\left(Ca^2b-5Aab^2\right)x^3+\left(Ca^3-5Aa^2b\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-4\left(Da^4-3Ba^3b\right)x-8\left(Bab^3x^5+2Ba^2b^2x^3+Ba^3bx\right)\log\left(bx^2+a\right)+16\left(Bab^3x^5+2Ba^2b^2x^3+Ba^3bx\right)\log(x)}{16\left(a^4b^3x^5+2a^5b^2x^3+a^6bx\right)},\ \frac{4Ba^2b^2x^3-8Aa^3b+3\left(Ca^2b^2-5Aab^3\right)x^4+5\left(Ca^3b-5Aa^2b^2\right)x^2+3\left(\left(Cab^2-5Ab^3\right)x^5+2\left(Ca^2b-5Aab^2\right)x^3+\left(Ca^3-5Aa^2b\right)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)-2\left(Da^4-3Ba^3b\right)x-4\left(Bab^3x^5+2Ba^2b^2x^3+Ba^3bx\right)\log\left(bx^2+a\right)+8\left(Bab^3x^5+2Ba^2b^2x^3+Ba^3bx\right)\log(x)}{8\left(a^4b^3x^5+2a^5b^2x^3+a^6bx\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(C
*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a
*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b^2*x
^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*
b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^2*x^3
- 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b^2)*x^2
 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^3 - 5*A*
a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*x - 4*(B*
a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*b^3*x^5 +
2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x
)]
```

giac [A] time = 0.49, size = 141, normalized size = 0.98

$$-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}+\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-5\,Aab^2\right)x^2-2\left(Da^3-3\,Ba^2b\right)x}{8\left(bx^2+a\right)^2a^3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x
^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/((b
*x^2 + a)^2*a^3*b*x)
```

maple [A] time = 0.02, size = 195, normalized size = 1.35

$$-\frac{7A\,b^2x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{3Cb\,x^3}{8\left(b\,x^2+a\right)^2a^2}+\frac{Bb\,x^2}{2\left(b\,x^2+a\right)^2a^2}-\frac{9Abx}{8\left(b\,x^2+a\right)^2a^2}+\frac{5Cx}{8\left(b\,x^2+a\right)^2a}-\frac{15Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^3}+\frac{3B}{4\left(b\,x^2+a\right)^2a}+\frac{3C\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}-\frac{D}{4\left(b\,x^2+a\right)^2b}+\frac{B\ln(x)}{a^3}-\frac{B\ln\left(b\,x^2+a\right)}{2a^3}-\frac{A}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -7/8/(b*x^2+a)^2*A/a^3*b^2*x^3+3/8/a^2/(b*x^2+a)^2*C*x^3*b+1/2/a^2/(b*x^2+a
)^2*B*x^2*b-9/8/(b*x^2+a)^2*A/a^2*b*x+5/8/a/(b*x^2+a)^2*C*x+3/4/(b*x^2+a)^2
*B/a-1/4/(b*x^2+a)^2/b*D-1/2*B/a^3*ln(b*x^2+a)-15/8/(a*b)^(1/2)*A/a^3*b*arc
tan(1/(a*b)^(1/2)*b*x)+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*C-A/a^
3/x+B/a^3*ln(x)
```

maxima [A] time = 2.99, size = 152, normalized size = 1.06

$$\frac{4\,Bab^2x^3+3\left(Cab^2-5\,Ab^3\right)x^4-8\,Aa^2b+5\left(Ca^2b-5\,Aab^2\right)x^2-2\left(Da^3-3\,Ba^2b\right)x}{8\left(a^3b^3x^5+2\,a^4b^2x^3+a^5bx\right)}-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log(x)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5
*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5
*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 1.40, size = 202, normalized size = 1.40

$$\frac{\frac{3\,B}{4\,a}+\frac{B\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{a}+\frac{25\,A\,b\,x^2}{8\,a^2}+\frac{15\,A\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{D}{4\,b\left(b\,x^2+a\right)^2}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{B\,\ln(x)}{a^3}-\frac{15\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] ((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(8
*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x^2
)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4*b*
(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^(1/2
)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/
(8*a^(5/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=174

$$-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}$$

Rubi [A] time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1805, 1802, 635, 205, 260}

$$-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{A}{2a^3x^2}-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*x^
2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5*b
*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*C)
*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx = -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4A-4Bx+4\left(\frac{Ab}{a}-C\right)x^2+3\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int \frac{8A+8Bx-8\left(\frac{2Ab}{a}-C\right)x^2-\left(7\ldots\right.}{x^3\left(a+bx^2\right)}}{8a^2}$$

$$= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int\left(\frac{8A}{ax^3}+\frac{8B}{ax^2}+\frac{8(-3Ab+\ldots}{a^2x}\right.}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{(3Ab-\ldots}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{(3Ab-\ldots}{}$$

$$= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{3(5bB-\ldots}{}$$

Mathematica [A] time = 0.16, size = 147, normalized size = 0.84

$$\frac{\frac{2a^2(a(C+Dx)-Ab-bBx)}{(a+bx^2)^2}+\frac{a(4aC+3aDx-8Ab-7bBx)}{a+bx^2}+4(3Ab-aC)\log\left(a+bx^2\right)+8\log(x)(aC-3Ab)-\frac{4aA}{x^2}+\frac{3\sqrt{a}(aD-5bB)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{8aB}{x}}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a + b
*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]*(
-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Log[x
] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3), x]
```

fricas [B] time = 0.83, size = 696, normalized size = 4.00

$$\left[-\frac{16Ba^3bx-6\left(Da^2b^2-5Bab^3\right)x^5+8Aa^3b-8\left(Ca^2b^2-3Aab^3\right)x^4-10\left(Da^3b-5Ba^2b^2\right)x^3-12\left(Ca^3b-3Aa^2b^2\right)x^2+3\left(\left(Dab^2-5Bb^3\right)x^6+2\left(Da^2b-5Bab^2\right)x^4+\left(Da^3-5Ba^2b\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+8\left(\left(Cab^3-3Ab^4\right)x^6+2\left(Ca^2b^2-3Aab^3\right)x^4+\left(Ca^3b-3Aa^2b^2\right)x^2\right)\log\left(bx^2+a\right)-16\left(\left(Cab^3-3Ab^4\right)x^6+2\left(Ca^2b^2-3Aab^3\right)x^4+\left(Ca^3b-3Aa^2b^2\right)x^2\right)\log(x)}{16\left(a^4b^3x^6+2a^5b^2x^4+a^6bx^2\right)},\ -\frac{8Ba^3bx-3\left(Da^2b^2-5Bab^3\right)x^5+4Aa^3b-4\left(Ca^2b^2-3Aab^3\right)x^4-5\left(Da^3b-5Ba^2b^2\right)x^3-6\left(Ca^3b-3Aa^2b^2\right)x^2-3\left(\left(Dab^2-5Bb^3\right)x^6+2\left(Da^2b-5Bab^2\right)x^4+\left(Da^3-5Ba^2b\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+4\left(\left(Cab^3-3Ab^4\right)x^6+2\left(Ca^2b^2-3Aab^3\right)x^4+\left(Ca^3b-3Aa^2b^2\right)x^2\right)\log\left(bx^2+a\right)-8\left(\left(Cab^3-3Ab^4\right)x^6+2\left(Ca^2b^2-3Aab^3\right)x^4+\left(Ca^3b-3Aa^2b^2\right)x^2\right)\log(x)}{8\left(a^4b^3x^6+2a^5b^2x^4+a^6bx^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^2
*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*A*
a^2*b^2)*x^2 + 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (
D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3
*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*
a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x^
6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 - 5*B*a*b^
3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b - 5*B*a^2*b
^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D
*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a
```

```
^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C
*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x
^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

giac [A] time = 0.38, size = 162, normalized size = 0.93

$$\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-4\,Aa^2}{8\left(bx^3+ax\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b)
*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 - 15
*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6*C*
a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

maple [A] time = 0.02, size = 250, normalized size = 1.44

$$-\frac{7B\,b^2x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{3Db\,x^3}{8\left(b\,x^2+a\right)^2a^2}-\frac{A\,b^2x^2}{\left(b\,x^2+a\right)^2a^3}+\frac{Cb\,x^2}{2\left(b\,x^2+a\right)^2a^2}-\frac{9Bbx}{8\left(b\,x^2+a\right)^2a^2}+\frac{5Dx}{8\left(b\,x^2+a\right)^2a}-\frac{5Ab}{4\left(b\,x^2+a\right)^2a^2}-\frac{15Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^3}+\frac{3C}{4\left(b\,x^2+a\right)^2a}+\frac{3D\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}-\frac{3Ab\ln(x)}{a^4}+\frac{3Ab\ln\left(b\,x^2+a\right)}{2a^4}+\frac{C\ln(x)}{a^3}-\frac{C\ln\left(b\,x^2+a\right)}{2a^3}-\frac{B}{a^3x}-\frac{A}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -7/8/a^3/(b*x^2+a)^2*B*x^3*b^2+3/8/a^2/(b*x^2+a)^2*D*x^3*b-1/a^3/(b*x^2+a)^
2*A*x^2*b^2+1/2/a^2/(b*x^2+a)^2*C*x^2*b-9/8/a^2/(b*x^2+a)^2*B*x*b+5/8/a/(b*
x^2+a)^2*D*x-5/4/(b*x^2+a)^2*A/a^2*b+3/4/a/(b*x^2+a)^2*C+3/2*A/a^4*b*ln(b*x
^2+a)-1/2/a^3*ln(b*x^2+a)*C-15/8/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
b*B+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*D-1/2*A/a^3/x^2-B/a^3/x-3
*A/a^4*b*ln(x)+1/a^3*ln(x)*C
```

maxima [A] time = 2.99, size = 172, normalized size = 0.99

$$\frac{3\left(Dab-5\,Bb^2\right)x^5+4\left(Cab-3\,Ab^2\right)x^4-8\,Ba^2x+5\left(Da^2-5\,Bab\right)x^3-4\,Aa^2+6\left(Ca^2-3\,Aab\right)x^2}{8\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*a
^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a^4
*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

mupad [B] time = 1.46, size = 229, normalized size = 1.32

$$\frac{\frac{3C}{4a}+\frac{C\,b\,x^2}{2a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{2a}+\frac{9\,A\,b\,x^2}{4a^2}+\frac{3\,A\,b^2\,x^4}{2a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{\frac{B}{a}+\frac{25\,B\,b\,x^2}{8a^2}+\frac{15\,B\,b^2\,x^4}{8a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{C\,\ln(x)}{a^3}+\frac{3\,A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{3\,A\,b\,\ln(x)}{a^4}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3}-\frac{15\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] ((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
(9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4
) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5
+ 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log(a +
 b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2, -(b*x
^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.106 $\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx$

Optimal. Leaf size=20

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx=\int \frac{x\left(-1+4x^2\right)}{\left(5+x^2\right)^2}\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{-1+4x}{(5+x)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{21}{(5+x)^2}+\frac{4}{5+x}\right)dx,x,x^2\right)$$
$$=\frac{21}{2\left(5+x^2\right)}+2\log\left(5+x^2\right)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + 4*x^3)/(5 + x^2)^2,x]
[Out] IntegrateAlgebraic[(-x + 4*x^3)/(5 + x^2)^2, x]
```

fricas [A] time = 0.80, size = 24, normalized size = 1.20

$$\frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

giac [A] time = 0.32, size = 25, normalized size = 1.25

$$-\frac{4x^2-1}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$2\ln\left(x^2+5\right)+\frac{21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x)/(x^2+5)^2,x)
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

maxima [A] time = 1.34, size = 18, normalized size = 0.90

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

mupad [B] time = 0.91, size = 20, normalized size = 1.00

$$2\ln\left(x^2+5\right)+\frac{21}{2\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 4*x^3)/(x^2 + 5)^2,x)
[Out] 2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.75

$$2\log\left(x^2+5\right)+\frac{21}{2x^2+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x)/(x**2+5)**2,x)
```

```
[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

3.107 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 444, 43}

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx &= \int \frac{x\left(-1+x^2\right)}{\sqrt{-2+x^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{\sqrt{-2+x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{-2+x}}+\sqrt{-2+x}\right)dx,x,x^2\right)\\ &= \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{x^2-2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{x^2-2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^3)/Sqrt[-2 + x^2],x]
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

fricas [A] time = 0.82, size = 14, normalized size = 0.61

$$\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

giac [A] time = 0.40, size = 17, normalized size = 0.74

$$\frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

maple [A] time = 0.01, size = 15, normalized size = 0.65

$$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x^2-2)^(1/2),x)
[Out] 1/3*(x^2+1)*(x^2-2)^(1/2)
```

maxima [A] time = 1.34, size = 22, normalized size = 0.96

$$\frac{1}{3}\sqrt{x^2-2}\,x^2+\frac{1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

mupad [B] time = 0.10, size = 14, normalized size = 0.61

$$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(x^2 - 2)^(1/2),x)
[Out] ((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

sympy [A] time = 0.69, size = 22, normalized size = 0.96

$$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(x**2-2)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

3.108 $\int \frac{-x^2+2x^4}{1+2x^2}\,dx$

Optimal. Leaf size=25

$$\frac{x^3}{3}-x+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1593, 459, 321, 203}

$$\frac{x^3}{3}-x+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x^2+2x^4}{1+2x^2}\,dx &= \int \frac{x^2\left(-1+2x^2\right)}{1+2x^2}\,dx\\
&= \frac{x^3}{3}-2\int \frac{x^2}{1+2x^2}\,dx\\
&= -x+\frac{x^3}{3}+\int \frac{1}{1+2x^2}\,dx\\
&= -x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3}-x+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
[Out] IntegrateAlgebraic[(-x^2 + 2*x^4)/(1 + 2*x^2), x]
```

fricas [A] time = 0.74, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

giac [A] time = 0.35, size = 20, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x^2)/(2*x^2+1),x)
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

maxima [A] time = 2.92, size = 20, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

mupad [B] time = 0.04, size = 20, normalized size = 0.80

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2} - x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

sympy [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x**2)/(2*x**2+1),x)
```

```
[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

3.109 $\int \frac{x^3+x^4}{1+x^2}\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 801, 635, 203, 260}

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3+x^4}{1+x^2}\,dx &= \int \frac{x^3(1+x)}{1+x^2}\,dx \\
&= \int \left(-1+x+x^2+\frac{1-x}{1+x^2}\right)dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1-x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + x^4)/(1 + x^2),x]
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+x^4}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3 + x^4)/(1 + x^2),x]
[Out] IntegrateAlgebraic[(x^3 + x^4)/(1 + x^2), x]
```

fricas [A] time = 0.83, size = 24, normalized size = 0.80

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-x+\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

giac [A] time = 0.39, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{x^3}{3} + \frac{x^2}{2} - x + \arctan(x) - \frac{\ln\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3)/(x^2+1),x)
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

maxima [A] time = 2.90, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\operatorname{atan}(x) - \frac{\ln\left(x^2 + 1\right)}{2} - x + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^4)/(x^2 + 1),x)
[Out] atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

sympy [A] time = 0.11, size = 22, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} - x - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3)/(x**2+1),x)
```

```
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

3.110 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2} dx$

Optimal. Leaf size=210

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{a^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}-\frac{ax^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^5}+\frac{x^5\left(a^3(-f)+a^2be\right.}{5b^4}$$

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^5\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{5b^4}-\frac{ax^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^5}+\frac{a^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{13/2}}+\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx = \int \left(\frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{b^5} + \frac{\left(b^3c-ab^2\right.}{}\right.$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5} + \frac{\left(b^3c-ab^2d\right.}{}$$

$$= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^6} - \frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^5} + \frac{\left(b^3c-ab^2d\right.}{}$$

Mathematica [A] time = 0.16, size = 210, normalized size = 1.00

$$\frac{x^7\left(a^2f-abe+b^2d\right)}{7b^3} - \frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^6} + \frac{ax^3\left(a^3f-a^2be+ab^2d-b^3c\right)}{3b^5} + \frac{x^5\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{5b^4} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{13/2}} + \frac{x^9(be-af)}{9b^2} + \frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9
*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2), x]
```

fricas [A] time = 0.86, size = 452, normalized size = 2.15

$$\left[\frac{630\,b^5fx^{11}+770\left(b^5e-ab^4f\right)x^9+990\left(b^5d-ab^4e+a^2b^3f\right)x^7+1386\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^5-2310\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^3-3465\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6930\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x}{6930\,b^6}, \frac{315\,b^5fx^{11}+385\left(b^5e-ab^4f\right)x^9+495\left(b^5d-ab^4e+a^2b^3f\right)x^7+693\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^5-1155\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^3-3465\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3465\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x}{3465\,b^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 231
```

```
0*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b
^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/3465*(31
5*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2*b^3*f
)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*b^4*c -
 a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d + a^4*b
*e - a^5*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x)/b^6]
```

giac [A] time = 0.46, size = 250, normalized size = 1.19

$$-\frac{\left(a^{3} b^{3} c - a^{4} b^{2} d - a^{6} f + a^{5} b e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{6}} + \frac{315 \, b^{10} f x^{11} - 385 \, a b^{9} f x^{9} + 385 \, b^{10} x^{9} e + 495 \, b^{10} d x^{7} + 495 \, a^{2} b^{8} f x^{7} - 495 \, a b^{9} x^{7} e + 693 \, b^{10} c x^{5} - 693 \, a b^{9} d x^{5} - 693 \, a^{3} b^{7} f x^{5} + 693 \, a^{2} b^{8} x^{5} e - 1155 \, a b^{9} c x^{3} + 1155 \, a^{2} b^{8} d x^{3} + 1155 \, a^{4} b^{6} f x^{3} - 1155 \, a^{3} b^{7} x^{3} e + 3465 \, a^{2} b^{8} c x - 3465 \, a^{3} b^{7} d x - 3465 \, a^{5} b^{5} f x + 3465 \, a^{4} b^{6} x e}{3465 \, b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^10*f*x^11 - 385*a*b^9*f*x^9 + 385*b^10*x^9*e + 495*b^
10*d*x^7 + 495*a^2*b^8*f*x^7 - 495*a*b^9*x^7*e + 693*b^10*c*x^5 - 693*a*b^9
*d*x^5 - 693*a^3*b^7*f*x^5 + 693*a^2*b^8*x^5*e - 1155*a*b^9*c*x^3 + 1155*a^
2*b^8*d*x^3 + 1155*a^4*b^6*f*x^3 - 1155*a^3*b^7*x^3*e + 3465*a^2*b^8*c*x -
3465*a^3*b^7*d*x - 3465*a^5*b^5*f*x + 3465*a^4*b^6*x*e)/b^11
```

maple [A] time = 0.00, size = 278, normalized size = 1.32

$$\frac{f x^{11}}{11 b}-\frac{a f x^{9}}{9 b^{2}}+\frac{e x^{9}}{9 b}+\frac{a^{2} f x^{7}}{7 b^{3}}-\frac{a e x^{7}}{7 b^{2}}+\frac{d x^{7}}{7 b}-\frac{a^{3} f x^{5}}{5 b^{4}}+\frac{a^{2} e x^{5}}{5 b^{3}}-\frac{a d x^{5}}{5 b^{2}}+\frac{c x^{5}}{5 b}+\frac{a^{4} f x^{3}}{3 b^{5}}-\frac{a^{3} e x^{3}}{3 b^{4}}+\frac{a^{2} d x^{3}}{3 b^{3}}-\frac{a c x^{3}}{3 b^{2}}+\frac{a^{6} f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{6}}-\frac{a^{5} e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{5}}+\frac{a^{4} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{4}}-\frac{a^{3} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{3}}-\frac{a^{5} f x}{b^{6}}+\frac{a^{4} e x}{b^{5}}-\frac{a^{3} d x}{b^{4}}+\frac{a^{2} c x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/11*f*x^11/b-1/9/b^2*x^9*a*f+1/9/b*x^9*e+1/7/b^3*x^7*a^2*f-1/7/b^2*x^7*a*e
+1/7/b*x^7*d-1/5/b^4*x^5*a^3*f+1/5/b^3*x^5*a^2*e-1/5/b^2*x^5*a*d+1/5/b*x^5*
c+1/3/b^5*x^3*a^4*f-1/3/b^4*x^3*a^3*e+1/3/b^3*x^3*a^2*d-1/3/b^2*x^3*a*c-1/b
^6*a^5*f*x+1/b^5*a^4*e*x-1/b^4*a^3*d*x+1/b^3*a^2*c*x+a^6/b^6/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*f-a^5/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+a
^4/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-a^3/b^3/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.90, size = 213, normalized size = 1.01

$$-\frac{\left(a^{3} b^{3} c - a^{4} b^{2} d + a^{5} b e - a^{6} f\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{6}} + \frac{315 \, b^{5} f x^{11} + 385 \, {\left(b^{5} e - a b^{4} f\right)} x^{9} + 495 \, {\left(b^{5} d - a b^{4} e + a^{2} b^{3} f\right)} x^{7} + 693 \, {\left(b^{5} c - a b^{4} d + a^{2} b^{3} e - a^{3} b^{2} f\right)} x^{5} - 1155 \, {\left(a b^{4} c - a^{2} b^{3} d + a^{3} b^{2} e - a^{4} b f\right)} x^{3} + 3465 \, {\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} x}{3465 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a
*b^4*e + a^2*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5
 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 3465*(a^2*b^3*c -
 a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

mupad [B] time = 0.93, size = 289, normalized size = 1.38

$$x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{13/2}}-\frac{a\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}+\frac{a^2\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*b))
 + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^11)/(11
*b) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))
/(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*
b*e))/b^(13/2) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b
) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b^2
```

sympy [A] time = 1.65, size = 384, normalized size = 1.83

$$x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)-\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{fx^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**9*(-a*f/(9*b**2) + e/(9*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) + d/(
7*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b))
+ x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2))
 + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) - sqrt(-a**5/
b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-a**5/b**13)*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a**3*b**2*d -
a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*b)
```

3.111 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=172

$$\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}-\frac{ax\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5}+\frac{x^3\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3b^4}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(\right.}{}$$

Rubi [A] time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x^3\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3b^4}-\frac{ax\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{11/2}}+\frac{x^5\left(a^2f-abe+b^2d\right)}{5b^3}+\frac{x^7(be-af)}{7b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx = \int \left(-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^2}{b^4} + \frac{\left(b^2d-abe\right.}{b^3}\right.$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4} + \frac{\left(b^2d-abe+\right.}{5b^3}$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5} + \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4} + \frac{\left(b^2d-abe+\right.}{5b^3}$$

Mathematica [A] time = 0.12, size = 162, normalized size = 0.94

$$\frac{x\left(315a^4f-105a^3b\left(3e+fx^2\right)+21a^2b^2\left(15d+5ex^2+3fx^4\right)-3ab^3\left(105c+35dx^2+21ex^4+15fx^6\right)+b^4x^2\left(105c+63dx^2+45ex^4+35fx^6\right)\right)}{315b^5}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f*
x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c +
63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2), x]
```

fricas [A] time = 0.67, size = 368, normalized size = 2.14

$$\left[\frac{70\,b^4fx^9+90\left(b^4e-ab^3f\right)x^7+126\left(b^4d-ab^3e+a^2b^2f\right)x^5+210\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^3-315\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x}{630\,b^5},\ \frac{35\,b^4fx^9+45\left(b^4e-ab^3f\right)x^7+63\left(b^4d-ab^3e+a^2b^2f\right)x^5+105\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^3+315\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x}{315\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(a*b^3
```

```
*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
- a)/(b*x^2 + a)) - 630*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5, 1/3
15*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

giac [A] time = 0.37, size = 200, normalized size = 1.16

$$\frac{\left(a^{2} b^{3} c - a^{3} b^{2} d - a^{5} f + a^{4} b e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{5}} + \frac{35 \, b^{8} f x^{9} - 45 \, a b^{7} f x^{7} + 45 \, b^{8} x^{7} e + 63 \, b^{8} d x^{5} + 63 \, a^{2} b^{6} f x^{5} - 63 \, a b^{7} x^{5} e + 105 \, b^{8} c x^{3} - 105 \, a b^{7} d x^{3} - 105 \, a^{3} b^{5} f x^{3} + 105 \, a^{2} b^{6} x^{3} e - 315 \, a b^{7} c x + 315 \, a^{2} b^{6} d x + 315 \, a^{4} b^{4} f x - 315 \, a^{3} b^{5} x e}{315 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^8*f*x^9 - 45*a*b^7*f*x^7 + 45*b^8*x^7*e + 63*b^8*d*x^5 +
 63*a^2*b^6*f*x^5 - 63*a*b^7*x^5*e + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 - 105*
a^3*b^5*f*x^3 + 105*a^2*b^6*x^3*e - 315*a*b^7*c*x + 315*a^2*b^6*d*x + 315*a
^4*b^4*f*x - 315*a^3*b^5*x*e)/b^9
```

maple [A] time = 0.01, size = 230, normalized size = 1.34

$$\frac{f x^{9}}{9 b} - \frac{a f x^{7}}{7 b^{2}} + \frac{e x^{7}}{7 b} + \frac{a^{2} f x^{5}}{5 b^{3}} - \frac{a e x^{5}}{5 b^{2}} + \frac{d x^{5}}{5 b} - \frac{a^{3} f x^{3}}{3 b^{4}} + \frac{a^{2} e x^{3}}{3 b^{3}} - \frac{a d x^{3}}{3 b^{2}} + \frac{c x^{3}}{3 b} - \frac{a^{5} f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{5}} + \frac{a^{4} e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{4}} - \frac{a^{3} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{3}} + \frac{a^{2} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{2}} + \frac{a^{4} f x}{b^{5}} - \frac{a^{3} e x}{b^{4}} + \frac{a^{2} d x}{b^{3}} - \frac{a c x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/9*f*x^9/b-1/7/b^2*x^7*a*f+1/7/b*x^7*e+1/5/b^3*x^5*a^2*f-1/5/b^2*x^5*a*e+1
/5/b*x^5*d-1/3/b^4*x^3*a^3*f+1/3/b^3*x^3*a^2*e-1/3/b^2*x^3*a*d+1/3/b*x^3*c+
1/b^5*a^4*f*x-1/b^4*a^3*e*x+1/b^3*a^2*d*x-1/b^2*a*c*x-a^5/b^5/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*f+a^4/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-
a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+a^2/b^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.93, size = 172, normalized size = 1.00

$$\frac{\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{5}} + \frac{35 \, b^{4} f x^{9} + 45 \, {\left(b^{4} e - a b^{3} f\right)} x^{7} + 63 \, {\left(b^{4} d - a b^{3} e + a^{2} b^{2} f\right)} x^{5} + 105 \, {\left(b^{4} c - a b^{3} d + a^{2} b^{2} e - a^{3} b f\right)} x^{3} - 315 \, {\left(a b^{3} c - a^{2} b^{2} d + a^{3} b e - a^{4} f\right)} x}{315 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e
```

```
 + a^2*b^2*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

mupad [B] time = 0.94, size = 243, normalized size = 1.41

$$x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)+x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b}-\frac{a\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^9)/(9*b
) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b - (a^(3/2)*atan((
a^(3/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^5*f - a^2*b^3*c +
 a^3*b^2*d - a^4*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(11/2)
```

sympy [B] time = 1.32, size = 337, normalized size = 1.96

$$x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^5\left(\frac{a^2f}{5b^3}-\frac{ae}{5b^2}+\frac{d}{5b}\right)+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)+\frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}+\frac{fx^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b))
+ x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b**11
)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a**3/b**11)*(a**3
*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**
3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log
(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b)
```

3.112 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$

Optimal. Leaf size=136

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^5(be-}{5b^2}$$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}+\frac{x^5(be-af)}{5b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx=\int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^2}{b^3}+\frac{(be-af)x^4}{b^2}+\frac{fx^6}{b}+-\right.$$

$$=\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{(a}{}$$

$$=\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^4}+\frac{\left(b^2d-abe+a^2f\right)x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{\sqrt{}}{}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 0.94

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{b^{9/2}}+\frac{x\left(-105a^3f+35a^2b\left(3e+fx^2\right)-7ab^2\left(15d+5ex^2+3fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^4
) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2), x]
```

fricas [A] time = 0.79, size = 286, normalized size = 2.10

$$\left[\frac{30\,b^3fx^7+42\left(b^3e-ab^2f\right)x^5+70\left(b^3d-ab^2e+a^2bf\right)x^3-105\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\left(b^3c-ab^2d+a^2be-a^3f\right)x}{210\,b^4},\ \frac{15\,b^3fx^7+21\left(b^3e-ab^2f\right)x^5+35\left(b^3d-ab^2e+a^2bf\right)x^3-105\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\left(b^3c-ab^2d+a^2be-a^3f\right)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b^2*e + a^2
*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a/b)*log((b*x^2 +
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b
^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4]
```

giac [A] time = 0.42, size = 152, normalized size = 1.12

$$-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6fx^7-21\,ab^5fx^5+21\,b^6x^5e+35\,b^6dx^3+35\,a^2b^4fx^3-35\,ab^5x^3e+105\,b^6cx-105\,ab^5dx-105\,a^3b^3fx+105\,a^2b^4xe}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^6*f*x^7 - 21*a*b^5*f*x^5 + 21*b^6*x^5*e + 35*b^6*d*x^3 +
35*a^2*b^4*f*x^3 - 35*a*b^5*x^3*e + 105*b^6*c*x - 105*a*b^5*d*x - 105*a^3*b
^3*f*x + 105*a^2*b^4*x*e)/b^7
```

maple [A]   time = 0.00, size = 182, normalized size = 1.34

$$\frac{fx^7}{7b}-\frac{afx^5}{5b^2}+\frac{ex^5}{5b}+\frac{a^2fx^3}{3b^3}-\frac{aex^3}{3b^2}+\frac{dx^3}{3b}+\frac{a^4f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}-\frac{a^3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{a^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{a^3fx}{b^4}+\frac{a^2ex}{b^3}-\frac{adx}{b^2}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/7*f*x^7/b-1/5/b^2*x^5*a*f+1/5/b*x^5*e+1/3/b^3*x^3*a^2*f-1/3/b^2*x^3*a*e+1
/3/b*x^3*d-1/b^4*a^3*f*x+1/b^3*a^2*e*x-1/b^2*a*d*x+1/b*c*x+a^4/b^4/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)*f-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*e+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-a/b/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*c
```

maxima [A]   time = 2.96, size = 133, normalized size = 0.98

$$-\frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^3fx^7+21\left(b^3e-ab^2f\right)x^5+35\left(b^3d-ab^2e+a^2bf\right)x^3+105\left(b^3c-ab^2d+a^2be-a^3f\right)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b^2*e
+ a^2*b*f)*x^3 + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

mupad [B]   time = 0.91, size = 193, normalized size = 1.42

$$x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)+\frac{f\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^7)/(7*b) + (a^(1/
2)*atan((a^(1/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^4*f + a^
2*b^2*d - a*b^3*c - a^3*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(9/2)
```

sympy [A] time = 1.13, size = 185, normalized size = 1.36

$$x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right)+x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right)-\frac{\sqrt{-\frac{a}{b^9}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2}+\frac{fx^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) - sqrt(-a/b**9)*(a*
*3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b**9) + x)/2 + sqrt(
-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4*sqrt(-a/b**9) + x
)/2 + f*x**7/(7*b)
```

3.113 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} dx$

Optimal. Leaf size=100

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1810, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}+\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx &= \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^2}{b^2}+\frac{fx^4}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{\left(b^2d-abe+a^2f\right)x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 98, normalized size = 0.98

$$\frac{x\left(15a^2f-5ab\left(3e+fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4)))/(15*b
^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2), x]
```

fricas [A] time = 1.14, size = 236, normalized size = 2.36

$$\left[\frac{6\,ab^3fx^5+10\left(ab^3e-a^2b^2f\right)x^3+15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+30\left(ab^3d-a^2b^2e+a^3bf\right)x}{30\,ab^4},\ \frac{3\,ab^3fx^5+5\left(ab^3e-a^2b^2f\right)x^3+15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\left(ab^3d-a^2b^2e+a^3bf\right)x}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*f*x^5 + 10*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a))
+ 30*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4), 1/15*(3*a*b^3*f*x^5 + 5*(a
*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 15*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4)]
```

giac [A] time = 0.45, size = 106, normalized size = 1.06

$$\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^4fx^5-5\,ab^3fx^3+5\,b^4x^3e+15\,b^4dx+15\,a^2b^2fx-15\,ab^3xe}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^4*f*x^5 - 5*a*b^3*f*x^3 + 5*b^4*x^3*e + 15*b^4*d*x + 15*a^2*b^2*
f*x - 15*a*b^3*x*e)/b^5
```

maple [A] time = 0.00, size = 135, normalized size = 1.35

$$\frac{f\,x^5}{5b}-\frac{af\,x^3}{3b^2}+\frac{e\,x^3}{3b}-\frac{a^3 f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{a^2 e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{a^2 f x}{b^3}-\frac{aex}{b^2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*f*x^5/b-1/3/b^2*x^3*a*f+1/3/b*x^3*e+1/b^3*a^2*f*x-1/b^2*a*e*x+1/b*d*x-1
/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*f+1/b^2/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x)*a^2*e-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.97, size = 94, normalized size = 0.94

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^2fx^5+5\left(b^2e-abf\right)x^3+15\left(b^2d-abe+a^2f\right)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^2*f*x^5 + 5*(b^2*e - a*b*f)*x^3 + 15*(b^2*d - a*b*e + a^2*f)*x)/
b^3
```

mupad [B] time = 0.94, size = 96, normalized size = 0.96

$$x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{\sqrt{a}\;b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^5)
/(5*b) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a
^(1/2)*b^(7/2))
```

sympy [A] time = 1.15, size = 160, normalized size = 1.60

$$x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right)+\frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}+\frac{fx^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + sqrt(-1
/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(-1/(a*b
**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b)
```

3.114 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Rubi [**A**] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^2\right)}\right)dx\\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{ab^2}\\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 1.25, size = 211, normalized size = 2.51

$$\left[\frac{2\,a^2b^2fx^4-6\,ab^3c+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\,x\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(a^2b^2e-a^3bf\right)x^2}{6\,a^2b^3x},\ \frac{a^2b^2fx^4-3\,ab^3c-3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(a^2b^2e-a^3bf\right)x^2}{3\,a^2b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*f*x^4 - 6*a*b^3*c + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*s
qrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a^2*b^2*e -
a^3*b*f)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a*b^3*c - 3*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(a^2*b^2*e - a
^3*b*f)*x^2)/(a^2*b^3*x)]
```

giac [A] time = 0.34, size = 86, normalized size = 1.02

$$-\frac{c}{ax}-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}+\frac{b^2fx^3-3\,abfx+3\,b^2xe}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a*b^2) + 1/3*(b^2*f*x^3 - 3*a*b*f*x + 3*b^2*x*e)/b^3
```

maple [A] time = 0.01, size = 114, normalized size = 1.36

$$\frac{f\,x^3}{3b}+\frac{a^2f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{afx}{b^2}+\frac{ex}{b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/b-1/b^2*a*f*x+1/b*e*x+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*f-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+1/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)*d-1/(a*b)^(1/2)/a*b*c*arctan(1/(a*b)^(1/2)*b*x)-1/a*c/x
```

maxima [A] time = 2.93, size = 80, normalized size = 0.95

$$\frac{bfx^3+3\left(be-af\right)x}{3\,b^2}-\frac{c}{ax}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b*f*x^3 + 3*(b*e - a*f)*x)/b^2 - c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 1.07, size = 76, normalized size = 0.90

$$x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^3}{3\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(a*x) + (f*x^3)/(3*b) - (atan((b^(1/2)*x)/a^(1/2))*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(3/2)*b^(5/2))
```

sympy [B] time = 1.64, size = 150, normalized size = 1.79

$$x\left(-\frac{af}{b^2}+\frac{e}{b}\right)-\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2}+\frac{fx^3}{3b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) - sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)) +
 x)/2 + f*x**3/(3*b) - c/(a*x)
```

3.115 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)} dx$

Optimal. Leaf size=82

$$\frac{bc-ad}{a^2x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{f}{b}+\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\int \frac{1}{a+bx^2}\,dx}{a^2b}\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 1.01

$$\frac{bc-ad}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 1.15, size = 216, normalized size = 2.63

$$\left[\frac{6\,a^3bfx^4+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\,x^3\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,a^2b^2c+6\left(ab^3c-a^2b^2d\right)x^2}{6\,a^3b^2x^3},\frac{3\,a^3bfx^4+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{ab}\,x^3\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-a^2b^2c+3\left(ab^3c-a^2b^2d\right)x^2}{3\,a^3b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^3*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*a^2*b^2*c + 6*(a*b^3*c -
a^2*b^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - a^2*b^2*c + 3*(a*b^3*c
 - a^2*b^2*d)*x^2)/(a^3*b^2*x^3)]
```

giac [A] time = 0.36, size = 81, normalized size = 0.99

$$\frac{fx}{b}+\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

maple [A] time = 0.01, size = 115, normalized size = 1.40

$$-\frac{af\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b}-\frac{bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a}+\frac{b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^2}+\frac{e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{fx}{b}-\frac{d}{ax}+\frac{bc}{a^2x}-\frac{c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] f*x/b-1/b*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*e-b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^2/a^2/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c
```

maxima [A] time = 2.94, size = 79, normalized size = 0.96

$$\frac{fx}{b}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)
```

mupad [B] time = 0.11, size = 80, normalized size = 0.98

$$\frac{f\,x}{b}-\frac{\frac{b\,c}{3\,a}+\frac{b\,x^2\,(a\,d-b\,c)}{a^2}}{b\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x)
```

```
[Out] (f*x)/b - ((b*c)/(3*a) + (b*x^2*(a*d - b*c))/a^2)/(b*x^3) + (atan((b^(1/2)*
x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(5/2)*b^(3/2))
```

sympy [B] time = 2.27, size = 151, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}+\frac{fx}{b}+\frac{-ac+x^2\,(-3ad+3bc)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*sq
rt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**
2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b + (-a*c + x**2
*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

3.116 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=104

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{a^2e-abd+b^2c}{a^3x}+\frac{bc-ad}{3a^2x^3}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -c/(5*a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x) -
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*
Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^2\right)}\right)dx$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{a^3}$$

$$= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.99

$$\frac{bc-ad}{3a^2x^3}+\frac{a^2(-e)+abd-b^2c}{a^3x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3*
x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)), x]
```

fricas [A] time = 1.20, size = 246, normalized size = 2.37

$$\left[\frac{15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}\,x^5\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-6\,a^3bc-30\left(ab^3c-a^2b^2d+a^3be\right)x^4+10\left(a^2b^2c-a^3bd\right)x^2}{30\,a^4bx^5}, -\frac{15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{ab}\,x^5\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,a^3bc+15\left(ab^3c-a^2b^2d+a^3be\right)x^4-5\left(a^2b^2c-a^3bd\right)x^2}{15\,a^4bx^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^5*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*a^3*b*c - 30*(a*b^3*c - a^2*b^2*d + a^3
*b*e)*x^4 + 10*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4*b*x^5), -1/15*(15*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) + 3*a^3*b*c +
 15*(a*b^3*c - a^2*b^2*d + a^3*b*e)*x^4 - 5*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4
*b*x^5)]
```

giac [A] time = 0.37, size = 105, normalized size = 1.01

$$-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{15\,b^2cx^4-15\,abdx^4+15\,a^2x^4e-5\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*x^4*e - 5*a*b*c*x^2 + 5*a^2*d*
x^2 + 3*a^2*c)/(a^3*x^5)
```

maple [A] time = 0.01, size = 142, normalized size = 1.37

$$-\frac{be\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{b^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{b^3c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{e}{ax}+\frac{bd}{a^2x}-\frac{b^2c}{a^3x}-\frac{d}{3a\,x^3}+\frac{bc}{3a^2x^3}-\frac{c}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*b*e+1/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2*d-1/a^3/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*c/a/x^5-1/3/a/x^3*d+1/3/a^2/x^3*b
*c-1/a/x*e+1/a^2/x*b*d-1/a^3/x*b^2*c
```

maxima [A] time = 3.09, size = 97, normalized size = 0.93

$$-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{15\left(b^2c-abd+a^2e\right)x^4+3\,a^2c-5\left(abc-a^2d\right)x^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)/(
a^3*x^5)
```

mupad [B] time = 1.20, size = 94, normalized size = 0.90

$$-\frac{\frac{c}{5\,a}+\frac{x^2\,\left(a\,d-b\,c\right)}{3\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}}{x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (c/(5*a) + (x^2*(a*d - b*c))/(3*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/a^3)
/x^5 - (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(
7/2)*b^(1/2))
```

sympy [A] time = 6.73, size = 167, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^7b}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}+\frac{-3a^2c+x^4\left(-15a^2e+15abd-15b^2c\right)+x^2\left(-5a^2d+5abc\right)}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(-
1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3
*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*a**2*c + x**4*(-15*a**2*e + 15*
a*b*d - 15*b**2*c) + x**2*(-5*a**2*d + 5*a*b*c))/(15*a**3*x**5)
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)} \, dx$

Optimal. Leaf size=137

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{a^4x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{9/2}}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{bc-ad}{5a^2x^5}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -c/(7*a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x^3
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^8} + \frac{-bc+ad}{a^2x^6} + \frac{b^2c-abd+a^2e}{a^3x^4} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2} - \frac{b\left(-b^3c+ab^2\right.}{a^4\left(a\right.}\right.$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{5a^2x^5} - \frac{b^2c-abd+a^2e}{3a^3x^3} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{\left(b\left(b^3c-ab^2d+a\right.\right.}{}$$

$$= -\frac{c}{7ax^7} + \frac{bc-ad}{5a^2x^5} - \frac{b^2c-abd+a^2e}{3a^3x^3} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{\sqrt{b}\left(b^3c-ab^2d+\right.}{}$$

Mathematica [A] time = 0.12, size = 139, normalized size = 1.01

$$\frac{bc-ad}{5a^2x^5} + \frac{a^2(-e)+abd-b^2c}{3a^3x^3} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{9/2}} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x} - \frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)), x]
```

fricas [A] time = 0.85, size = 292, normalized size = 2.13

$$\left[-\frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-210\left(b^3c-ab^2d+a^2be-a^3f\right)x^6+70\left(ab^2c-a^2bd+a^3e\right)x^4+30a^3c-42\left(a^2bc-a^3d\right)x^2}{210a^4x^7},\ \frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\left(b^3c-ab^2d+a^2be-a^3f\right)x^6-35\left(ab^2c-a^2bd+a^3e\right)x^4-15a^3c+21\left(a^2bc-a^3d\right)x^2}{105a^4x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*log((b*x^2
- 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b^2*d + a^2*b*e - a^3
```

```
*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4 + 30*a^3*c - 42*(a^2*b*c - a^3
*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 3
5*(a*b^2*c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a
^4*x^7)]
```

giac [A] time = 0.46, size = 151, normalized size = 1.10

$$\frac{\left(b^4c - ab^3d - a^3bf + a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} + \frac{105\,b^3cx^6 - 105\,ab^2dx^6 - 105\,a^3fx^6 + 105\,a^2bx^6e - 35\,ab^2cx^4 + 35\,a^2bdx^4 - 35\,a^3x^4e + 21\,a^2bcx^2 - 21\,a^3dx^2 - 15\,a^3c}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 - 105*a^3*f*x^6 + 105*a^2*b*x^6
*e - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*x^4*e + 21*a^2*b*c*x^2 - 21*a
^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

maple [A] time = 0.01, size = 190, normalized size = 1.39

$$-\frac{bf\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{b^2e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} - \frac{b^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3} + \frac{b^4c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4} - \frac{f}{ax} + \frac{be}{a^2x} - \frac{b^2d}{a^3x} + \frac{b^3c}{a^4x} - \frac{e}{3a\,x^3} + \frac{bd}{3a^2x^3} - \frac{b^2c}{3a^3x^3} - \frac{d}{5a\,x^5} + \frac{bc}{5a^2x^5} - \frac{c}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x)
```

```
[Out] -b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+b^2/a^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*e-b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^4/a^4/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7*c/a/x^7-1/5/a/x^5*d+1/5/a^2/x^5*
b*c-1/3/a/x^3*e+1/3/a^2/x^3*b*d-1/3/a^3/x^3*b^2*c-1/a/x*f+1/a^2/x*b*e-1/a^3
/x*b^2*d+1/a^4/x*b^3*c
```

maxima [A] time = 3.03, size = 134, normalized size = 0.98

$$\frac{\left(b^4c - ab^3d + a^2b^2e - a^3bf\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} + \frac{105\left(b^3c - ab^2d + a^2be - a^3f\right)x^6 - 35\left(ab^2c - a^2bd + a^3e\right)x^4 - 15\,a^3c + 21\left(a^2bc - a^3d\right)x^2}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*c - a^2
*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)
```

mupad [B] time = 0.98, size = 127, normalized size = 0.93

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{9/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{5\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
9/2) - (c/(7*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^2*(a*d
 - b*c))/(5*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^7
```

sympy [B] time = 21.65, size = 301, normalized size = 2.20

$$\frac{\sqrt{-\frac{b}{a^9}}\,\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}\,\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^9}}\,\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}\,\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}+\frac{-15a^3c+x^6\left(-105a^3f+105a^2be-105ab^2d+105b^3c\right)+x^4\left(-35a^3e+35a^2bd-35ab^2c\right)+x^2\left(-21a^3d+21a^2bc\right)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a**
9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3
*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
 a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 + (-15*a**3*c + x**6*(-105*a**3*f
+ 105*a**2*b*e - 105*a*b**2*d + 105*b**3*c) + x**4*(-35*a**3*e + 35*a**2*b*
d - 35*a*b**2*c) + x**2*(-21*a**3*d + 21*a**2*b*c))/(105*a**4*x**7)
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$

Optimal. Leaf size=175

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{11/2}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-$$

Rubi [A] time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$\frac{a^2be+a^3(-f)-ab^2d+b^3c}{3a^4x^3}-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^5x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{11/2}}-\frac{a^2e-abd+b^2c}{5a^3x^5}+\frac{bc-ad}{7a^2x^7}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -c/(9*a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x^5
) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^{10}} + \frac{-bc+ad}{a^2x^8} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^4} - \frac{b\left(-b^3c+ab\right.}{a}\right.$$

$$= -\frac{c}{9ax^9} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3} - \frac{b\left(b^3c-ab^2d+a^2\right.}{a^5x}$$

$$= -\frac{c}{9ax^9} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3} - \frac{b\left(b^3c-ab^2d+a^2\right.}{a^5x}$$

Mathematica [A] time = 0.14, size = 174, normalized size = 0.99

$$\frac{bc-ad}{7a^2x^7} + \frac{a^2(-e)+abd-b^2c}{5a^3x^5} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^{11/2}} + \frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3} - \frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a^
3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e
 + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)), x]
```

fricas [A] time = 0.95, size = 374, normalized size = 2.14

$$\left[-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-210\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^6+70a^4c+126\left(a^2b^2c-a^3bd+a^4e\right)x^4-90\left(a^3bc-a^4d\right)x^2}{630a^5x^9}, -\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-105\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x^6+35a^4c+63\left(a^2b^2c-a^3bd+a^4e\right)x^4-45\left(a^3bc-a^4d\right)x^2}{315a^5x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c - a*b^3*d + a^2*b^2*e
```

```
 - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 70*a^4*
c + 126*(a^2*b^2*c - a^3*b*d + a^4*e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(a^5*
x^9), -1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a)*arc
tan(x*sqrt(b/a)) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)]
```

giac [A] time = 0.42, size = 201, normalized size = 1.15

$$-\frac{\left(b^{5} c-a b^{4} d-a^{3} b^{2} f+a^{2} b^{3} e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{5}}-\frac{315\, b^{4} c x^{8}-315\, a b^{3} d x^{8}-315\, a^{3} b f x^{8}+315\, a^{2} b^{2} x^{8} e-105\, a b^{3} c x^{6}+105\, a^{2} b^{2} d x^{6}+105\, a^{4} f x^{6}-105\, a^{3} b x^{6} e+63\, a^{2} b^{2} c x^{4}-63\, a^{3} b d x^{4}+63\, a^{4} x^{4} e-45\, a^{3} b c x^{2}+45\, a^{4} d x^{2}+35\, a^{4} c}{315\, a^{5} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 - 315*a^3*b*f*x^8 + 315*a^2*
b^2*x^8*e - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 + 105*a^4*f*x^6 - 105*a^3*b
*x^6*e + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*x^4*e - 45*a^3*b*c*x^2
+ 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

maple [A] time = 0.01, size = 238, normalized size = 1.36

$$\frac{b^{2} f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{2}}-\frac{b^{3} e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{3}}+\frac{b^{4} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{4}}-\frac{b^{5} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{5}}+\frac{b f}{a^{2} x}-\frac{b^{2} e}{a^{3} x}+\frac{b^{3} d}{a^{4} x}-\frac{b^{4} c}{a^{5} x}-\frac{f}{3 a x^{3}}+\frac{b e}{3 a^{2} x^{3}}-\frac{b^{2} d}{3 a^{3} x^{3}}+\frac{b^{3} c}{3 a^{4} x^{3}}-\frac{e}{5 a x^{5}}+\frac{b d}{5 a^{2} x^{5}}-\frac{b^{2} c}{5 a^{3} x^{5}}-\frac{d}{7 a x^{7}}+\frac{b c}{7 a^{2} x^{7}}-\frac{c}{9 a x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x)
```

```
[Out] b^2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-b^3/a^3/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*e+b^4/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-b^5/a^
5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/9*c/a/x^9-1/7/a/x^7*d+1/7/a^2/x
^7*b*c-1/5/a/x^5*e+1/5/a^2/x^5*b*d-1/5/a^3/x^5*b^2*c-1/3/a/x^3*f+1/3/a^2/x^
3*b*e-1/3/a^3/x^3*b^2*d+1/3/a^4/x^3*b^3*c+1/a^2*b/x*f-1/a^3*b^2/x*e+1/a^4*b
^3/x*d-1/a^5*b^4/x*c
```

maxima [A] time = 3.01, size = 175, normalized size = 1.00

$$-\frac{\left(b^{5} c-a b^{4} d+a^{2} b^{3} e-a^{3} b^{2} f\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{5}}-\frac{315\, \left(b^{4} c-a b^{3} d+a^{2} b^{2} e-a^{3} b f\right) x^{8}-105\, \left(a b^{3} c-a^{2} b^{2} d+a^{3} b e-a^{4} f\right) x^{6}+35\, a^{4} c+63\, \left(a^{2} b^{2} c-a^{3} b d+a^{4} e\right) x^{4}-45\, \left(a^{3} b c-a^{4} d\right) x^{2}}{315\, a^{5} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a*b^3
```

```
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b*d
+ a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
```

mupad [B] time = 1.02, size = 161, normalized size = 0.92

$$-\frac{\frac{c}{9a}-\frac{x^6\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^2\,(a\,d-b\,c)}{7\,a^2}+\frac{x^4\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}+\frac{b\,x^8\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^9}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x)
```

```
[Out] - (c/(9*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^2*(a*d
- b*c))/(7*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(5*a^3) + (b*x^8*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/a^5)/x^9 - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(11/2)
```

sympy [B] time = 32.72, size = 354, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{-35a^4c+x^8\left(315a^3bf-315a^2b^2e+315ab^3d-315b^4c\right)+x^6\left(-105a^4f+105a^3be-105a^2b^2d+105ab^3c\right)+x^4\left(-63a^4e+63a^3bd-63a^2b^2c\right)+x^2\left(-45a^4d+45a^3bc\right)}{315a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(-
b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b**
3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4
*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x*
*6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**4*(-6
3*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a**3*b*c))
/(315*a**5*x**9)
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=211

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\,(}{}$$

Rubi [A] time = 0.18, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1802, 205}

$$-\frac{b\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^2be+a^3(-f)-ab^2d+b^3c}{5a^4x^5}+\frac{b^2\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^6x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{a^{13/2}}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{bc-ad}{9a^2x^9}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -c/(11*a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3*x
^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
)/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(13/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^{10}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b\left(-b^3c+ab\right.}{}\right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{3a^5x^3}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b\left(b^3c-ab^2d+\right.}{3a^5x^3}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 1.00

$$\frac{bc-ad}{9a^2x^9} - \frac{a^2e-abd+b^2c}{7a^3x^7} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}} + \frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x} + \frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{3a^5x^3} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5} - \frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/a^(13/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)), x]
```

fricas [A] time = 1.08, size = 458, normalized size = 2.17

$$\left[-\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 6930\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10} + 2310\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^8 - 1386\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x^6 + 630a^5c + 990\left(a^3b^2c-a^4bd+a^5e\right)x^4 - 770\left(a^4bc-a^5d\right)x^2}{6930a^6x^{11}}, \frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{10} - 1155\left(ab^4c-a^2b^3d+a^3b^2e-a^4bf\right)x^8 + 693\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x^6 - 315a^5c - 495\left(a^3b^2c-a^4bd+a^5e\right)x^4 + 385\left(a^4bc-a^5d\right)x^2}{3465a^6x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(-b/a)*lo
g((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b^5*c - a*b^4*d + a^2
*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)
*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 + 630*a^5*c + 990
*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11),
 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a)*arct
an(x*sqrt(b/a)) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 115
5*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^
2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*
x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)]
```

giac [A] time = 0.36, size = 249, normalized size = 1.18

$$\frac{\left(b^{6} c - a b^{5} d - a^{3} b^{3} f + a^{2} b^{4} e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} a^{6}} + \frac{3465 \, b^{5} c x^{10} - 3465 \, a b^{4} d x^{10} - 3465 \, a^{3} b^{2} f x^{10} + 3465 \, a^{2} b^{3} x^{10} e - 1155 \, a b^{4} c x^{8} + 1155 \, a^{2} b^{3} d x^{8} + 1155 \, a^{4} b f x^{8} - 1155 \, a^{3} b^{2} x^{8} e + 693 \, a^{2} b^{3} c x^{6} - 693 \, a^{3} b^{2} d x^{6} - 693 \, a^{5} f x^{6} + 693 \, a^{4} b x^{6} e - 495 \, a^{3} b^{2} c x^{4} + 495 \, a^{4} b d x^{4} - 495 \, a^{5} x^{4} e + 385 \, a^{4} b c x^{2} - 385 \, a^{5} d x^{2} - 315 \, a^{5} c}{3465 \, a^{6} x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 - 3465*a^3*b^2*f*x^10 +
3465*a^2*b^3*x^10*e - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 + 1155*a^4*b*f*
x^8 - 1155*a^3*b^2*x^8*e + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 - 693*a^5*
f*x^6 + 693*a^4*b*x^6*e - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*x^4
*e + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

maple [A] time = 0.01, size = 286, normalized size = 1.36

$$-\frac{b^{3} f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{3}} + \frac{b^{4} e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{4}} - \frac{b^{5} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{5}} + \frac{b^{6} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{6}} - \frac{b^{2} f}{a^{3} x} + \frac{b^{3} e}{a^{4} x} - \frac{b^{4} d}{a^{5} x} + \frac{b^{5} c}{a^{6} x} + \frac{b f}{3 a^{2} x^{3}} - \frac{b^{2} e}{3 a^{3} x^{3}} + \frac{b^{3} d}{3 a^{4} x^{3}} - \frac{b^{4} c}{3 a^{5} x^{3}} - \frac{f}{5 a x^{5}} + \frac{b e}{5 a^{2} x^{5}} - \frac{b^{2} d}{5 a^{3} x^{5}} + \frac{b^{3} c}{5 a^{4} x^{5}} - \frac{e}{7 a x^{7}} + \frac{b d}{7 a^{2} x^{7}} - \frac{b^{2} c}{7 a^{3} x^{7}} - \frac{d}{9 a x^{9}} + \frac{b c}{9 a^{2} x^{9}} - \frac{c}{11 a x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x)
```

```
[Out] -b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+b^4/a^4/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*e-b^5/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^6/a
^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/11*c/a/x^11-1/9/a/x^9*d+1/9/a^
2/x^9*b*c-1/7/a/x^7*e+1/7/a^2/x^7*b*d-1/7/a^3/x^7*b^2*c-1/5/a/x^5*f+1/5/a^2
/x^5*b*e-1/5/a^3/x^5*b^2*d+1/5/a^4/x^5*b^3*c-1/a^3*b^2/x*f+1/a^4*b^3/x*e-1/
a^5*b^4/x*d+1/a^6*b^5/x*c+1/3/a^2*b/x^3*f-1/3/a^3*b^2/x^3*e+1/3/a^4*b^3/x^3
*d-1/3/a^5*b^4/x^3*c
```

maxima [A] time = 2.95, size = 214, normalized size = 1.01

$$\frac{\left(b^{6} c - a b^{5} d + a^{2} b^{4} e - a^{3} b^{3} f\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} a^{6}} + \frac{3465 \, {\left(b^{5} c - a b^{4} d + a^{2} b^{3} e - a^{3} b^{2} f\right)} x^{10} - 1155 \, {\left(a b^{4} c - a^{2} b^{3} d + a^{3} b^{2} e - a^{4} b f\right)} x^{8} + 693 \, {\left(a^{2} b^{3} c - a^{3} b^{2} d + a^{4} b e - a^{5} f\right)} x^{6} - 315 \, a^{5} c - 495 \, {\left(a^{3} b^{2} c - a^{4} b d + a^{5} e\right)} x^{4} + 385 \, {\left(a^{4} b c - a^{5} d\right)} x^{2}}{3465 \, a^{6} x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 1155*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4
 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)
```

mupad [B] time = 0.99, size = 197, normalized size = 0.93

$$\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{13/2}}-\frac{\frac{c}{11\,a}-\frac{x^6\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^4\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^8\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}-\frac{b^2\,x^{10}\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
13/2) - (c/(11*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^
2*(a*d - b*c))/(9*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^8*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^5) - (b^2*x^10*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/a^6)/x^11
```

sympy [A] time = 84.14, size = 398, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}+\frac{-315a^5c+x^{10}\left(-3465a^3b^2f+3465a^2b^3e-3465ab^4d+3465b^5c\right)+x^8\left(1155a^4bf-1155a^3b^2e+1155a^2b^3d-1155ab^4c\right)+x^6\left(-693a^5f+693a^4be-693a^3b^2d+693a^2b^3c\right)+x^4\left(-495a^5e+495a^4bd-495a^3b^2c\right)+x^2\left(-385a^5d+385a^4bc\right)}{3465a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-b
**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b**4
*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 + (-315*a**5
*c + x**10*(-3465*a**3*b**2*f + 3465*a**2*b**3*e - 3465*a*b**4*d + 3465*b**
5*c) + x**8*(1155*a**4*b*f - 1155*a**3*b**2*e + 1155*a**2*b**3*d - 1155*a*b
**4*c) + x**6*(-693*a**5*f + 693*a**4*b*e - 693*a**3*b**2*d + 693*a**2*b**3
*c) + x**4*(-495*a**5*e + 495*a**4*b*d - 495*a**3*b**2*c) + x**2*(-385*a**5
*d + 385*a**4*b*c))/(3465*a**6*x**11)
```

3.120 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=240

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{ax\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{2b^6}+\frac{x^3\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{6b^5}-\frac{x^5\left(-11a^3f\right.}{}$$

Rubi [A] time = 0.29, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^5\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{10ab^4}+\frac{x^3\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{6b^5}-\frac{ax\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{2b^6}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(9a^2be-11a^3f-7ab^2d+5b^3c\right)}{2b^{13/2}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/(2*b^6) + ((5*b^3*c - 7
*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a
^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/
(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (
a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^5\left(\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^6\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \left(\frac{a^2\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)}{b^5} - \frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^2}{b^4} \cdots\right.}{\cdots}$$

$$= -\frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x}{2b^6} + \frac{\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^3}{6b^5} - \cdots$$

$$= -\frac{a\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x}{2b^6} + \frac{\left(5b^3c-7ab^2d+9a^2be-11a^3f\right)x^3}{6b^5} - \cdots$$

Mathematica [A] time = 0.13, size = 227, normalized size = 0.95

$$\frac{x^5\left(3a^2f-2abe+b^2d\right)}{5b^4}+\frac{ax\left(5a^3f-4a^2be+3ab^2d-2b^3c\right)}{b^6}+\frac{x^3\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(11a^3f-9a^2be+7ab^2d-5b^3c\right)}{2b^{13/2}}-\frac{x\left(a^5(-f)+a^4be-a^3b^2d+a^2b^3c\right)}{2b^6\left(a+bx^2\right)}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(
5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*
b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(c + dx^2 + ex^4 + fx^6\right)}{\left(a + bx^2\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.87, size = 572, normalized size = 2.38

$$\left[\frac{140 \, b^{5} f x^{11} + 20 \, {\left(9 \, b^{5} e - 11 \, a b^{4} f\right)} x^{9} + 36 \, {\left(7 \, b^{5} d - 9 \, a b^{4} e + 11 \, a^{2} b^{3} f\right)} x^{7} + 84 \, {\left(5 \, b^{5} c - 7 \, a b^{4} d + 9 \, a^{2} b^{3} e - 11 \, a^{3} b^{2} f\right)} x^{5} - 420 \, {\left(5 \, a b^{4} c - 7 \, a^{2} b^{3} d + 9 \, a^{3} b^{2} e - 11 \, a^{4} b f\right)} x^{3} - 315 \, {\left(5 \, a^{2} b^{3} c - 7 \, a^{3} b^{2} d + 9 \, a^{4} b e - 11 \, a^{5} f + {\left(5 \, a b^{4} c - 7 \, a^{2} b^{3} d + 9 \, a^{3} b^{2} e - 11 \, a^{4} b f\right)} x^{2}\right)} \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2} - 2 \, b x \sqrt{-\frac{a}{b}} - a}{b x^{2} + a}\right) - 630 \, {\left(5 \, a^{2} b^{3} c - 7 \, a^{3} b^{2} d + 9 \, a^{4} b e - 11 \, a^{5} f\right)} x}{1260 \, {\left(b^{7} x^{2} + a b^{6}\right)}}, \frac{70 \, b^{5} f x^{11} + 10 \, {\left(9 \, b^{5} e - 11 \, a b^{4} f\right)} x^{9} + 18 \, {\left(7 \, b^{5} d - 9 \, a b^{4} e + 11 \, a^{2} b^{3} f\right)} x^{7} + 42 \, {\left(5 \, b^{5} c - 7 \, a b^{4} d + 9 \, a^{2} b^{3} e - 11 \, a^{3} b^{2} f\right)} x^{5} - 210 \, {\left(5 \, a b^{4} c - 7 \, a^{2} b^{3} d + 9 \, a^{3} b^{2} e - 11 \, a^{4} b f\right)} x^{3} + 315 \, {\left(5 \, a^{2} b^{3} c - 7 \, a^{3} b^{2} d + 9 \, a^{4} b e - 11 \, a^{5} f + {\left(5 \, a b^{4} c - 7 \, a^{2} b^{3} d + 9 \, a^{3} b^{2} e - 11 \, a^{4} b f\right)} x^{2}\right)} \sqrt{\frac{a}{b}} \arctan\left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right) - 315 \, {\left(5 \, a^{2} b^{3} c - 7 \, a^{3} b^{2} d + 9 \, a^{4} b e - 11 \, a^{5} f\right)} x}{630 \, {\left(b^{7} x^{2} + a b^{6}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b^5*d - 9*a
*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3
*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3
- 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^
2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*
a^5*f)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11*a*b^4*
f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c - 7*a*b^
4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^
3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f)*x)/(b^7*x^2 + a*b^6)]
```

giac [A] time = 0.39, size = 252, normalized size = 1.05

$$\frac{{\left(5 \, a^{2} b^{3} c - 7 \, a^{3} b^{2} d - 11 \, a^{5} f + 9 \, a^{4} b e\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} b^{6}} - \frac{a^{2} b^{3} c x - a^{3} b^{2} d x - a^{5} f x + a^{4} b x e}{2 \, {\left(b x^{2} + a\right)} b^{6}} + \frac{35 \, b^{16} f x^{9} - 90 \, a b^{15} f x^{7} + 45 \, b^{16} x^{7} e + 63 \, b^{16} d x^{5} + 189 \, a^{2} b^{14} f x^{5} - 126 \, a b^{15} x^{5} e + 105 \, b^{16} c x^{3} - 210 \, a b^{15} d x^{3} - 420 \, a^{3} b^{13} f x^{3} + 315 \, a^{2} b^{14} x^{3} e - 630 \, a b^{15} c x + 945 \, a^{2} b^{14} d x + 1575 \, a^{4} b^{12} f x - 1260 \, a^{3} b^{13} x e}{315 \, b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/(
(b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 - 90*a*b^15*f*x^7 + 45*b^16*x^7*e +
 63*b^16*d*x^5 + 189*a^2*b^14*f*x^5 - 126*a*b^15*x^5*e + 105*b^16*c*x^3 - 2
10*a*b^15*d*x^3 - 420*a^3*b^13*f*x^3 + 315*a^2*b^14*x^3*e - 630*a*b^15*c*x
+ 945*a^2*b^14*d*x + 1575*a^4*b^12*f*x - 1260*a^3*b^13*x*e)/b^18
```

maple [A] time = 0.01, size = 309, normalized size = 1.29

$$\frac{f x^9}{9b^2}-\frac{2af x^7}{7b^3}+\frac{e x^7}{7b^2}+\frac{3a^2f x^5}{5b^4}-\frac{2ae x^5}{5b^3}+\frac{d x^5}{5b^2}-\frac{4a^3f x^3}{3b^5}+\frac{a^2e x^3}{b^4}-\frac{2ad x^3}{3b^3}+\frac{c x^3}{3b^2}+\frac{a^5fx}{2\left(b x^2+a\right)b^6}-\frac{11a^5f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^6}-\frac{a^4ex}{2\left(b x^2+a\right)b^5}+\frac{9a^4e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5}+\frac{a^3dx}{2\left(b x^2+a\right)b^4}-\frac{7a^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4}-\frac{a^2cx}{2\left(b x^2+a\right)b^3}+\frac{5a^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2dx}{b^4}-\frac{2acx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/9*f*x^9/b^2-2/7/b^3*x^7*a*f+1/7/b^2*x^7*e+3/5/b^4*x^5*a^2*f-2/5/b^3*x^5*a
*e+1/5/b^2*x^5*d-4/3/b^5*x^3*a^3*f+1/b^4*x^3*a^2*e-2/3/b^3*x^3*a*d+1/3/b^2*
x^3*c+5/b^6*a^4*f*x-4/b^5*a^3*e*x+3/b^4*a^2*d*x-2/b^3*a*c*x+1/2*a^5/b^6*x/(
b*x^2+a)*f-1/2*a^4/b^5*x/(b*x^2+a)*e+1/2*a^3/b^4*x/(b*x^2+a)*d-1/2*a^2/b^3*
x/(b*x^2+a)*c-11/2*a^5/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+9/2*a^4/
b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-7/2*a^3/b^4/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*d+5/2*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.01, size = 227, normalized size = 0.95

$$-\frac{\left(a^2b^3c-a^3b^2d+a^4be-a^5f\right)x}{2\left(b^7x^2+ab^6\right)}+\frac{\left(5\,a^2b^3c-7\,a^3b^2d+9\,a^4be-11\,a^5f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{35\,b^4fx^9+45\left(b^4e-2\,ab^3f\right)x^7+63\left(b^4d-2\,ab^3e+3\,a^2b^2f\right)x^5+105\left(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf\right)x^3-315\left(2\,ab^3c-3\,a^2b^2d+4\,a^3be-5\,a^4f\right)x}{315\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^2 + a*b^6) + 1/2*(5
*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^6) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 2*a*b^3*f)*x^7 + 63*(b^4*d
- 2*a*b^3*e + 3*a^2*b^2*f)*x^5 + 105*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a
^3*b*f)*x^3 - 315*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6
```

mupad [B] time = 0.10, size = 413, normalized size = 1.72

$$x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b^2}+\frac{x\left(\frac{f\,a^5}{2}-\frac{e\,a^4\,b}{2}+\frac{d\,a^3\,b^2}{2}-\frac{c\,a^2\,b^3}{2}\right)}{b^7\,x^2+a\,b^6}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{11\,f\,a^5-9\,e\,a^4\,b+7\,d\,a^3\,b^2-5\,c\,a^2\,b^3}\right)\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{2\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^5
*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^3*(c
/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2
+ (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) + (f*x^9)/(9*b^2) + (x*((a^5*f)/2
- (a^2*b^3*c)/2 + (a^3*b^2*d)/2 - (a^4*b*e)/2))/(a*b^6 + b^7*x^2) - (a^(3/2
)*atan((a^(3/2)*b^(1/2)*x*(5*b^3*c - 11*a^3*f - 7*a*b^2*d + 9*a^2*b*e))/(11
*a^5*f - 5*a^2*b^3*c + 7*a^3*b^2*d - 9*a^4*b*e))*(5*b^3*c - 11*a^3*f - 7*a*
b^2*d + 9*a^2*b*e))/(2*b^(13/2))
```

sympy [A] time = 3.07, size = 444, normalized size = 1.85

$$x^{7}\left(-\frac{2af}{7b^{3}}+\frac{e}{7b^{2}}\right)+x^{5}\left(\frac{3a^{2}f}{5b^{4}}-\frac{2ae}{5b^{3}}+\frac{d}{5b^{2}}\right)+x^{3}\left(-\frac{4a^{3}f}{3b^{5}}+\frac{a^{2}e}{b^{4}}-\frac{2ad}{3b^{3}}+\frac{c}{3b^{2}}\right)+x\left(\frac{5a^{4}f}{b^{6}}-\frac{4a^{3}e}{b^{5}}+\frac{3a^{2}d}{b^{4}}-\frac{2ac}{b^{3}}\right)+\frac{x\left(a^{5}f-a^{4}be+a^{3}b^{2}d-a^{2}b^{3}c\right)}{2ab^{6}+2b^{7}x^{2}}+\frac{\sqrt{-\frac{a^{3}}{b^{13}}}\left(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c\right)\log\left(-\frac{b^{6}\sqrt{-\frac{a^{3}}{b^{13}}}\left(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c\right)}{11a^{4}f-9a^{3}be+7a^{2}b^{2}d-5ab^{3}c}+x\right)}{4}-\frac{\sqrt{-\frac{a^{3}}{b^{13}}}\left(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c\right)\log\left(\frac{b^{6}\sqrt{-\frac{a^{3}}{b^{13}}}\left(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c\right)}{11a^{4}f-9a^{3}be+7a^{2}b^{2}d-5ab^{3}c}+x\right)}{4}+\frac{fx^{9}}{9b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b*
*3) + d/(5*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3)
 + c/(3*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b
**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b**7
*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)
*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*
c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sqrt(-a**
3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a*
*3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a
**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2)
```

3.121 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^{11/2}}+\frac{x\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^5}-\frac{x^3}{}$$

Rubi [A] time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x^3\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{2b^5}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(7a^2be-9a^3f-5ab^2d+3b^3c\right)}{2b^{11/2}}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int\left(-\frac{a\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)}{b^4}+\frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^2}{b^3} \ldots\right.}{2ab \ldots}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be- \ldots}{5 \ldots}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be- \ldots}{5 \ldots}$$

Mathematica [A] time = 0.10, size = 187, normalized size = 0.93

$$\frac{x^3\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)}{2b^{11/2}}+\frac{x\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{b^5}+\frac{x\left(a^4(-f)+a^3be-a^2b^2d+ab^3c\right)}{2b^5\left(a+bx^2\right)}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*b^
```

```
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3*b^
3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(1
1/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2, x]
```

fricas [A] time = 1.17, size = 478, normalized size = 2.37

$$\left[\frac{60\,b^4fx^9+12\left(7\,b^4e-9\,ab^3f\right)x^7+28\left(5\,b^4d-7\,ab^3e+9\,a^2b^2f\right)x^5+140\left(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3bf\right)x^3-105\left(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f+\left(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3bf\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\left(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f\right)x}{420\left(b^6x^2+ab^5\right)},\ \frac{30\,b^4fx^9+6\left(7\,b^4e-9\,ab^3f\right)x^7+14\left(5\,b^4d-7\,ab^3e+9\,a^2b^2f\right)x^5+70\left(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3bf\right)x^3-105\left(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f+\left(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3bf\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\left(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f\right)x}{210\left(b^6x^2+ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d - 7*a*b^3
*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)
*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f + (3*b^4*c - 5*a*
b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)
*x)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*x^7 +
14*(5*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d + 7*a^
2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4
*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*x)/(
b^6*x^2 + a*b^5)]
```

giac [A] time = 0.42, size = 201, normalized size = 1.00

$$-\frac{\left(3\,ab^3c-5\,a^2b^2d-9\,a^4f+7\,a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{ab^3cx-a^2b^2dx-a^4fx+a^3bxe}{2\left(bx^2+a\right)b^5}+\frac{15\,b^{12}fx^7-42\,ab^{11}fx^5+21\,b^{12}x^5e+35\,b^{12}dx^3+105\,a^2b^{10}fx^3-70\,ab^{11}x^3e+105\,b^{12}cx-210\,ab^{11}dx-420\,a^3b^9fx+315\,a^2b^{10}xe}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x
^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 - 42*a*b^11*f*x^5 + 21*b^12*x^5*e + 35*
```

```
b^12*d*x^3 + 105*a^2*b^10*f*x^3 - 70*a*b^11*x^3*e + 105*b^12*c*x - 210*a*b^
11*d*x - 420*a^3*b^9*f*x + 315*a^2*b^10*x*e)/b^14
```

maple [A] time = 0.01, size = 258, normalized size = 1.28

$$\frac{f x^7}{7b^2} - \frac{2af x^5}{5b^3} + \frac{e x^5}{5b^2} + \frac{a^2 f x^3}{b^4} - \frac{2ae x^3}{3b^3} + \frac{d x^3}{3b^2} - \frac{a^4 fx}{2\left(b x^2 + a\right) b^5} + \frac{9a^4 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\, b^5} + \frac{a^3 ex}{2\left(b x^2 + a\right) b^4} - \frac{7a^3 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\, b^4} - \frac{a^2 dx}{2\left(b x^2 + a\right) b^3} + \frac{5a^2 d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\, b^3} + \frac{acx}{2\left(b x^2 + a\right) b^2} - \frac{3ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\, b^2} - \frac{4a^3 fx}{b^5} + \frac{3a^2 ex}{b^4} - \frac{2adx}{b^3} + \frac{cx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*f*x^7/b^2-2/5/b^3*x^5*a*f+1/5/b^2*x^5*e+1/b^4*x^3*a^2*f-2/3/b^3*x^3*a*e
+1/3/b^2*x^3*d-4/b^5*a^3*f*x+3/b^4*a^2*e*x-2/b^3*a*d*x+1/b^2*c*x-1/2*a^4/b^
5*x/(b*x^2+a)*f+1/2*a^3/b^4*x/(b*x^2+a)*e-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b
^2*x/(b*x^2+a)*c+9/2*a^4/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-7/2*a^
3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+5/2*a^2/b^3/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*d-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.01, size = 183, normalized size = 0.91

$$\frac{\left(ab^3 c - a^2 b^2 d + a^3 be - a^4 f\right)x}{2\left(b^6 x^2 + ab^5\right)} - \frac{\left(3\, ab^3 c - 5\, a^2 b^2 d + 7\, a^3 be - 9\, a^4 f\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}\, b^5} + \frac{15\, b^3 f x^7 + 21\left(b^3 e - 2\, ab^2 f\right)x^5 + 35\left(b^3 d - 2\, ab^2 e + 3\, a^2 bf\right)x^3 + 105\left(b^3 c - 2\, ab^2 d + 3\, a^2 be - 4\, a^3 f\right)x}{105\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^2 + a*b^5) - 1/2*(3*a*
b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 2*a*b^2*f)*x^5 + 35*(b^3*d - 2*a*
b^2*e + 3*a^2*b*f)*x^3 + 105*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b
^5
```

mupad [B] time = 0.97, size = 288, normalized size = 1.43

$$x^5 \left(\frac{e}{5\, b^2} - \frac{2\, a\, f}{5\, b^3}\right) + x \left(\frac{c}{b^2} - \frac{a^2 \left(\frac{e}{b^2} - \frac{2\, a\, f}{b^3}\right)}{b^2} + \frac{2\, a \left(\frac{a^2\, f}{b^4} - \frac{d}{b^2} + \frac{2\, a \left(\frac{e}{b^2} - \frac{2\, a\, f}{b^3}\right)}{b}\right)}{b}\right) - x^3 \left(\frac{a^2\, f}{3\, b^4} - \frac{d}{3\, b^2} + \frac{2\, a \left(\frac{e}{b^2} - \frac{2\, a\, f}{b^3}\right)}{3\, b}\right) - \frac{x \left(\frac{f\, a^4}{2} - \frac{e\, a^3\, b}{2} + \frac{d\, a^2\, b^2}{2} - \frac{c\, a\, b^3}{2}\right)}{b^6\, x^2 + a\, b^5} + \frac{f\, x^7}{7\, b^2} + \frac{\sqrt{a}\, \mathrm{atan}\left(\frac{\sqrt{a}\, \sqrt{b}\, x \left(-9\, f\, a^3 + 7\, e\, a^2\, b - 5\, d\, a\, b^2 + 3\, c\, b^3\right)}{9\, f\, a^4 - 7\, e\, a^3\, b + 5\, d\, a^2\, b^2 - 3\, c\, a\, b^3}\right) \left(-9\, f\, a^3 + 7\, e\, a^2\, b - 5\, d\, a\, b^2 + 3\, c\, b^3\right)}{2\, b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^3*
((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(3*b)) - (x*((a^
4*f)/2 + (a^2*b^2*d)/2 - (a*b^3*c)/2 - (a^3*b*e)/2))/(a*b^5 + b^6*x^2) + (f
*x^7)/(7*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*b^3*c - 9*a^3*f - 5*a*b
```

```
^2*d + 7*a^2*b*e))/(9*a^4*f + 5*a^2*b^2*d - 3*a*b^3*c - 7*a^3*b*e))*(3*b^3*
c - 9*a^3*f - 5*a*b^2*d + 7*a^2*b*e))/(2*b^(11/2))
```

sympy [A] time = 4.76, size = 257, normalized size = 1.27

$$x^5\left(-\frac{2af}{5b^3}+\frac{e}{5b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right)+x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x\left(-a^4f+a^3be-a^2b^2d+ab^3c\right)}{2ab^5+2b^6x^2}-\frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)\log\left(-b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^{11}}}\left(9a^3f-7a^2be+5ab^2d-3b^3c\right)\log\left(b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{fx^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) +
d/(3*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2) + x*
(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) - sq
rt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*sqrt
(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*
b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2)
```

3.122 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=163

$$\frac{x^3\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-7a^3f+5a^2be-3ab^2d+b^3c\right)}{2\sqrt{a}\,b^{9/2}}-\frac{x\left(-7a^3f+5a^2be-3ab^2d+b^3c\right)}{2ab^4}+\frac{x^3(be-}{3b}$$

Rubi [A] time = 0.23, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1804, 1585, 1261, 205}

$$\frac{x^3\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{x\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2ab^4}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(5a^2be-7a^3f-3ab^2d+b^3c\right)}{2\sqrt{a}\,b^{9/2}}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(2*a*b^4) + ((b*e - 2*a*f)*x
^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)
/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x\left(\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^2\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \left(c-\frac{a\left(3b^2d-5abe+7a^2f\right)}{b^3}-\frac{2a(be-2af)x^2}{b^2}-\frac{2afx^4}{b}+\frac{-ab^3c+3a^2}{b^3}\right.}{2ab}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.91

$$\frac{x\left(3a^2f-2abe+b^2d\right)}{b^4} - \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(7a^3f-5a^2be+3ab^2d-b^3c\right)}{2\sqrt{a}\,b^{9/2}} - \frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2b^4\left(a+bx^2\right)} + \frac{x^3(be-2af)}{3b^3} + \frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5)
/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) - ((
-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*Sqrt[a]*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2, x]
```

fricas [A] time = 1.17, size = 418, normalized size = 2.56

$$\left[\frac{12\,ab^4fx^7+4\left(5\,ab^4e-7\,a^2b^3f\right)x^5+20\left(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f\right)x^3+15\left(ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+\left(b^4c-3\,ab^3d+5\,a^2b^2e-7\,a^3bf\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-30\left(ab^4c-3\,a^2b^3d+5\,a^3b^2e-7\,a^4bf\right)x}{60\left(ab^6x^2+a^2b^5\right)},\ \frac{6\,ab^4fx^7+2\left(5\,ab^4e-7\,a^2b^3f\right)x^5+10\left(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f\right)x^3+15\left(ab^3c-3\,a^2b^2d+5\,a^3be-7\,a^4f+\left(b^4c-3\,ab^3d+5\,a^2b^2e-7\,a^3bf\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-15\left(ab^4c-3\,a^2b^3d+5\,a^3b^2e-7\,a^4bf\right)x}{30\left(ab^6x^2+a^2b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*f*x^7 + 4*(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 20*(3*a*b^4*d - 5
*a^2*b^3*e + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a
^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(-a*b)*log((b
*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d + 5*a^3
*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x^7 + 2*(5*a*
b^4*e - 7*a^2*b^3*f)*x^5 + 10*(3*a*b^4*d - 5*a^2*b^3*e + 7*a^3*b^2*f)*x^3 +
 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c - 3*a*b^3*d + 5*a
^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c -
3*a^2*b^3*d + 5*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.37, size = 152, normalized size = 0.93

$$\frac{\left(b^3c-3\,ab^2d-7\,a^3f+5\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)b^4}+\frac{3\,b^8fx^5-10\,ab^7fx^3+5\,b^8x^3e+15\,b^8dx+45\,a^2b^6fx-30\,ab^7xe}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*f*x^5 - 10*a*b^7*f*x^3 + 5*b^8*x^3*e + 15*b^8*d*x + 45*a^2*
b^6*f*x - 30*a*b^7*x*e)/b^10
```

maple [A] time = 0.01, size = 212, normalized size = 1.30

$$\frac{f\,x^{5}}{5b^{2}}-\frac{2af\,x^{3}}{3b^{3}}+\frac{e\,x^{3}}{3b^{2}}+\frac{a^{3}fx}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{7a^{3}f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{4}}-\frac{a^{2}ex}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5a^{2}e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{adx}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}-\frac{cx}{2\left(b\,x^{2}+a\right)b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{3a^{2}fx}{b^{4}}-\frac{2aex}{b^{3}}+\frac{dx}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*f*x^5/b^2-2/3/b^3*x^3*a*f+1/3/b^2*x^3*e+3/b^4*a^2*f*x-2/b^3*a*e*x+1/b^2
*d*x+1/2/b^4*x/(b*x^2+a)*a^3*f-1/2/b^3*x/(b*x^2+a)*a^2*e+1/2/b^2*x/(b*x^2+a
)*a*d-1/2/(b*x^2+a)/b*c*x-7/2/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3
*f+5/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*e-3/2/b^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*a*d+1/2/(a*b)^(1/2)/b*c*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 140, normalized size = 0.86

$$-\frac{\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x}{2\left(b^{5}x^{2}+ab^{4}\right)}+\frac{\left(b^{3}c-3\,ab^{2}d+5\,a^{2}be-7\,a^{3}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{3\,b^{2}fx^{5}+5\left(b^{2}e-2\,abf\right)x^{3}+15\left(b^{2}d-2\,abe+3\,a^{2}f\right)x}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^2 + a*b^4) + 1/2*(b^3*c -
 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1
/15*(3*b^2*f*x^5 + 5*(b^2*e - 2*a*b*f)*x^3 + 15*(b^2*d - 2*a*b*e + 3*a^2*f)
*x)/b^4
```

mupad [B] time = 1.00, size = 153, normalized size = 0.94

$$x^{3}\left(\frac{e}{3\,b^{2}}-\frac{2\,a\,f}{3\,b^{3}}\right)-x\left(\frac{a^{2}\,f}{b^{4}}-\frac{d}{b^{2}}+\frac{2\,a\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)}{b}\right)-\frac{x\left(-\frac{f\,a^{3}}{2}+\frac{e\,a^{2}\,b}{2}-\frac{d\,a\,b^{2}}{2}+\frac{c\,b^{3}}{2}\right)}{b^{5}\,x^{2}+a\,b^{4}}+\frac{f\,x^{5}}{5\,b^{2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-7\,f\,a^{3}+5\,e\,a^{2}\,b-3\,d\,a\,b^{2}+c\,b^{3}\right)}{2\,\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 -
(2*a*f)/b^3))/b) - (x*((b^3*c)/2 - (a^3*f)/2 - (a*b^2*d)/2 + (a^2*b*e)/2))/
(a*b^4 + b^5*x^2) + (f*x^5)/(5*b^2) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - 7
*a^3*f - 3*a*b^2*d + 5*a^2*b*e))/(2*a^(1/2)*b^(9/2))
```

sympy [A] time = 3.04, size = 221, normalized size = 1.36

$$x^{3}\left(-\frac{2af}{3b^{3}}+\frac{e}{3b^{2}}\right)+x\left(\frac{3a^{2}f}{b^{4}}-\frac{2ae}{b^{3}}+\frac{d}{b^{2}}\right)+\frac{x\left(a^{3}f-a^{2}be+ab^{2}d-b^{3}c\right)}{2ab^{4}+2b^{5}x^{2}}+\frac{\sqrt{-\frac{1}{ab^{9}}}\left(7a^{3}f-5a^{2}be+3ab^{2}d-b^{3}c\right)\log\left(-ab^{4}\sqrt{-\frac{1}{ab^{9}}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^{9}}}\left(7a^{3}f-5a^{2}be+3ab^{2}d-b^{3}c\right)\log\left(ab^{4}\sqrt{-\frac{1}{ab^{9}}}+x\right)}{4}+\frac{fx^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**
2) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) + s
qrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b**4*
sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b
**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=118

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 1153, 205}

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^2be+5a^3f+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-\frac{b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{2a(be-af)x^2}{b^2}-\frac{2afx^4}{b}}{a+bx^2}\,dx}{2a}$$

$$=\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2a(be-2af)}{b^3}-\frac{2afx^2}{b^2}+\frac{-b^3c-ab^2d+3a^2be-5a^3f}{b^3\left(a+bx^2\right)}\right)dx}{2a}$$

$$=\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\int \frac{1}{a+bx^2}}{2ab^3}$$

$$=\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}\left(\right.}{2a^{3/2}b^{7/2}}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 1.03

$$-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2ab^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2, x]
```

fricas [A] time = 1.04, size = 364, normalized size = 3.08

$$\left[\frac{4\,a^2b^3fx^5+4\left(3\,a^2b^3e-5\,a^3b^2f\right)x^3-3\left(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+\left(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(ab^4c-a^2b^3d+3\,a^3b^2e-5\,a^4bf\right)x}{12\left(a^2b^5x^2+a^3b^4\right)},\ \frac{2\,a^2b^3fx^5+2\left(3\,a^2b^3e-5\,a^3b^2f\right)x^3+3\left(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+\left(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(ab^4c-a^2b^3d+3\,a^3b^2e-5\,a^4bf\right)x}{6\left(a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*f*x^5 + 4*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 - 3*(a*b^3*c + a
^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f
)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*
c - a^2*b^3*d + 3*a^3*b^2*e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2
*a^2*b^3*f*x^5 + 2*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 + 3*(a*b^3*c + a^2*b^2*d
 - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*x^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d + 3*a^3*b^2*e - 5*a
^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.40, size = 126, normalized size = 1.07

$$\frac{\left(b^3c+ab^2d+5\,a^3f-3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^3}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\left(bx^2+a\right)ab^3}+\frac{b^4fx^3-6\,ab^3fx+3\,b^4xe}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a*b
^3) + 1/3*(b^4*f*x^3 - 6*a*b^3*f*x + 3*b^4*x*e)/b^6
```

maple [A] time = 0.01, size = 177, normalized size = 1.50

$$\frac{f\,x^3}{3b^2}-\frac{a^2fx}{2\left(b\,x^2+a\right)b^3}+\frac{5a^2f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{aex}{2\left(b\,x^2+a\right)b^2}-\frac{3ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{cx}{2\left(b\,x^2+a\right)a}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{dx}{2\left(b\,x^2+a\right)b}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{2afx}{b^3}+\frac{ex}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*f*x^3/b^2-2/b^3*a*f*x+1/b^2*e*x-1/2/b^3*a^2*x/(b*x^2+a)*f+1/2/b^2*a*x/(
b*x^2+a)*e-1/2/b*x/(b*x^2+a)*d+1/2/(b*x^2+a)/a*c*x+5/2/b^3*a^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*f-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*e+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2/(a*b)^(1/2)/a*c*arctan
(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 117, normalized size = 0.99

$$\frac{\left(b^3c - ab^2d + a^2be - a^3f\right)x}{2\left(ab^4x^2 + a^2b^3\right)} + \frac{bfx^3 + 3\left(be - 2\,af\right)x}{3\,b^3} + \frac{\left(b^3c + ab^2d - 3\,a^2be + 5\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(b*f*
x^3 + 3*(b*e - 2*a*f)*x)/b^3 + 1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

mupad [B] time = 0.10, size = 113, normalized size = 0.96

$$x\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right) + \frac{f\,x^3}{3\,b^2} + \frac{x\,\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{2\,a\,\left(b^4\,x^2 + a\,b^3\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(5\,f\,a^3 - 3\,e\,a^2\,b + d\,a\,b^2 + c\,b^3\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^3)/(3*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c + 5*a^
3*f + a*b^2*d - 3*a^2*b*e))/(2*a^(3/2)*b^(7/2))
```

sympy [A] time = 2.88, size = 201, normalized size = 1.70

$$x\left(-\frac{2af}{b^3} + \frac{e}{b^2}\right) + \frac{x\left(-a^3f + a^2be - ab^2d + b^3c\right)}{2a^2b^3 + 2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f - 3a^2be + ab^2d + b^3c\right)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}\left(5a^3f - 3a^2be + ab^2d + b^3c\right)\log\left(a^2b^3\sqrt{-\frac{1}{a^3b^7}} + x\right)}{4} + \frac{fx^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(2*a*
*2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*
b**2*d + b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a**3
*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(-1/(
a**3*b**7)) + x)/4 + f*x**3/(3*b**2)
```

3.124 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=112

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{c}{a^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1261, 205}

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-a^2be+3a^3f-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}-\frac{c}{a^2x}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2c+\left(\frac{bc}{a}-d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{2afx^4}{b}}{x^2\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2af}{b^2}-\frac{2c}{ax^2}+\frac{3b^3c-ab^2d-a^2be+3a^3f}{ab^2\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}$$

$$= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

Mathematica [A] time = 0.07, size = 115, normalized size = 1.03

$$-\frac{c}{a^2x}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^2b^2\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^
2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 1.02, size = 354, normalized size = 3.16

$$\left[\frac{4\,a^3b^2fx^4-4\,a^2b^3c-2\left(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf\right)x^2-\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b^2d-a^3be+3\,a^4f\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^3b^4x^3+a^4b^3x\right)},\ \frac{2\,a^3b^2fx^4-2\,a^2b^3c-\left(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf\right)x^2-\left(\left(3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf\right)x^3+\left(3\,ab^3c-a^2b^2d-a^3be+3\,a^4f\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^3b^4x^3+a^4b^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*f*x^4 - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*b^3*d + a^3*b^2*e
- 3*a^4*b*f)*x^2 - ((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*
b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*f*x^4 -
2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d + a^3*b^2*e - 3*a^4*b*f)*x^2 - ((3*b^4
*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d - a^3*b*
e + 3*a^4*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

giac [A] time = 0.36, size = 122, normalized size = 1.09

$$\frac{fx}{b^2}-\frac{\left(3\,b^3c-ab^2d+3\,a^3f-a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}-\frac{3\,b^3cx^2-ab^2dx^2-a^3fx^2+a^2bx^2e+2\,ab^2c}{2\left(bx^3+ax\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x
^2*e + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

maple [A] time = 0.01, size = 165, normalized size = 1.47

$$\frac{afx}{2\left(b\,x^2+a\right)b^2}-\frac{3af\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{dx}{2\left(b\,x^2+a\right)a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{bcx}{2\left(b\,x^2+a\right)a^2}-\frac{3bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}-\frac{ex}{2\left(b\,x^2+a\right)b}+\frac{e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{fx}{b^2}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] f*x/b^2+1/2*a/b^2*x/(b*x^2+a)*f-1/2/b*x/(b*x^2+a)*e+1/2/a*x/(b*x^2+a)*d-1/2
/(b*x^2+a)/a^2*b*c*x-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/2/
b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*d-3/2/(a*b)^(1/2)/a^2*b*c*arctan(1/(a*b)^(1/2)*b*x)-1/a^2*c/x
```

maxima [A] time = 2.99, size = 117, normalized size = 1.04

$$-\frac{2\,ab^2c + \left(3\,b^3c - ab^2d + a^2be - a^3f\right)x^2}{2\left(a^2b^3x^3 + a^3b^2x\right)} + \frac{fx}{b^2} - \frac{\left(3\,b^3c - ab^2d - a^2be + 3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b^2*c + (3*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^3*x^3 +
 a^3*b^2*x) + f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [B] time = 1.00, size = 112, normalized size = 1.00

$$\frac{f\,x}{b^2} - \frac{\frac{x^2\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^2} + \frac{b^2\,c}{a}}{b^3\,x^3 + a\,b^2\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3\,f\,a^3 - e\,a^2\,b - d\,a\,b^2 + 3\,c\,b^3\right)}{2\,a^{5/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^2*(3*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^2) + (b^2*c)/
a)/(b^3*x^3 + a*b^2*x) - (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c + 3*a^3*f - a*
b^2*d - a^2*b*e))/(2*a^(5/2)*b^(5/2))
```

sympy [A] time = 9.46, size = 197, normalized size = 1.76

$$\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f - a^2be - ab^2d + 3b^3c\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^3f - a^2be - ab^2d + 3b^3c\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}} + x\right)}{4} + \frac{-2ab^2c + x^2\left(a^3f - a^2be + ab^2d - 3b^3c\right)}{2a^3b^2x + 2a^2b^3x^3} + \frac{fx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3*
b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*
e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2*a*
b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2
*a**2*b**3*x**3) + f*x/b**2
```

3.125 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=121

$$\frac{2bc-ad}{a^3x}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}-\frac{c}{3a^2x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1261, 205}

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^2be+a^3f-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{b^2c}{a^2}+\frac{bd}{a}-e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)}\,dx}{2a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^4}-\frac{2(-2bc+ad)}{a^2x^2}+\frac{-5b^3c+3ab^2d-a^2be-a^3f}{a^2b\left(a+bx^2\right)}\right)dx}{2a}$$

$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\int \frac{1}{a+bx^2}}{2a^3b}$$

$$= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\tan^{-1}\left(\right.}{2a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.08, size = 125, normalized size = 1.03

$$\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}-\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^3b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2), x]
```

fricas [A] time = 1.02, size = 378, normalized size = 3.12

$$\left[-\frac{4\,a^{3}b^{2}c-6\left(5\,ab^{4}c-3\,a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{4}-4\left(5\,a^{2}b^{3}c-3\,a^{3}b^{2}d\right)x^{2}+3\left(\left(5\,b^{4}c-3\,ab^{3}d+a^{2}b^{2}e+a^{3}bf\right)x^{5}+\left(5\,ab^{3}c-3\,a^{2}b^{2}d+a^{3}be+a^{4}f\right)x^{3}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\sqrt{-ab}x-a}{bx^{2}+a}\right)}{12\left(a^{4}b^{3}x^{5}+a^{5}b^{2}x^{3}\right)},\ -\frac{2\,a^{3}b^{2}c-3\left(5\,ab^{4}c-3\,a^{2}b^{3}d+a^{3}b^{2}e-a^{4}bf\right)x^{4}-2\left(5\,a^{2}b^{3}c-3\,a^{3}b^{2}d\right)x^{2}-3\left(\left(5\,b^{4}c-3\,ab^{3}d+a^{2}b^{2}e+a^{3}bf\right)x^{5}+\left(5\,ab^{3}c-3\,a^{2}b^{2}d+a^{3}be+a^{4}f\right)x^{3}\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{6\left(a^{4}b^{3}x^{5}+a^{5}b^{2}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c - 6*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4
 - 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
+ a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3
), -1/6*(2*a^3*b^2*c - 3*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
4 - 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
 + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(a*b
)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^3)]
```

giac [A] time = 0.48, size = 123, normalized size = 1.02

$$\frac{\left(5\,b^{3}c-3\,ab^{2}d+a^{3}f+a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}b}+\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{2\left(bx^{2}+a\right)a^{3}b}+\frac{6\,bcx^{2}-3\,adx^{2}-ac}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^3
*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

maple [A] time = 0.02, size = 182, normalized size = 1.50

$$\frac{ex}{2\left(b\,x^{2}+a\right)a}+\frac{e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{bdx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{b^{2}cx}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{fx}{2\left(b\,x^{2}+a\right)b}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{d}{a^{2}x}+\frac{2bc}{a^{3}x}-\frac{c}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2+a)*f+1/2/a*x/(b*x^2+a)*e-1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^
2*x/(b*x^2+a)*c+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+1/2/a/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a^2/x^3-1
/a^2/x*d+2/a^3/x*b*c
```

maxima [A] time = 3.02, size = 130, normalized size = 1.07

$$\frac{3\left(5\,b^3c-3\,ab^2d+a^2be-a^3f\right)x^4-2\,a^2bc+2\left(5\,ab^2c-3\,a^2bd\right)x^2}{6\left(a^3b^2x^5+a^4bx^3\right)}+\frac{\left(5\,b^3c-3\,ab^2d+a^2be+a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 3*a*b^2*d + a^2*b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^2
*c - 3*a^2*b*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2*(5*b^3*c - 3*a*b^2*d +
 a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

mupad [B] time = 0.13, size = 119, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}+\frac{x^2\,(3\,a\,d-5\,b\,c)}{3\,a^2}-\frac{x^4\,\left(-f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^3\,b}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(5*b^3*c + a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^(7
/2)*b^(3/2)) - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) - (x^4*(5*b^3*c - a
^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^3*b))/(a*x^3 + b*x^5)
```

sympy [A] time = 25.99, size = 212, normalized size = 1.75

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\left(a^3f+a^2be-3ab^2d+5b^3c\right)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{-2a^2bc+x^4\left(-3a^3f+3a^2be-9ab^2d+15b^3c\right)+x^2\left(-6a^2bd+10ab^2c\right)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**4
*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e -
3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + (-2*a**2*b*
c + x**4*(-3*a**3*f + 3*a**2*b*e - 9*a*b**2*d + 15*b**3*c) + x**2*(-6*a**2*
b*d + 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

3.126 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^2} \, dx$

Optimal. Leaf size=152

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b\right.}{2a^4\left(a+bx^2\right)}$$

Rubi [A] time = 0.21, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^2be+a^3(-f)-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -c/(5*a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/(a
^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7*b
^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/
2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^6}-\frac{2(-2bc+ad)}{a^2x^4}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^2}+\frac{7b^3c-5ab^2d+}{a^3\left(a+b\right.}\right.}{2a}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-\right.}{}$$

$$= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-\right.}{}$$

Mathematica [A] time = 0.09, size = 151, normalized size = 0.99

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}+\frac{a^2(-e)+2abd-3b^2c}{a^4x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f-3a^2be+5ab^2d-7b^3c\right)}{2a^{9/2}\sqrt{b}}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2)) +
 ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
2*a^(9/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2), x]
```

fricas [A] time = 0.96, size = 438, normalized size = 2.88

$$\left[-\frac{30\left(7\,ab^{4}c-5\,a^{2}b^{3}d+3\,a^{3}b^{2}e-a^{4}bf\right)x^{6}+12\,a^{4}bc+20\left(7\,a^{2}b^{3}c-5\,a^{3}b^{2}d+3\,a^{4}be\right)x^{4}-4\left(7\,a^{3}b^{2}c-5\,a^{4}bd\right)x^{2}-15\left(\left(7\,b^{4}c-5\,ab^{3}d+3\,a^{2}b^{2}e-a^{3}bf\right)x^{7}+\left(7\,ab^{3}c-5\,a^{2}b^{2}d+3\,a^{3}be-a^{4}f\right)x^{5}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}x-a}{bx^{2}+a}\right)}{60\left(a^{5}b^{2}x^{7}+a^{6}bx^{5}\right)},\ -\frac{15\left(7\,ab^{4}c-5\,a^{2}b^{3}d+3\,a^{3}b^{2}e-a^{4}bf\right)x^{6}+6\,a^{4}bc+10\left(7\,a^{2}b^{3}c-5\,a^{3}b^{2}d+3\,a^{4}be\right)x^{4}-2\left(7\,a^{3}b^{2}c-5\,a^{4}bd\right)x^{2}+15\left(\left(7\,b^{4}c-5\,ab^{3}d+3\,a^{2}b^{2}e-a^{3}bf\right)x^{7}+\left(7\,ab^{3}c-5\,a^{2}b^{2}d+3\,a^{3}be-a^{4}f\right)x^{5}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{30\left(a^{5}b^{2}x^{7}+a^{6}bx^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 12*a^4*b
*c + 20*(7*a^2*b^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 4*(7*a^3*b^2*c - 5*a^
4*b*d)*x^2 - 15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b
^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^2*x^7 + a^6*b*x^5), -1/30*(15*(7*a*b^4*c
 - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b^3*c -
 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^
4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d + 3
*a^3*b*e - a^4*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^2*x^7 + a^6*
b*x^5)]
```

giac [A] time = 0.49, size = 151, normalized size = 0.99

$$-\frac{\left(7\,b^{3}c-5\,ab^{2}d-a^{3}f+3\,a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}-\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{2\left(bx^{2}+a\right)a^{4}}-\frac{45\,b^{2}cx^{4}-30\,abdx^{4}+15\,a^{2}x^{4}e-10\,abcx^{2}+5\,a^{2}dx^{2}+3\,a^{2}c}{15\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*x^4*e - 10*a*b*c*x^2 + 5*a^
2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

maple [A] time = 0.02, size = 219, normalized size = 1.44

$$\frac{fx}{2\left(b\,x^{2}+a\right)a}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{bex}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3be\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{b^{2}dx}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{b^{3}cx}{2\left(b\,x^{2}+a\right)a^{4}}-\frac{7b^{3}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{4}}-\frac{e}{a^{2}x}+\frac{2bd}{a^{3}x}-\frac{3b^{2}c}{a^{4}x}-\frac{d}{3a^{2}x^{3}}+\frac{2bc}{3a^{3}x^{3}}-\frac{c}{5a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a*x/(b*x^2+a)*f-1/2/a^2*x/(b*x^2+a)*b*e+1/2/a^3*x/(b*x^2+a)*b^2*d-1/2/a
^4*x/(b*x^2+a)*b^3*c+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-3/2/a^2/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b*e+5/2/a^3/(a*b)^(1/2)*arctan(1/(a*b
```

```
)^(1/2)*b*x)*b^2*d-7/2/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*
c/a^2/x^5-1/3/a^2/x^3*d+2/3/a^3/x^3*b*c-1/a^2/x*e+2/a^3/x*b*d-3/a^4/x*b^2*c
```

maxima [A] time = 2.99, size = 151, normalized size = 0.99

$$-\frac{15\left(7\,b^{3}c-5\,ab^{2}d+3\,a^{2}be-a^{3}f\right)x^{6}+10\left(7\,ab^{2}c-5\,a^{2}bd+3\,a^{3}e\right)x^{4}+6\,a^{3}c-2\left(7\,a^{2}bc-5\,a^{3}d\right)x^{2}}{30\left(a^{4}bx^{7}+a^{5}x^{5}\right)}-\frac{\left(7\,b^{3}c-5\,ab^{2}d+3\,a^{2}be-a^{3}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c - 5
*a^2*b*d + 3*a^3*e)*x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)/(a^4*b*x^7
 + a^5*x^5) - 1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 1.00, size = 145, normalized size = 0.95

$$-\frac{\frac{c}{5\,a}+\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^4}+\frac{x^2\,\left(5\,a\,d-7\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(3\,e\,a^2-5\,d\,a\,b+7\,c\,b^2\right)}{3\,a^3}}{b\,x^7+a\,x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (c/(5*a) + (x^6*(7*b^3*c - a^3*f - 5*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^2
*(5*a*d - 7*b*c))/(15*a^2) + (x^4*(7*b^2*c + 3*a^2*e - 5*a*b*d))/(3*a^3))/(
a*x^5 + b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(7*b^3*c - a^3*f - 5*a*b^2*d +
3*a^2*b*e))/(2*a^(9/2)*b^(1/2))
```

sympy [A] time = 32.02, size = 226, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^{9}b}}\left(a^{3}f-3a^{2}be+5ab^{2}d-7b^{3}c\right)\log\left(-a^{5}\sqrt{-\frac{1}{a^{9}b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{9}b}}\left(a^{3}f-3a^{2}be+5ab^{2}d-7b^{3}c\right)\log\left(a^{5}\sqrt{-\frac{1}{a^{9}b}}+x\right)}{4}+\frac{-6a^{3}c+x^{6}\left(15a^{3}f-45a^{2}be+75ab^{2}d-105b^{3}c\right)+x^{4}\left(-30a^{3}e+50a^{2}bd-70ab^{2}c\right)+x^{2}\left(-10a^{3}d+14a^{2}bc\right)}{30a^{5}x^{5}+30a^{4}bx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5*
sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**
2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(15*a
**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50*a**
2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*
a**4*b*x**7)
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^3f+5a^2be-7ab^2d+9b^3c\right)}{2a^{11/2}}+\frac{bx\left(a^3(-f)+a^2be-ab^2\right.}{2a^5\left(a+bx^2\right)}$$

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{a^5x}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(5a^2be-3a^3f-7ab^2d+9b^3c\right)}{2a^{11/2}}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -c/(7*a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/(3
*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c -
 a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c - 7*
a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{b\left(b^3\right.}{}}{x^8\left(a+bx^2\right)}}{2a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^8}-\frac{2(-2bc+ad)}{a^2x^6}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^4}-\frac{2\left(-4b^3c+3ab^2\right.}{a^4}\right.}{2a}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right.}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3c-\right.}{}$$

Mathematica [A] time = 0.11, size = 190, normalized size = 1.01

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}+\frac{a^2(-e)+2abd-3b^2c}{3a^4x^3}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-5a^2be+7ab^2d-9b^3c\right)}{2a^{11/2}}-\frac{bx\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*b^3
*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11
/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2), x]
```

fricas [A] time = 1.02, size = 488, normalized size = 2.58

$$\left[\frac{210\left(9 b^{4} c-7 a b^{3} d+5 a^{2} b^{2} e-3 a^{3} b f\right) x^{8}+140\left(9 a b^{3} c-7 a^{2} b^{2} d+5 a^{3} b e-3 a^{4} f\right) x^{6}-60 a^{4} c-28\left(9 a^{2} b^{2} c-7 a^{3} b d+5 a^{4} e\right) x^{4}+12\left(9 a^{3} b c-7 a^{4} d\right) x^{2}-105\left(\left(9 b^{4} c-7 a b^{3} d+5 a^{2} b^{2} e-3 a^{3} b f\right) x^{9}+\left(9 a b^{3} c-7 a^{2} b^{2} d+5 a^{3} b e-3 a^{4} f\right) x^{7}\right) \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2}-2 a x \sqrt{-\frac{b}{a}}-a}{b x^{2}+a}\right)}{420\left(a^{5} b x^{9}+a^{6} x^{7}\right)}, \frac{105\left(9 b^{4} c-7 a b^{3} d+5 a^{2} b^{2} e-3 a^{3} b f\right) x^{8}+70\left(9 a b^{3} c-7 a^{2} b^{2} d+5 a^{3} b e-3 a^{4} f\right) x^{6}-30 a^{4} c-14\left(9 a^{2} b^{2} c-7 a^{3} b d+5 a^{4} e\right) x^{4}+6\left(9 a^{3} b c-7 a^{4} d\right) x^{2}+105\left(\left(9 b^{4} c-7 a b^{3} d+5 a^{2} b^{2} e-3 a^{3} b f\right) x^{9}+\left(9 a b^{3} c-7 a^{2} b^{2} d+5 a^{3} b e-3 a^{4} f\right) x^{7}\right) \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right)}{210\left(a^{5} b x^{9}+a^{6} x^{7}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 140*(9*a*
b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2*c
 - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c - 7*a^4*d)*x^2 - 105*((9*b^4*c
- 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a
^3*b*e - 3*a^4*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*
e - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6
 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7
*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9
*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(
b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

giac [A] time = 0.43, size = 201, normalized size = 1.06

$$\frac{\left(9 b^{4} c-7 a b^{3} d-3 a^{3} b f+5 a^{2} b^{2} e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b} a^{5}}+\frac{b^{4} c x-a b^{3} d x-a^{3} b f x+a^{2} b^{2} x e}{2\left(b x^{2}+a\right) a^{5}}+\frac{420 b^{3} c x^{6}-315 a b^{2} d x^{6}-105 a^{3} f x^{6}+210 a^{2} b x^{6} e-105 a b^{2} c x^{4}+70 a^{2} b d x^{4}-35 a^{3} x^{4} e+42 a^{2} b c x^{2}-21 a^{3} d x^{2}-15 a^{3} c}{105 a^{5} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^
2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 - 105*a^3*f*x^6 + 210*
a^2*b*x^6*e - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*x^4*e + 42*a^2*b*c*
x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

maple [A] time = 0.02, size = 268, normalized size = 1.42

$$-\frac{b f x}{2\left(b x^{2}+a\right) a^{2}}-\frac{3 b f \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, a^{2}}+\frac{b^{2} e x}{2\left(b x^{2}+a\right) a^{3}}+\frac{5 b^{2} e \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, a^{3}}-\frac{b^{3} d x}{2\left(b x^{2}+a\right) a^{4}}-\frac{7 b^{3} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, a^{4}}+\frac{b^{4} c x}{2\left(b x^{2}+a\right) a^{5}}+\frac{9 b^{4} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\, a^{5}}-\frac{f}{a^{2} x}+\frac{2 b e}{a^{3} x}-\frac{3 b^{2} d}{a^{4} x}+\frac{4 b^{3} c}{a^{5} x}-\frac{e}{3 a^{2} x^{3}}+\frac{2 b d}{3 a^{3} x^{3}}-\frac{b^{2} c}{a^{4} x^{3}}-\frac{d}{5 a^{2} x^{5}}+\frac{2 b c}{5 a^{3} x^{5}}-\frac{c}{7 a^{2} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/2*b/a^2*x/(b*x^2+a)*f+1/2*b^2/a^3*x/(b*x^2+a)*e-1/2*b^3/a^4*x/(b*x^2+a)*
d+1/2*b^4/a^5*x/(b*x^2+a)*c-3/2*b/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*f+5/2*b^2/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-7/2*b^3/a^4/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)*d+9/2*b^4/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
```

```
)*b*x)*c-1/7*c/a^2/x^7-1/5/a^2/x^5*d+2/5/a^3/x^5*b*c-1/3/a^2/x^3*e+2/3/a^3/
x^3*b*d-1/a^4/x^3*b^2*c-1/a^2/x*f+2/a^3/x*b*e-3/a^4/x*b^2*d+4/a^5/x*b^3*c
```

maxima [A] time = 3.05, size = 194, normalized size = 1.03

$$\frac{105\left(9\,b^{4}c-7\,ab^{3}d+5\,a^{2}b^{2}e-3\,a^{3}bf\right)x^{8}+70\left(9\,ab^{3}c-7\,a^{2}b^{2}d+5\,a^{3}be-3\,a^{4}f\right)x^{6}-30\,a^{4}c-14\left(9\,a^{2}b^{2}c-7\,a^{3}bd+5\,a^{4}e\right)x^{4}+6\left(9\,a^{3}bc-7\,a^{4}d\right)x^{2}}{210\left(a^{5}bx^{9}+a^{6}x^{7}\right)}+\frac{\left(9\,b^{4}c-7\,ab^{3}d+5\,a^{2}b^{2}e-3\,a^{3}bf\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 70*(9*a*b^
3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c -
 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2)/(a^5*b*x^9 + a^6*x
^7) + 1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 0.99, size = 181, normalized size = 0.96

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^{11/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(7\,a\,d-9\,b\,c\right)}{35\,a^2}+\frac{x^4\left(5\,e\,a^2-7\,d\,a\,b+9\,c\,b^2\right)}{15\,a^3}-\frac{b\,x^8\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^5}}{b\,x^9+a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b
*e))/(2*a^(11/2)) - (c/(7*a) - (x^6*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*
b*e))/(3*a^4) + (x^2*(7*a*d - 9*b*c))/(35*a^2) + (x^4*(9*b^2*c + 5*a^2*e -
7*a*b*d))/(15*a^3) - (b*x^8*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b*e))/(2
*a^5))/(a*x^7 + b*x^9)
```

sympy [B] time = 99.02, size = 394, normalized size = 2.08

$$\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^{3}f-5a^{2}be+7ab^{2}d-9b^{3}c\right)\log\left(-\frac{a^{6}\sqrt{-\frac{b}{a^{11}}}\left(3a^{3}f-5a^{2}be+7ab^{2}d-9b^{3}c\right)}{3a^{3}bf-5a^{2}b^{2}e+7ab^{3}d-9b^{4}c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^{11}}}\left(3a^{3}f-5a^{2}be+7ab^{2}d-9b^{3}c\right)\log\left(\frac{a^{6}\sqrt{-\frac{b}{a^{11}}}\left(3a^{3}f-5a^{2}be+7ab^{2}d-9b^{3}c\right)}{3a^{3}bf-5a^{2}b^{2}e+7ab^{3}d-9b^{4}c}+x\right)}{4}+\frac{-30a^{4}c+x^{8}\left(-315a^{3}bf+525a^{2}b^{2}e-735ab^{3}d+945b^{4}c\right)+x^{6}\left(-210a^{4}f+350a^{3}be-490a^{2}b^{2}d+630ab^{3}c\right)+x^{4}\left(-70a^{4}e+98a^{3}bd-126a^{2}b^{2}c\right)+x^{2}\left(-42a^{4}d+54a^{3}bc\right)}{210a^{6}x^{7}+210a^{5}bx^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(-a**6*sq
rt(-b/a**11)*(3*a**3*f - 5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f -
5*a**2*b**2*e + 7*a*b**3*d - 9*b**4*c) + x)/4 - sqrt(-b/a**11)*(3*a**3*f -
5*a**2*b*e + 7*a*b**2*d - 9*b**3*c)*log(a**6*sqrt(-b/a**11)*(3*a**3*f - 5*a
**2*b*e + 7*a*b**2*d - 9*b**3*c)/(3*a**3*b*f - 5*a**2*b**2*e + 7*a*b**3*d -
 9*b**4*c) + x)/4 + (-30*a**4*c + x**8*(-315*a**3*b*f + 525*a**2*b**2*e - 7
35*a*b**3*d + 945*b**4*c) + x**6*(-210*a**4*f + 350*a**3*b*e - 490*a**2*b**
2*d + 630*a*b**3*c) + x**4*(-70*a**4*e + 98*a**3*b*d - 126*a**2*b**2*c) + x
**2*(-42*a**4*d + 54*a**3*b*c))/(210*a**6*x**7 + 210*a**5*b*x**9)
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2} \, dx$

Optimal. Leaf size=230

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x\left(a^3(-f)+a^2be-a\right.}{2a^6\left(a+bx^2\right.}$$

Rubi [A] time = 0.38, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{2a^2be+a^3(-f)-3ab^2d+4b^3c}{3a^5x^3}-\frac{b\left(3a^2be-2a^3f-4ab^2d+5b^3c\right)}{a^6x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(7a^2be-5a^3f-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}+\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -c/(9*a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/(5
*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5*b
^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d +
a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d +
7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-}{x^{10}\left(a+bx^2\right.}}{2a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^6\left(a+bx^2\right)} - \frac{\int\left(-\frac{2c}{ax^{10}}-\frac{2(-2bc+ad)}{a^2x^8}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-\frac{2\left(-4b^3c+\right.}{}\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b\left(5b^3c\right.}{}$$

Mathematica [A] time = 0.12, size = 230, normalized size = 1.00

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}+\frac{a^2(-e)+2abd-3b^2c}{5a^4x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(5a^3f-7a^2be+9ab^2d-11b^3c\right)}{2a^{13/2}}+\frac{b^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{2a^6\left(a+bx^2\right)}+\frac{b\left(2a^3f-3a^2be+4ab^2d-5b^3c\right)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(
-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*
b^2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2), x]
```

fricas [A] time = 0.87, size = 582, normalized size = 2.53

$$\left[-\frac{630\,\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{10}+420\,\left(11\,ab^{4}c-9\,a^{2}b^{3}d+7\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{8}-84\,\left(11\,a^{2}b^{3}c-9\,a^{3}b^{2}d+7\,a^{4}be-5\,a^{5}f\right)x^{6}+140\,a^{5}c+36\,\left(11\,a^{3}b^{2}c-9\,a^{4}bd+7\,a^{5}e\right)x^{4}-20\,\left(11\,a^{4}bc-9\,a^{5}d\right)x^{2}+315\,\left(\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{11}+\left(11\,ab^{4}c-9\,a^{2}b^{3}d+7\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{9}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{1260\,\left(a^{6}bx^{11}+a^{7}x^{9}\right)},\ -\frac{315\,\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{10}+210\,\left(11\,ab^{4}c-9\,a^{2}b^{3}d+7\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{8}-42\,\left(11\,a^{2}b^{3}c-9\,a^{3}b^{2}d+7\,a^{4}be-5\,a^{5}f\right)x^{6}+70\,a^{5}c+18\,\left(11\,a^{3}b^{2}c-9\,a^{4}bd+7\,a^{5}e\right)x^{4}-10\,\left(11\,a^{4}bc-9\,a^{5}d\right)x^{2}+315\,\left(\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{11}+\left(11\,ab^{4}c-9\,a^{2}b^{3}d+7\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{9}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{630\,\left(a^{6}bx^{11}+a^{7}x^{9}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 420
*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^3*
c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 140*a^5*c + 36*(11*a^3*b^2*c -
 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c
- 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d +
 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*b^4*d +
 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^
2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f
)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4
*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*
f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

giac [A] time = 0.38, size = 252, normalized size = 1.10

$$-\frac{\left(11\,b^{5}c-9\,ab^{4}d-5\,a^{3}b^{2}f+7\,a^{2}b^{3}e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{6}}-\frac{b^{5}cx-ab^{4}dx-a^{3}b^{2}fx+a^{2}b^{3}xe}{2\,\left(bx^{2}+a\right)a^{6}}-\frac{1575\,b^{4}cx^{8}-1260\,ab^{3}dx^{8}-630\,a^{3}bfx^{8}+945\,a^{2}b^{2}x^{8}e-420\,ab^{3}cx^{6}+315\,a^{2}b^{2}dx^{6}+105\,a^{4}fx^{6}-210\,a^{3}bx^{6}e+189\,a^{2}b^{2}cx^{4}-126\,a^{3}bdx^{4}+63\,a^{4}x^{4}e-90\,a^{3}bcx^{2}+45\,a^{4}dx^{2}+35\,a^{4}c}{315\,a^{6}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x - a^3*b^2*f*x + a^2*b^3*x*e)/
((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 - 630*a^3*b*f*
x^8 + 945*a^2*b^2*x^8*e - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 + 105*a^4*f*x
^6 - 210*a^3*b*x^6*e + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*x^4*e -
 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

maple [A] time = 0.02, size = 318, normalized size = 1.38

$$\frac{b^{2}fx}{2\left(bx^{2}+a\right)a^{3}}+\frac{5b^{2}f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{3}}-\frac{b^{3}ex}{2\left(bx^{2}+a\right)a^{4}}-\frac{7b^{3}e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{4}}+\frac{b^{4}dx}{2\left(bx^{2}+a\right)a^{5}}+\frac{9b^{4}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{5}}-\frac{b^{5}cx}{2\left(bx^{2}+a\right)a^{6}}-\frac{11b^{5}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{6}}+\frac{2bf}{a^{3}x}-\frac{3b^{2}e}{a^{4}x}+\frac{4b^{3}d}{a^{5}x}-\frac{5b^{4}c}{a^{6}x}-\frac{f}{3a^{2}x^{3}}+\frac{2be}{3a^{3}x^{3}}-\frac{b^{2}d}{a^{4}x^{3}}+\frac{4b^{3}c}{3a^{5}x^{3}}-\frac{e}{5a^{2}x^{5}}+\frac{2bd}{5a^{3}x^{5}}-\frac{3b^{2}c}{5a^{4}x^{5}}-\frac{d}{7a^{2}x^{7}}+\frac{2bc}{7a^{3}x^{7}}-\frac{c}{9a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a^3*b^2*x/(b*x^2+a)*f-1/2/a^4*b^3*x/(b*x^2+a)*e+1/2/a^5*b^4*x/(b*x^2+a)
*d-1/2/a^6*b^5*x/(b*x^2+a)*c+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*f-7/2/a^4*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+9/2/a^5*b^4/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-11/2/a^6*b^5/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*c-1/9*c/a^2/x^9-1/7/a^2/x^7*d+2/7/a^3/x^7*b*c-1/5/a^2/x^5*e+2/5/
a^3/x^5*b*d-3/5/a^4/x^5*b^2*c-1/3/a^2/x^3*f+2/3/a^3/x^3*b*e-1/a^4/x^3*b^2*d
+4/3/a^5/x^3*b^3*c+2*b/a^3/x*f-3*b^2/a^4/x*e+4*b^3/a^5/x*d-5*b^4/a^6/x*c
```

maxima [A] time = 2.99, size = 238, normalized size = 1.03

$$-\frac{315\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)x^{10}+210\left(11\,ab^{4}c-9\,a^{2}b^{3}d+7\,a^{3}b^{2}e-5\,a^{4}bf\right)x^{8}-42\left(11\,a^{2}b^{3}c-9\,a^{3}b^{2}d+7\,a^{4}be-5\,a^{5}f\right)x^{6}+70\,a^{5}c+18\left(11\,a^{3}b^{2}c-9\,a^{4}bd+7\,a^{5}e\right)x^{4}-10\left(11\,a^{4}bc-9\,a^{5}d\right)x^{2}}{630\left(a^{6}bx^{11}+a^{7}x^{9}\right)}-\frac{\left(11\,b^{5}c-9\,ab^{4}d+7\,a^{2}b^{3}e-5\,a^{3}b^{2}f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/630*(315*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(
11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c
- 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*
a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2)/(a^6*b*x^11 + a^7*x
^9) - 1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 1.01, size = 219, normalized size = 0.95

$$-\frac{\frac{c}{9\,a}-\frac{x^{6}\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{15\,a^{4}}+\frac{x^{2}\left(9\,a\,d-11\,b\,c\right)}{63\,a^{2}}+\frac{x^{4}\left(7\,e\,a^{2}-9\,d\,a\,b+11\,c\,b^{2}\right)}{35\,a^{3}}+\frac{b\,x^{8}\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{3\,a^{5}}+\frac{b^{2}\,x^{10}\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{2\,a^{6}}}{b\,x^{11}+a\,x^{9}}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{2\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x)
```

```
[Out] - (c/(9*a) - (x^6*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(15*a^4) +
(x^2*(9*a*d - 11*b*c))/(63*a^2) + (x^4*(11*b^2*c + 7*a^2*e - 9*a*b*d))/(35*
a^3) + (b*x^8*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(3*a^5) + (b^2*
x^10*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(2*a^6))/(a*x^9 + b*x^11
) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*
a^2*b*e))/(2*a^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.129 $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=287

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{a^2x\left(-17a^3f+13a^2be-9ab^2d+5b^3c\right)}{8b^7\left(a+bx^2\right)}-\frac{ax\left(-63a^3f+43a^2be-27ab^2d+15b^3c\right)}{4b^7}+\frac{x^3\left(-2\right.}{}$$

Rubi [A] time = 0.49, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x^5\left(17a^2be-29a^3f-9ab^2d+5b^3c\right)}{20ab^5}+\frac{x^3\left(15a^2be-23a^3f-9ab^2d+5b^3c\right)}{6b^6}-\frac{a^2x\left(13a^2be-17a^3f-9ab^2d+5b^3c\right)}{8b^7\left(a+bx^2\right)}-\frac{ax\left(43a^2be-63a^3f-27ab^2d+15b^3c\right)}{4b^7}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^2be-143a^3f-63ab^2d+35b^3c\right)}{8b^{15/2}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/(4*b^7) + ((5*b^3*c
- 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^8 \left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^7\left(\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^8\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int \frac{a^3\left(5b^3c-9ab^2d\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x}{8b^7\left(a+bx^2\right)} + \frac{\int \left(-2a^2\left(15b^3\cdots\right.\right.}{\cdots}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3\cdots}{6b^6}$$

$$= -\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7} + \frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3\cdots}{6b^6}$$

<u>Mathematica</u> [A] time = 0.19, size = 272, normalized size = 0.95

$$\frac{x^5\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{a^2x\left(25a^3f-21a^2be+17ab^2d-13b^3c\right)}{8b^7\left(a+bx^2\right)}+\frac{a^3x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4b^7\left(a+bx^2\right)^2}+\frac{ax\left(15a^3f-10a^2be+6ab^2d-3b^3c\right)}{b^7}+\frac{x^3\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{3b^6}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{8b^{15/2}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5
)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a
*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*
c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(15/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(c + dx^2 + ex^4 + fx^6\right)}{\left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3, x]
```

fricas [A] time = 1.08, size = 762, normalized size = 2.66

$$\left[\frac{560\,b^{6}fx^{13} + 80\,(9\,b^{6}e - 13\,ab^{5}f)x^{11} + 16\,(63\,b^{6}d - 99\,ab^{5}e + 143\,a^{2}b^{4}f)x^{9} + 48\,(35\,b^{6}c - 63\,ab^{5}d + 99\,a^{2}b^{4}e - 143\,a^{3}b^{3}f)x^{7} - 336\,(35\,ab^{5}c - 63\,a^{2}b^{4}d + 99\,a^{3}b^{3}e - 143\,a^{4}b^{2}f)x^{5} - 1050\,(35\,a^{2}b^{4}c - 63\,a^{3}b^{3}d + 99\,a^{4}b^{2}e - 143\,a^{5}bf)x^{3} - 315\,(35\,a^{3}b^{3}c - 63\,a^{4}b^{2}d + 99\,a^{5}be - 143\,a^{6}f + (35\,ab^{5}c - 63\,a^{2}b^{4}d + 99\,a^{3}b^{3}e - 143\,a^{4}b^{2}f)x^{4} + 2\,(35\,a^{2}b^{4}c - 63\,a^{3}b^{3}d + 99\,a^{4}b^{2}e - 143\,a^{5}bf)x^{2})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2} - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right) - 630\,(35\,a^{3}b^{3}c - 63\,a^{4}b^{2}d + 99\,a^{5}be - 143\,a^{6}f)x}{5040\,(b^{9}x^{4} + 2\,ab^{8}x^{2} + a^{2}b^{7})}, \frac{280\,b^{6}fx^{13} + 40\,(9\,b^{6}e - 13\,ab^{5}f)x^{11} + 8\,(63\,b^{6}d - 99\,ab^{5}e + 143\,a^{2}b^{4}f)x^{9} + 24\,(35\,b^{6}c - 63\,ab^{5}d + 99\,a^{2}b^{4}e - 143\,a^{3}b^{3}f)x^{7} - 168\,(35\,ab^{5}c - 63\,a^{2}b^{4}d + 99\,a^{3}b^{3}e - 143\,a^{4}b^{2}f)x^{5} - 525\,(35\,a^{2}b^{4}c - 63\,a^{3}b^{3}d + 99\,a^{4}b^{2}e - 143\,a^{5}bf)x^{3} + 315\,(35\,a^{3}b^{3}c - 63\,a^{4}b^{2}d + 99\,a^{5}be - 143\,a^{6}f + (35\,ab^{5}c - 63\,a^{2}b^{4}d + 99\,a^{3}b^{3}e - 143\,a^{4}b^{2}f)x^{4} + 2\,(35\,a^{2}b^{4}c - 63\,a^{3}b^{3}d + 99\,a^{4}b^{2}e - 143\,a^{5}bf)x^{2})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 315\,(35\,a^{3}b^{3}c - 63\,a^{4}b^{2}d + 99\,a^{5}be - 143\,a^{6}f)x}{2520\,(b^{9}x^{4} + 2\,ab^{8}x^{2} + a^{2}b^{7})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63*b^6*d - 9
9*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e -
 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a
^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*
b*f)*x^3 - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*
a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*
c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 -
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^2*d + 99
*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520*(280*b^6
*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11 + 8*(63*b^6*d - 99*a*b^5*e + 143*a
^2*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x
^7 - 168*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 5
25*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 + 315*(35
*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c - 63*a^2*b
^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d +
 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(
35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8
*x^2 + a^2*b^7)]
```

giac [A] time = 0.40, size = 301, normalized size = 1.05

$$\frac{\left(35\,a^{2}b^{3}c - 63\,a^{3}b^{2}d - 143\,a^{5}f + 99\,a^{4}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{7}} - \frac{13\,a^{2}b^{4}cx^{3} - 17\,a^{3}b^{3}dx^{3} - 25\,a^{5}bfx^{3} + 21\,a^{4}b^{2}x^{3}e + 11\,a^{3}b^{3}cx - 15\,a^{4}b^{2}dx - 23\,a^{6}fx + 19\,a^{5}bxe}{8\,\left(bx^{2} + a\right)^{2}b^{7}} + \frac{35\,b^{24}fx^{9} - 135\,ab^{23}fx^{7} + 45\,b^{24}x^{7}e + 63\,b^{24}dx^{5} + 378\,a^{2}b^{22}fx^{5} - 189\,ab^{23}x^{5}e + 105\,b^{24}cx^{3} - 315\,ab^{23}dx^{3} - 1050\,a^{3}b^{21}fx^{3} + 630\,a^{2}b^{22}x^{3}e - 945\,ab^{23}cx + 1890\,a^{2}b^{22}dx + 4725\,a^{4}b^{20}fx - 3150\,a^{3}b^{21}xe}{315\,b^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 - 25*a^5*b
*f*x^3 + 21*a^4*b^2*x^3*e + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x - 23*a^6*f*x +
19*a^5*b*x*e)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 - 135*a*b^23*f*x^7
 + 45*b^24*x^7*e + 63*b^24*d*x^5 + 378*a^2*b^22*f*x^5 - 189*a*b^23*x^5*e +
105*b^24*c*x^3 - 315*a*b^23*d*x^3 - 1050*a^3*b^21*f*x^3 + 630*a^2*b^22*x^3*
e - 945*a*b^23*c*x + 1890*a^2*b^22*d*x + 4725*a^4*b^20*f*x - 3150*a^3*b^21*
x*e)/b^27
```

maple [A] time = 0.02, size = 394, normalized size = 1.37

$$\frac{f x^9}{9b^3} - \frac{3af x^7}{7b^4} + \frac{e x^7}{7b^3} + \frac{25a^5 f x^3}{8\left(b x^2+a\right)^2 b^6} - \frac{21a^4 e x^3}{8\left(b x^2+a\right)^2 b^5} + \frac{17a^3 d x^3}{8\left(b x^2+a\right)^2 b^4} - \frac{13a^2 c x^3}{8\left(b x^2+a\right)^2 b^3} + \frac{6a^2 f x^5}{5b^5} - \frac{3ae x^5}{5b^4} + \frac{d x^5}{5b^3} + \frac{23a^6 f x}{8\left(b x^2+a\right)^2 b^7} - \frac{19a^5 e x}{8\left(b x^2+a\right)^2 b^6} + \frac{15a^4 d x}{8\left(b x^2+a\right)^2 b^5} - \frac{11a^3 c x}{8\left(b x^2+a\right)^2 b^4} - \frac{10a^3 f x^3}{3b^6} + \frac{2a^2 e x^3}{b^5} - \frac{ad x^3}{b^4} + \frac{c x^3}{3b^3} - \frac{143a^5 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^7} + \frac{99a^4 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^6} - \frac{63a^3 d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^5} + \frac{35a^2 c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^4} + \frac{15a^4 f x}{b^7} - \frac{10a^3 e x}{b^6} + \frac{6a^2 d x}{b^5} - \frac{3acx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3/b^3*x^3*c-3/7/b^4*x^7*a*f+6/5/b^5*x^5*a^2*f-3/5/b^4*x^5*a*e-10/3/b^6*x^
3*a^3*f+2/b^5*x^3*a^2*e-1/b^4*x^3*a*d+15/b^7*a^4*f*x-10/b^6*a^3*e*x+6/b^5*a
^2*d*x-3/b^4*a*c*x+1/9*f*x^9/b^3+25/8*a^5/b^6/(b*x^2+a)^2*x^3*f+23/8*a^6/b^
7/(b*x^2+a)^2*f*x-19/8*a^5/b^6/(b*x^2+a)^2*e*x+15/8*a^4/b^5/(b*x^2+a)^2*d*x
-11/8*a^3/b^4/(b*x^2+a)^2*c*x-143/8*a^5/b^7/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*f+99/8*a^4/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-63/8*a^3/b^5/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+35/8*a^2/b^4/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*c+1/7/b^3*x^7*e+1/5/b^3*x^5*d-21/8*a^4/b^5/(b*x^2+a)^2*x^3*
e+17/8*a^3/b^4/(b*x^2+a)^2*x^3*d-13/8*a^2/b^3/(b*x^2+a)^2*x^3*c
```

maxima [A] time = 3.10, size = 281, normalized size = 0.98

$$-\frac{\left(13\,a^2b^4c - 17\,a^3b^3d + 21\,a^4b^2e - 25\,a^5bf\right)x^3 + \left(11\,a^3b^3c - 15\,a^4b^2d + 19\,a^5be - 23\,a^6f\right)x}{8\left(b^9x^4 + 2\,ab^8x^2 + a^2b^7\right)} + \frac{\left(35\,a^2b^3c - 63\,a^3b^2d + 99\,a^4be - 143\,a^5f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^7} + \frac{35\,b^4fx^9 + 45\left(b^4e - 3\,ab^3f\right)x^7 + 63\left(b^4d - 3\,ab^3e + 6\,a^2b^2f\right)x^5 + 105\left(b^4c - 3\,ab^3d + 6\,a^2b^2e - 10\,a^3bf\right)x^3 - 315\left(3\,ab^3c - 6\,a^2b^2d + 10\,a^3be - 15\,a^4f\right)x}{315\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*a^2*b^4*c - 17*a^3*b^3*d + 21*a^4*b^2*e - 25*a^5*b*f)*x^3 + (11*a
^3*b^3*c - 15*a^4*b^2*d + 19*a^5*b*e - 23*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2
+ a^2*b^7) + 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 3*a*
b^3*f)*x^7 + 63*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^5 + 105*(b^4*c - 3*a*b^
3*d + 6*a^2*b^2*e - 10*a^3*b*f)*x^3 - 315*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3
*b*e - 15*a^4*f)*x)/b^7
```

mupad [B] time = 1.00, size = 506, normalized size = 1.76

$$x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)+\frac{x\left(\frac{23\,f\,a^6}{8}-\frac{19\,e\,a^5\,b}{8}+\frac{15\,d\,a^4\,b^2}{8}-\frac{11\,c\,a^3\,b^3}{8}\right)-x^3\left(-\frac{25\,f\,a^5\,b}{8}+\frac{21\,e\,a^4\,b^2}{8}-\frac{17\,d\,a^3\,b^3}{8}+\frac{13\,c\,a^2\,b^4}{8}\right)}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}-x\left(\frac{3\,a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{3\,a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right)-x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^9}{9\,b^3}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{143\,f\,a^5-99\,e\,a^4\,b+63\,d\,a^3\,b^2-35\,c\,a^2\,b^3}\right)\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{8\,b^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*
a*f)/b^4))/b))/b) + (x*((23*a^6*f)/8 - (11*a^3*b^3*c)/8 + (15*a^4*b^2*d)/8
- (19*a^5*b*e)/8) - x^3*((13*a^2*b^4*c)/8 - (17*a^3*b^3*d)/8 + (21*a^4*b^2*
e)/8 - (25*a^5*b*f)/8))/(a^2*b^7 + b^9*x^4 + 2*a*b^8*x^2) - x*((3*a*(c/b^3
- (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d
/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3*a^2*f)/b^5 - d/b^
3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/b^3)
- x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) +
 (f*x^9)/(9*b^3) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(35*b^3*c - 143*a^3*f -
 63*a*b^2*d + 99*a^2*b*e))/(143*a^5*f - 35*a^2*b^3*c + 63*a^3*b^2*d - 99*a^
4*b*e))*(35*b^3*c - 143*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(8*b^(15/2))
```

sympy [A] time = 16.43, size = 503, normalized size = 1.75

$$x^7\left(-\frac{3af}{7b^4}+\frac{e}{7b^3}\right)+x^5\left(\frac{6a^2f}{5b^5}-\frac{3ae}{5b^4}+\frac{d}{5b^3}\right)+x^3\left(-\frac{10a^3f}{3b^6}+\frac{2a^2e}{b^5}-\frac{ad}{b^4}+\frac{c}{3b^3}\right)+x\left(\frac{15a^4f}{b^7}-\frac{10a^3e}{b^6}+\frac{6a^2d}{b^5}-\frac{3ac}{b^4}\right)+\frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16}-\frac{\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)\log\left(\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\left(143a^3f-99a^2be+63ab^2d-35b^3c\right)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16}+\frac{x^3\left(25a^5bf-21a^4b^2e+17a^3b^3d-13a^2b^4c\right)+x\left(23a^6f-19a^5be+15a^4b^2d-11a^3b^3c\right)}{8a^2b^7+16ab^8x^2+8b^9x^4}+\frac{fx^9}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**7*(-3*a*f/(7*b**4) + e/(7*b**3)) + x**5*(6*a**2*f/(5*b**5) - 3*a*e/(5*b*
*4) + d/(5*b**3)) + x**3*(-10*a**3*f/(3*b**6) + 2*a**2*e/b**5 - a*d/b**4 +
c/(3*b**3)) + x*(15*a**4*f/b**7 - 10*a**3*e/b**6 + 6*a**2*d/b**5 - 3*a*c/b*
*4) + sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*
b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 -
 sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log
(b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3
*(25*a**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a
**6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b**7 + 16*a
*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3)
```

3.130 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=247

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-99a^3f+63a^2be-35ab^2d+15b^3c\right)}{8b^{13/2}}+\frac{ax\left(-15a^3f+11a^2be-7ab^2d+3\right.}{8b^6(a+bx^2)}$$

Rubi [A] time = 0.41, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x^3\left(15a^2be-27a^3f-7ab^2d+3b^3c\right)}{12ab^5}+\frac{ax\left(11a^2be-15a^3f-7ab^2d+3b^3c\right)}{8b^6(a+bx^2)}+\frac{x\left(13a^2be-21a^3f-7ab^2d+3b^3c\right)}{2b^6}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(63a^2be-99a^3f-35ab^2d+15b^3c\right)}{8b^{13/2}}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6 \left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^5\left(\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^6\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \frac{\int \frac{-a^2\left(3b^3c-7ab^2d\right.}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a\left(3b^3c-7ab^2d+11a^2be-15a^3f\right)x}{8b^6\left(a+bx^2\right)} + \frac{\int \left(4a\left(3b^3c-7\right.\right.}{}$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} + \frac{(b}{}$$

$$= \frac{\left(3b^3c-7ab^2d+13a^2be-21a^3f\right)x}{2b^6} - \frac{\left(3b^3c-7ab^2d+15a^2be-27a^3f\right)x^3}{12ab^5} + \frac{(b}{}$$

Mathematica [A] time = 0.15, size = 232, normalized size = 0.94

$$\frac{x^3\left(6a^2f-3abe+b^2d\right)}{3b^5}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^3f-63a^2be+35ab^2d-15b^3c\right)}{8b^{13/2}}+\frac{ax\left(-21a^3f+17a^2be-13ab^2d+9b^3c\right)}{8b^6\left(a+bx^2\right)}+\frac{a^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4b^6\left(a+bx^2\right)^2}+\frac{x\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{b^6}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^2*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*b^3
*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*
(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3, x]
```

fricas [A] time = 1.14, size = 668, normalized size = 2.70

$$\left[\frac{240\,b^5fx^{11}+48\left(7b^5e-11\,ab^4f\right)x^9+16\left(35\,b^5d-63\,ab^4e+99\,a^2b^3f\right)x^7+112\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-99\,a^3b^2f\right)x^5+350\left(15\,ab^4c-35\,a^2b^3d+63\,a^3b^2e-99\,a^4bf\right)x^3-105\left(15\,a^2b^3c-35\,a^3b^2d+63\,a^4be-99\,a^5f+\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-99\,a^3b^2f\right)x^4+2\left(15\,ab^4c-35\,a^2b^3d+63\,a^3b^2e-99\,a^4bf\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\left(15\,a^2b^3c-35\,a^3b^2d+63\,a^4be-99\,a^5f\right)x}{1680\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)},\ \frac{120\,b^5fx^{11}+24\left(7b^5e-11\,ab^4f\right)x^9+8\left(35\,b^5d-63\,ab^4e+99\,a^2b^3f\right)x^7+56\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-99\,a^3b^2f\right)x^5+175\left(15\,ab^4c-35\,a^2b^3d+63\,a^3b^2e-99\,a^4bf\right)x^3-105\left(15\,a^2b^3c-35\,a^3b^2d+63\,a^4be-99\,a^5f+\left(15\,b^5c-35\,ab^4d+63\,a^2b^3e-99\,a^3b^2f\right)x^4+2\left(15\,ab^4c-35\,a^2b^3d+63\,a^3b^2e-99\,a^4bf\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\left(15\,a^2b^3c-35\,a^3b^2d+63\,a^4be-99\,a^5f\right)x}{840\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*b^5*d - 63
*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e -
99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*
b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b
^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a
^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e
- 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*b^5*f*x^11 + 2
4*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*b^4*e + 99*a^2*b^3*f)*x^7
 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 + 175*(15*a
*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c
- 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^2*b^
3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*
a^4*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(15*a^2*b^3*c - 35*a^
3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)]
```

giac [A] time = 0.47, size = 250, normalized size = 1.01

$$-\frac{\left(15\,ab^3c-35\,a^2b^2d-99\,a^4f+63\,a^3be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{9\,ab^4cx^3-13\,a^2b^3dx^3-21\,a^4bfx^3+17\,a^3b^2x^3e+7\,a^2b^3cx-11\,a^3b^2dx-19\,a^5fx+15\,a^4bxe}{8\left(bx^2+a\right)^2b^6}+\frac{15\,b^{18}fx^7-63\,ab^{17}fx^5+21\,b^{18}x^5e+35\,b^{18}dx^3+210\,a^2b^{16}fx^3-105\,ab^{17}x^3e+105\,b^{18}cx-315\,ab^{17}dx-1050\,a^3b^{15}fx+630\,a^2b^{16}xe}{105\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 - 21*a^4*b*f*x^
3 + 17*a^3*b^2*x^3*e + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x - 19*a^5*f*x + 15*a^4
```

```
*b*x*e)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 - 63*a*b^17*f*x^5 + 21*b
^18*x^5*e + 35*b^18*d*x^3 + 210*a^2*b^16*f*x^3 - 105*a*b^17*x^3*e + 105*b^1
8*c*x - 315*a*b^17*d*x - 1050*a^3*b^15*f*x + 630*a^2*b^16*x*e)/b^21
```

maple [A] time = 0.02, size = 343, normalized size = 1.39

$$\frac{f x^7}{7b^3}-\frac{21a^4 f x^3}{8\left(b x^2+a\right)^2 b^5}+\frac{17a^3 e x^3}{8\left(b x^2+a\right)^2 b^4}-\frac{13a^2 d x^3}{8\left(b x^2+a\right)^2 b^3}+\frac{9ac x^3}{8\left(b x^2+a\right)^2 b^2}-\frac{3af x^5}{5b^4}+\frac{e x^5}{5b^3}-\frac{19a^5 f x}{8\left(b x^2+a\right)^2 b^6}+\frac{15a^4 ex}{8\left(b x^2+a\right)^2 b^5}-\frac{11a^3 dx}{8\left(b x^2+a\right)^2 b^4}+\frac{7a^2 cx}{8\left(b x^2+a\right)^2 b^3}+\frac{2a^2 f x^3}{b^5}-\frac{ae x^3}{b^4}+\frac{d x^3}{3b^3}+\frac{99a^4 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^6}-\frac{63a^3 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^5}+\frac{35a^2 d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^4}-\frac{15ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^3}-\frac{10a^3 f x}{b^6}+\frac{6a^2 ex}{b^5}-\frac{3adx}{b^4}+\frac{cx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*f*x^7/b^3-3/5/b^4*x^5*a*f+1/5/b^3*x^5*e+2/b^5*x^3*a^2*f-1/b^4*x^3*a*e+1
/3/b^3*x^3*d-10/b^6*a^3*f*x+6/b^5*a^2*e*x-3/b^4*a*d*x+1/b^3*c*x-21/8*a^4/b^
5/(b*x^2+a)^2*x^3*f+17/8*a^3/b^4/(b*x^2+a)^2*x^3*e-13/8*a^2/b^3/(b*x^2+a)^2
*x^3*d+9/8*a/b^2/(b*x^2+a)^2*x^3*c-19/8*a^5/b^6/(b*x^2+a)^2*f*x+15/8*a^4/b^
5/(b*x^2+a)^2*e*x-11/8*a^3/b^4/(b*x^2+a)^2*d*x+7/8*a^2/b^3/(b*x^2+a)^2*c*x+
99/8*a^4/b^6/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-63/8*a^3/b^5/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*e+35/8*a^2/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*d-15/8*a/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.99, size = 237, normalized size = 0.96

$$\frac{\left(9\,ab^4c-13\,a^2b^3d+17\,a^3b^2e-21\,a^4bf\right)x^3+\left(7\,a^2b^3c-11\,a^3b^2d+15\,a^4be-19\,a^5f\right)x}{8\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}-\frac{\left(15\,ab^3c-35\,a^2b^2d+63\,a^3be-99\,a^4f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{15\,b^3fx^7+21\left(b^3e-3\,ab^2f\right)x^5+35\left(b^3d-3\,ab^2e+6\,a^2bf\right)x^3+105\left(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f\right)x}{105\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*a*b^4*c - 13*a^2*b^3*d + 17*a^3*b^2*e - 21*a^4*b*f)*x^3 + (7*a^2*b^
3*c - 11*a^3*b^2*d + 15*a^4*b*e - 19*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2
*b^6) - 1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 3*a*b^2*f)*x
^5 + 35*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^3 + 105*(b^3*c - 3*a*b^2*d + 6*a^
2*b*e - 10*a^3*f)*x)/b^6
```

mupad [B] time = 0.11, size = 348, normalized size = 1.41

$$x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)-x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)-\frac{\left(\frac{21\,f\,a^4\,b}{8}-\frac{17\,e\,a^3\,b^2}{8}+\frac{13\,d\,a^2\,b^3}{8}-\frac{9\,c\,a\,b^4}{8}\right)x^3+\left(\frac{19\,f\,a^5}{8}-\frac{15\,e\,a^4\,b}{8}+\frac{11\,d\,a^3\,b^2}{8}-\frac{7\,c\,a^2\,b^3}{8}\right)x}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}+\frac{f\,x^7}{7\,b^3}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{99\,f\,a^4-63\,e\,a^3\,b+35\,d\,a^2\,b^2-15\,c\,a\,b^3}\right)\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{8\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
```

```
^4))/b))/b) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/b) -
 (x*((19*a^5*f)/8 - (7*a^2*b^3*c)/8 + (11*a^3*b^2*d)/8 - (15*a^4*b*e)/8) +
x^3*((13*a^2*b^3*d)/8 - (17*a^3*b^2*e)/8 - (9*a*b^4*c)/8 + (21*a^4*b*f)/8))
/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + (f*x^7)/(7*b^3) + (a^(1/2)*atan((a^(1/
2)*b^(1/2)*x*(15*b^3*c - 99*a^3*f - 35*a*b^2*d + 63*a^2*b*e))/(99*a^4*f + 3
5*a^2*b^2*d - 15*a*b^3*c - 63*a^3*b*e))*(15*b^3*c - 99*a^3*f - 35*a*b^2*d +
 63*a^2*b*e))/(8*b^(13/2))
```

sympy [A] time = 18.22, size = 316, normalized size = 1.28

$$x^{5}\left(-\frac{3af}{5b^{4}}+\frac{e}{5b^{3}}\right)+x^{3}\left(\frac{2a^{2}f}{b^{5}}-\frac{ae}{b^{4}}+\frac{d}{3b^{3}}\right)+x\left(-\frac{10a^{3}f}{b^{6}}+\frac{6a^{2}e}{b^{5}}-\frac{3ad}{b^{4}}+\frac{c}{b^{3}}\right)-\frac{\sqrt{-\frac{a}{b^{13}}}\left(99a^{3}f-63a^{2}be+35ab^{2}d-15b^{3}c\right)\log\left(-b^{6}\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}+\frac{\sqrt{-\frac{a}{b^{13}}}\left(99a^{3}f-63a^{2}be+35ab^{2}d-15b^{3}c\right)\log\left(b^{6}\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}+\frac{x^{3}\left(-21a^{4}bf+17a^{3}b^{2}e-13a^{2}b^{3}d+9ab^{4}c\right)+x\left(-19a^{5}f+15a^{4}be-11a^{3}b^{2}d+7a^{2}b^{3}c\right)}{8a^{2}b^{6}+16ab^{7}x^{2}+8b^{8}x^{4}}+\frac{fx^{7}}{7b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**5*(-3*a*f/(5*b**4) + e/(5*b**3)) + x**3*(2*a**2*f/b**5 - a*e/b**4 + d/(3
*b**3)) + x*(-10*a**3*f/b**6 + 6*a**2*e/b**5 - 3*a*d/b**4 + c/b**3) - sqrt(
-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b**6*sqr
t(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d
 - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**3*(-21*a**4*b*f + 17*a*
*3*b**2*e - 13*a**2*b**3*d + 9*a*b**4*c) + x*(-19*a**5*f + 15*a**4*b*e - 11
*a**3*b**2*d + 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4)
 + f*x**7/(7*b**3)
```

3.131 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}\,b^{11/2}}-\frac{x\left(-13a^3f+9a^2be-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(\right.}{}$$

Rubi [A] time = 0.33, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1810, 205}

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(9a^2be-13a^3f-5ab^2d+b^3c\right)}{8b^5\left(a+bx^2\right)}-\frac{x\left(13a^2be-25a^3f-5ab^2d+b^3c\right)}{4ab^5}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35a^2be-63a^3f-15ab^2d+3b^3c\right)}{8\sqrt{a}\,b^{11/2}}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(4*a*b^5) + ((b*e - 3*a*f)
*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(8*
b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^3\left(\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^4\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int \frac{a\left(b^3c-5ab^2d+9a^2be-1\right.}{}}{}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)} + \frac{\int\left(-2\left(b^3c-5ab^2d\right.\right.}{}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right.}{4a\left(a+bx^2\right)^2}$$

$$= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right.}{4a\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.18, size = 176, normalized size = 0.85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{8\sqrt{a}\,b^{11/2}} + \frac{x\left(945a^4f-525a^3b\left(e-3fx^2\right)+a^2b^2\left(225d-875ex^2+504fx^4\right)-ab^3\left(45c-375dx^2+280ex^4+72fx^6\right)+b^4x^2\left(8\left(15dx^2+5ex^4+3fx^6\right)-75c\right)\right)}{120b^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504*
f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c +
 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c -
15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(c + dx^2 + ex^4 + fx^6\right)}{\left(a + bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.94, size = 614, normalized size = 2.97

$$\left[\frac{48\,ab^5fx^9 + 16\left(5\,ab^5e - 9\,a^2b^4f\right)x^7 + 16\left(15\,ab^5d - 35\,a^2b^4e + 63\,a^3b^3f\right)x^5 - 50\left(3\,ab^5c - 15\,a^2b^4d + 35\,a^3b^3e - 63\,a^4b^2f\right)x^3 + 15\left(3\,a^2b^3c - 15\,a^3b^2d + 35\,a^4be - 63\,a^5f + \left(3\,b^5c - 15\,ab^4d + 35\,a^2b^3e - 63\,a^3b^2f\right)x^4 + 2\left(3\,ab^4c - 15\,a^2b^3d + 35\,a^3b^2e - 63\,a^4bf\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2 + 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 30\left(3\,a^2b^4c - 15\,a^3b^3d + 35\,a^4b^2e - 63\,a^5bf\right)x}{240\left(ab^8x^4 + 2\,a^2b^7x^2 + a^3b^6\right)}, \frac{24\,ab^5fx^9 + 8\left(5\,ab^5e - 9\,a^2b^4f\right)x^7 + 8\left(15\,ab^5d - 35\,a^2b^4e + 63\,a^3b^3f\right)x^5 - 25\left(3\,ab^5c - 15\,a^2b^4d + 35\,a^3b^3e - 63\,a^4b^2f\right)x^3 + 15\left(3\,a^2b^3c - 15\,a^3b^2d + 35\,a^4be - 63\,a^5f + \left(3\,b^5c - 15\,ab^4d + 35\,a^2b^3e - 63\,a^3b^2f\right)x^4 + 2\left(3\,ab^4c - 15\,a^2b^3d + 35\,a^3b^2e - 63\,a^4bf\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 15\left(3\,a^2b^4c - 15\,a^3b^3d + 35\,a^4b^2e - 63\,a^5bf\right)x}{120\left(ab^8x^4 + 2\,a^2b^7x^2 + a^3b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
[Out] [1/240*(48*a*b^5*f*x^9 + 16*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 16*(15*a*b^5*d
- 35*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 50*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*
b^3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 6
3*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a
*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x
^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(3*a^2*b^4*c - 15*a^3*b^3*d + 35
*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1/120*(2
4*a*b^5*f*x^9 + 8*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 8*(15*a*b^5*d - 35*a^2*b^
4*e + 63*a^3*b^3*f)*x^5 - 25*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*b^3*e - 63*
a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f + (
3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c - 15
*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) - 15*(3*a^2*b^4*c - 15*a^3*b^3*d + 35*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x
^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

giac [A] time = 0.51, size = 200, normalized size = 0.97

$$\frac{\left(3\,b^3c - 15\,ab^2d - 63\,a^3f + 35\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5} - \frac{5\,b^4cx^3 - 9\,ab^3dx^3 - 17\,a^3bfx^3 + 13\,a^2b^2x^3e + 3\,ab^3cx - 7\,a^2b^2dx - 15\,a^4fx + 11\,a^3bxe}{8\left(bx^2 + a\right)^2b^5} + \frac{3\,b^{12}fx^5 - 15\,ab^{11}fx^3 + 5\,b^{12}x^3e + 15\,b^{12}dx + 90\,a^2b^{10}fx - 45\,ab^{11}xe}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 - 17*a^3*b*f*x^3 + 13*a^2*
b^2*x^3*e + 3*a*b^3*c*x - 7*a^2*b^2*d*x - 15*a^4*f*x + 11*a^3*b*x*e)/((b*x^
```

```
2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 - 15*a*b^11*f*x^3 + 5*b^12*x^3*e + 15*b^
12*d*x + 90*a^2*b^10*f*x - 45*a*b^11*x*e)/b^15
```

maple [A] time = 0.01, size = 294, normalized size = 1.42

$$\frac{17a^3 f x^3}{8\left(b x^2+a\right)^2 b^4}-\frac{13a^2 e x^3}{8\left(b x^2+a\right)^2 b^3}+\frac{9ad x^3}{8\left(b x^2+a\right)^2 b^2}-\frac{5c x^3}{8\left(b x^2+a\right)^2 b}+\frac{f x^5}{5b^3}+\frac{15a^4 f x}{8\left(b x^2+a\right)^2 b^5}-\frac{11a^3 ex}{8\left(b x^2+a\right)^2 b^4}+\frac{7a^2 dx}{8\left(b x^2+a\right)^2 b^3}-\frac{3acx}{8\left(b x^2+a\right)^2 b^2}-\frac{af x^3}{b^4}+\frac{e x^3}{3b^3}-\frac{63a^3 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^5}+\frac{35a^2 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^4}-\frac{15ad \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^3}+\frac{3c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^2}+\frac{6a^2 f x}{b^5}-\frac{3aex}{b^4}+\frac{dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*f*x^5/b^3-1/b^4*x^3*a*f+1/3/b^3*x^3*e+6/b^5*a^2*f*x-3/b^4*a*e*x+1/b^3*d
*x+17/8/b^4/(b*x^2+a)^2*x^3*a^3*f-13/8/b^3/(b*x^2+a)^2*x^3*a^2*e+9/8/b^2/(b
*x^2+a)^2*x^3*a*d-5/8/b/(b*x^2+a)^2*x^3*c+15/8/b^5/(b*x^2+a)^2*a^4*f*x-11/8
/b^4/(b*x^2+a)^2*a^3*e*x+7/8/b^3/(b*x^2+a)^2*a^2*d*x-3/8/b^2/(b*x^2+a)^2*a*
c*x-63/8/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*f+35/8/b^4/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*e-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*a*d+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.04, size = 193, normalized size = 0.93

$$-\frac{\left(5 b^4 c-9 a b^3 d+13 a^2 b^2 e-17 a^3 b f\right) x^3+\left(3 a b^3 c-7 a^2 b^2 d+11 a^3 b e-15 a^4 f\right) x}{8\left(b^7 x^4+2 a b^6 x^2+a^2 b^5\right)}+\frac{\left(3 b^3 c-15 a b^2 d+35 a^2 b e-63 a^3 f\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^5}+\frac{3 b^2 f x^5+5\left(b^2 e-3 a b f\right) x^3+15\left(b^2 d-3 a b e+6 a^2 f\right) x}{15 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 9*a*b^3*d + 13*a^2*b^2*e - 17*a^3*b*f)*x^3 + (3*a*b^3*c -
7*a^2*b^2*d + 11*a^3*b*e - 15*a^4*f)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) +
 1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^5) + 1/15*(3*b^2*f*x^5 + 5*(b^2*e - 3*a*b*f)*x^3 + 15*(b^2*d -
3*a*b*e + 6*a^2*f)*x)/b^5
```

mupad [B] time = 0.95, size = 206, normalized size = 1.00

$$x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)-\frac{x^3\left(-\frac{17\,f\,a^3\,b}{8}+\frac{13\,e\,a^2\,b^2}{8}-\frac{9\,d\,a\,b^3}{8}+\frac{5\,c\,b^4}{8}\right)-x\left(\frac{15\,f\,a^4}{8}-\frac{11\,e\,a^3\,b}{8}+\frac{7\,d\,a^2\,b^2}{8}-\frac{3\,c\,a\,b^3}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}+\frac{f\,x^5}{5\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-63\,f\,a^3+35\,e\,a^2\,b-15\,d\,a\,b^2+3\,c\,b^3\right)}{8\,\sqrt{a}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a
*f)/b^4))/b) - (x^3*((5*b^4*c)/8 + (13*a^2*b^2*e)/8 - (9*a*b^3*d)/8 - (17*a
^3*b*f)/8) - x*((15*a^4*f)/8 + (7*a^2*b^2*d)/8 - (3*a*b^3*c)/8 - (11*a^3*b*
e)/8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + (f*x^5)/(5*b^3) + (atan((b^(1/2)
```

```
*x)/a^(1/2))*(3*b^3*c - 63*a^3*f - 15*a*b^2*d + 35*a^2*b*e))/(8*a^(1/2)*b^(
11/2))
```

sympy [A] time = 17.84, size = 280, normalized size = 1.35

$$x^3\left(-\frac{af}{b^4}+\frac{e}{3b^3}\right)+x\left(\frac{6a^2f}{b^5}-\frac{3ae}{b^4}+\frac{d}{b^3}\right)+\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{ab^{11}}}\left(63a^3f-35a^2be+15ab^2d-3b^3c\right)\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}+\frac{x^3\left(17a^3bf-13a^2b^2e+9ab^3d-5b^4c\right)+x\left(15a^4f-11a^3be+7a^2b^2d-3ab^3c\right)}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{fx^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*f/b**4 + e/(3*b**3)) + x*(6*a**2*f/b**5 - 3*a*e/b**4 + d/b**3) + s
qrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(-a
*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*
b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (x**3
*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 11
*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*
b**7*x**4) + f*x**5/(5*b**3)
```

3.132 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=167

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x(11a^3f-7a^2be+3ab^2d+b^3c)}{8ab^4(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}}+\frac{x(be-3}{b^4}$$

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1804, 1585, 1257, 1153, 205}

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{x(-7a^2be+11a^3f+3ab^2d+b^3c)}{8ab^4(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-15a^2be+35a^3f+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}}+\frac{x(be-3af)}{b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
```

```
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x\left(-\left(\left(bc+3ad-\frac{3a^2e}{b}+\frac{3a^3f}{b^2}\right)x\right)-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^2\left(-bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \frac{b^3c+3ab^2d-7a^2be+11\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int\left(8a(be-3af)+\cdots\right.}{\cdots}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x\cdots}{8ab^4\left(a+bx^2\right)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x\cdots}{8ab^4\left(a+bx^2\right)}$$

Mathematica [A] time = 0.13, size = 156, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^3f-15a^2be+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}} + \frac{x\left(-105a^4f+5a^3b\left(9e-35fx^2\right)+a^2b^2\left(-9d+75ex^2-56fx^4\right)+ab^3\left(-3c-15dx^2+24ex^4+8fx^6\right)+3b^4cx^2\right)}{24ab^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d + 7
5*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*a*
b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx^2 + ex^4 + fx^6\right)}{\left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.97, size = 555, normalized size = 3.32

$$\left[\frac{16\,a^2b^4fx^7 + 16\left(3\,a^2b^4e - 7\,a^3b^3f\right)x^5 + 2\left(3\,ab^5c - 15\,a^2b^4d + 75\,a^3b^3e - 175\,a^4b^2f\right)x^3 - 3\left(a^2b^3c + 3\,a^3b^2d - 15\,a^4be + 35\,a^5f + \left(b^5c + 3\,ab^4d - 15\,a^2b^3e + 35\,a^3b^2f\right)x^4 + 2\left(ab^4c + 3\,a^2b^3d - 15\,a^3b^2e + 35\,a^4bf\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 6\left(a^2b^4c + 3\,a^3b^3d - 15\,a^4b^2e + 35\,a^5bf\right)x}{48\left(a^2b^7x^4 + 2\,a^3b^6x^2 + a^4b^5\right)}, \frac{8\,a^2b^4fx^7 + 8\left(3\,a^2b^4e - 7\,a^3b^3f\right)x^5 + \left(3\,ab^5c - 15\,a^2b^4d + 75\,a^3b^3e - 175\,a^4b^2f\right)x^3 + 3\left(a^2b^3c + 3\,a^3b^2d - 15\,a^4be + 35\,a^5f + \left(b^5c + 3\,ab^4d - 15\,a^2b^3e + 35\,a^3b^2f\right)x^4 + 2\left(ab^4c + 3\,a^2b^3d - 15\,a^3b^2e + 35\,a^4bf\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 3\left(a^2b^4c + 3\,a^3b^3d - 15\,a^4b^2e + 35\,a^5bf\right)x}{24\left(a^2b^7x^4 + 2\,a^3b^6x^2 + a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^2*b^4*f*x^7 + 16*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + 2*(3*a*b^5*c
 - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 - 3*(a^2*b^3*c + 3*a^3*
b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*
b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c + 3*a^
3*b^3*d - 15*a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5), 1/24*(8*a^2*b^4*f*x^7 + 8*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + (3*a*b^5*
c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3
*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3
*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d - 15*a^4*b^2*e +
 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.45, size = 173, normalized size = 1.04

$$\frac{\left(b^3c + 3\,ab^2d + 35\,a^3f - 15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^4} + \frac{b^4cx^3 - 5\,ab^3dx^3 - 13\,a^3bfx^3 + 9\,a^2b^2x^3e - ab^3cx - 3\,a^2b^2dx - 11\,a^4fx + 7\,a^3bxe}{8\left(bx^2 + a\right)^2ab^4} + \frac{b^6fx^3 - 9\,ab^5fx + 3\,b^6xe}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 - 13*a^3*b*f*x^3 + 9*a^2*b^2*
x^3*e - a*b^3*c*x - 3*a^2*b^2*d*x - 11*a^4*f*x + 7*a^3*b*x*e)/((b*x^2 + a)^
2*a*b^4) + 1/3*(b^6*f*x^3 - 9*a*b^5*f*x + 3*b^6*x*e)/b^9
```

maple [A] time = 0.01, size = 259, normalized size = 1.55

$$-\frac{13a^2 f x^3}{8\left(b x^2+a\right)^2 b^3}+\frac{9ae x^3}{8\left(b x^2+a\right)^2 b^2}+\frac{c x^3}{8\left(b x^2+a\right)^2 a}-\frac{5d x^3}{8\left(b x^2+a\right)^2 b}-\frac{11a^3 f x}{8\left(b x^2+a\right)^2 b^4}+\frac{7a^2 ex}{8\left(b x^2+a\right)^2 b^3}-\frac{3adx}{8\left(b x^2+a\right)^2 b^2}-\frac{cx}{8\left(b x^2+a\right)^2 b}+\frac{f x^3}{3b^3}+\frac{35a^2 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^4}-\frac{15ae \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^3}+\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, ab}+\frac{3d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^2}-\frac{3afx}{b^4}+\frac{ex}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*f*x^3/b^3-3/b^4*a*f*x+1/b^3*e*x-13/8/b^3/(b*x^2+a)^2*x^3*a^2*f+9/8/b^2/
(b*x^2+a)^2*x^3*a*e-5/8/b/(b*x^2+a)^2*x^3*d+1/8/(b*x^2+a)^2/a*x^3*c-11/8/b^
4/(b*x^2+a)^2*a^3*f*x+7/8/b^3/(b*x^2+a)^2*a^2*e*x-3/8/b^2/(b*x^2+a)^2*a*d*x
-1/8/b/(b*x^2+a)^2*c*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f
-15/8/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+3/8/b^2/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*d+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.02, size = 169, normalized size = 1.01

$$\frac{\left(b^4 c - 5\,ab^3 d + 9\,a^2 b^2 e - 13\,a^3 bf\right)x^3 - \left(ab^3 c + 3\,a^2 b^2 d - 7\,a^3 be + 11\,a^4 f\right)x}{8\left(ab^6 x^4 + 2\,a^2 b^5 x^2 + a^3 b^4\right)} + \frac{bfx^3 + 3\left(be - 3\,af\right)x}{3\,b^4} + \frac{\left(b^3 c + 3\,ab^2 d - 15\,a^2 be + 35\,a^3 f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^4*c - 5*a*b^3*d + 9*a^2*b^2*e - 13*a^3*b*f)*x^3 - (a*b^3*c + 3*a^2*
b^2*d - 7*a^3*b*e + 11*a^4*f)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) + 1/
3*(b*f*x^3 + 3*(b*e - 3*a*f)*x)/b^4 + 1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*e +
 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

mupad [B] time = 1.02, size = 163, normalized size = 0.98

$$x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{11\,f\,a^3}{8}-\frac{7\,e\,a^2\,b}{8}+\frac{3\,d\,a\,b^2}{8}+\frac{c\,b^3}{8}\right)-\frac{x^3\left(-13\,f\,a^3\,b+9\,e\,a^2\,b^2-5\,d\,a\,b^3+c\,b^4\right)}{8\,a}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{f\,x^3}{3\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35\,f\,a^3-15\,e\,a^2\,b+3\,d\,a\,b^2+c\,b^3\right)}{8\,a^{3/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/8 + (11*a^3*f)/8 + (3*a*b^2*d)/8 - (7
*a^2*b*e)/8) - (x^3*(b^4*c + 9*a^2*b^2*e - 5*a*b^3*d - 13*a^3*b*f))/(8*a))/
(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (f*x^3)/(3*b^3) + (atan((b^(1/2)*x)/a^(
1/2))*(b^3*c + 35*a^3*f + 3*a*b^2*d - 15*a^2*b*e))/(8*a^(3/2)*b^(9/2))
```

sympy [A] time = 13.07, size = 260, normalized size = 1.56

$$x\left(-\frac{3af}{b^4}+\frac{e}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^9}}\left(35a^3f-15a^2be+3ab^2d+b^3c\right)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{x^3\left(-13a^3bf+9a^2b^2e-5ab^3d+b^4c\right)+x\left(-11a^4f+7a^3be-3a^2b^2d-ab^3c\right)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}+\frac{fx^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*f/b**4 + e/b**3) - sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e +
3*a*b**2*d + b**3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/
(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b**4*
sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(-13*a**3*b*f + 9*a**2*b**2*e - 5*a*b*
*3*d + b**4*c) + x*(-11*a**4*f + 7*a**3*b*e - 3*a**2*b**2*d - a*b**3*c))/(8
*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4) + f*x**3/(3*b**3)
```

3.133 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=147

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{x\left(9a^3f-5a^2be+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1814, 1157, 388, 205}

$$\frac{x\left(-5a^2be+9a^3f+ab^2d+3b^3c\right)}{8a^2b^3\left(a+bx^2\right)}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^2be-15a^3f+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
```

```
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-\frac{3b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{4a(be-af)x^2}{b^2}-\frac{4afx^4}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\int \frac{\frac{3b^3c+ab^2d+3a^2be-7a^3f}{b^3}+\frac{8a^2fx^2}{b^2}}{a+bx^2}}{8a^2}$$

$$= \frac{fx}{b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\left(3b^3c+ab^2d+3a\right.}{}$$

$$= \frac{fx}{b^3} + \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)} + \frac{\left(3b^3c+ab^2d+3a\right.}{8}$$

Mathematica [A] time = 0.12, size = 141, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}} + \frac{x\left(15a^4f+a^3b\left(25fx^2-3e\right)-a^2b^2\left(d+5ex^2-8fx^4\right)+ab^3\left(5c+dx^2\right)+3b^4cx^2\right)}{8a^2b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
- a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c +
 a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^
(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3, x]
```

fricas [A] time = 1.17, size = 504, normalized size = 3.43

$$\left[\frac{16\,a^3b^3fx^5+2\left(3\,ab^5c+a^2b^4d-5\,a^3b^3e+25\,a^4b^2f\right)x^3+\left(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+\left(3\,b^5c+ab^4d+3\,a^2b^3e-15\,a^3b^2f\right)x^4+2\left(3\,ab^4c+a^2b^3d+3\,a^3b^2e-15\,a^4bf\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(5\,a^2b^4c-a^3b^3d-3\,a^4b^2e+15\,a^5bf\right)x}{16\left(a^3b^6x^4+2\,a^4b^5x^2+a^5b^4\right)},\ \frac{8\,a^3b^3fx^5+\left(3\,ab^5c+a^2b^4d-5\,a^3b^3e+25\,a^4b^2f\right)x^3+\left(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+\left(3\,b^5c+ab^4d+3\,a^2b^3e-15\,a^3b^2f\right)x^4+2\left(3\,ab^4c+a^2b^3d+3\,a^3b^2e-15\,a^4bf\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(5\,a^2b^4c-a^3b^3d-3\,a^4b^2e+15\,a^5bf\right)x}{8\left(a^3b^6x^4+2\,a^4b^5x^2+a^5b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*b
^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*c + a*
b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*
b^2*e - 15*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 2*(5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^6
*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^5*c + a^2*b^
4*d - 5*a^3*b^3*e + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*
e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*
a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^
6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

giac [A] time = 0.46, size = 149, normalized size = 1.01

$$\frac{fx}{b^3}+\frac{\left(3\,b^3c+ab^2d-15\,a^3f+3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}+\frac{3\,b^4cx^3+ab^3dx^3+9\,a^3bfx^3-5\,a^2b^2x^3e+5\,ab^3cx-a^2b^2dx+7\,a^4fx-3\,a^3bxe}{8\left(bx^2+a\right)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 + 9*a^3*b*f*x^3 -
```

```
5*a^2*b^2*x^3*e + 5*a*b^3*c*x - a^2*b^2*d*x + 7*a^4*f*x - 3*a^3*b*x*e)/((b*
x^2 + a)^2*a^2*b^3)
```

maple [A] time = 0.01, size = 234, normalized size = 1.59

$$\frac{9afx^3}{8\left(bx^2+a\right)^2b^2}+\frac{dx^3}{8\left(bx^2+a\right)^2a}+\frac{3bcx^3}{8\left(bx^2+a\right)^2a^2}-\frac{5ex^3}{8\left(bx^2+a\right)^2b}+\frac{7a^2fx}{8\left(bx^2+a\right)^2b^3}-\frac{3aex}{8\left(bx^2+a\right)^2b^2}+\frac{5cx}{8\left(bx^2+a\right)^2a}-\frac{dx}{8\left(bx^2+a\right)^2b}-\frac{15af\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{3c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}+\frac{3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}+\frac{fx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] f*x/b^3+9/8/b^2/(b*x^2+a)^2*x^3*a*f-5/8/b/(b*x^2+a)^2*x^3*e+1/8/(b*x^2+a)^2
/a*x^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c+7/8/b^3/(b*x^2+a)^2*a^2*f*x-3/8/b^2/(b
*x^2+a)^2*a*e*x-1/8/b/(b*x^2+a)^2*d*x+5/8/(b*x^2+a)^2/a*x*c-15/8/b^3*a/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*b*x)*e+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+3/8/a^2/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 3.00, size = 154, normalized size = 1.05

$$\frac{\left(3\,b^4c+ab^3d-5\,a^2b^2e+9\,a^3bf\right)x^3+\left(5\,ab^3c-a^2b^2d-3\,a^3be+7\,a^4f\right)x}{8\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}+\frac{fx}{b^3}+\frac{\left(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^4*c + a*b^3*d - 5*a^2*b^2*e + 9*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*b
^2*d - 3*a^3*b*e + 7*a^4*f)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + f*
x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^3)
```

mupad [B] time = 1.05, size = 148, normalized size = 1.01

$$\frac{\frac{x\left(7\,f\,a^3-3\,e\,a^2\,b-d\,a\,b^2+5\,c\,b^3\right)}{8\,a}+\frac{x^3\left(9\,f\,a^3\,b-5\,e\,a^2\,b^2+d\,a\,b^3+3\,c\,b^4\right)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{f\,x}{b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15\,f\,a^3+3\,e\,a^2\,b+d\,a\,b^2+3\,c\,b^3\right)}{8\,a^{5/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x)
```

```
[Out] ((x*(5*b^3*c + 7*a^3*f - a*b^2*d - 3*a^2*b*e))/(8*a) + (x^3*(3*b^4*c - 5*a^
2*b^2*e + a*b^3*d + 9*a^3*b*f))/(8*a^2))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2)
+ (f*x)/b^3 + (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c - 15*a^3*f + a*b^2*d + 3*
a^2*b*e))/(8*a^(5/2)*b^(7/2))
```

sympy [A] time = 10.09, size = 243, normalized size = 1.65

$$\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^7}}\left(15a^3f-3a^2be-ab^2d-3b^3c\right)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}+\frac{x^3\left(9a^3bf-5a^2b^2e+ab^3d+3b^4c\right)+x\left(7a^4f-3a^3be-a^2b^2d+5ab^3c\right)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{fx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a*
*3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*
a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16
+ (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*f -
3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 +
8*a**2*b**5*x**4) + f*x/b**3
```

3.134 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{c}{a^3x}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f-a^2be-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}$$

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 453, 205}

$$-\frac{x\left(-a^2be+5a^3f-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-a^2be-3a^3f-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}-\frac{c}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
```

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+\left(\frac{3bc}{a}-3d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{4afx^4}{b}}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)} + \frac{\int \frac{8ab^2c-\left(7b^3c-3ab^2d-a^2be-3a\right.}{x^2\left(a+bx^2\right)}}{8a^3b^2}$$

$$= -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)} - \frac{\left(15b^3c-3ab^2d-\right.}{}$$

$$= -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)} - \frac{\left(15b^3c-3ab^2d-\right.}{8}$$

Mathematica [A] time = 0.13, size = 155, normalized size = 1.01

$$-\frac{c}{a^3x} - \frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)} + \frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^2b^2\left(a+bx^2\right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f+a^2be+3ab^2d-15b^3c\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a + b
*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*
x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(7/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^2 \left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3), x]
```

fricas [A] time = 1.17, size = 517, normalized size = 3.38

$$\left[-\frac{16\,a^3b^3c + 2\left(15\,ab^5c - 3\,a^2b^4d - a^3b^3e + 5\,a^4b^2f\right)x^4 + 2\left(25\,a^2b^4c - 5\,a^3b^3d + a^4b^2e + 3\,a^5bf\right)x^2 - \left(\left(15\,b^5c - 3\,ab^4d - a^2b^3e - 3\,a^3b^2f\right)x^5 + 2\left(15\,ab^4c - 3\,a^2b^3d - a^3b^2e - 3\,a^4bf\right)x^3 + \left(15\,a^2b^3c - 3\,a^3b^2d - a^4be - 3\,a^5f\right)x\right)\sqrt{-ab}\log\left(\frac{bx^2 - 2\,\sqrt{-ab}x - a}{bx^2 + a}\right)}{16\left(a^4b^5x^5 + 2\,a^5b^4x^3 + a^6b^3x\right)}, -\frac{8\,a^3b^3c + \left(15\,ab^5c - 3\,a^2b^4d - a^3b^3e + 5\,a^4b^2f\right)x^4 + \left(25\,a^2b^4c - 5\,a^3b^3d + a^4b^2e + 3\,a^5bf\right)x^2 + \left(\left(15\,b^5c - 3\,ab^4d - a^2b^3e - 3\,a^3b^2f\right)x^5 + 2\left(15\,ab^4c - 3\,a^2b^3d - a^3b^2e - 3\,a^4bf\right)x^3 + \left(15\,a^2b^3c - 3\,a^3b^2d - a^4be - 3\,a^5f\right)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{8\left(a^4b^5x^5 + 2\,a^5b^4x^3 + a^6b^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*
f)*x^4 + 2*(25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 - ((15*
b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^
3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e -
3*a^5*f)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*
b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^5*c - 3*a
^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*d + a^4
*b^2*e + 3*a^5*b*f)*x^2 + ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)
*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b
^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

giac [A] time = 0.45, size = 153, normalized size = 1.00

$$-\frac{c}{a^3x} - \frac{\left(15\,b^3c - 3\,ab^2d - 3\,a^3f - a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3b^2} - \frac{7\,b^4cx^3 - 3\,ab^3dx^3 + 5\,a^3bfx^3 - a^2b^2x^3e + 9\,ab^3cx - 5\,a^2b^2dx + 3\,a^4fx + a^3bxe}{8\left(bx^2 + a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 + 5*a^3*b*f*x
^3 - a^2*b^2*x^3*e + 9*a*b^3*c*x - 5*a^2*b^2*d*x + 3*a^4*f*x + a^3*b*x*e)/(
(b*x^2 + a)^2*a^3*b^2)
```

maple [A] time = 0.01, size = 237, normalized size = 1.55

$$\frac{e x^3}{8 \left(b x^2+a\right)^2 a}+\frac{3bd x^3}{8 \left(b x^2+a\right)^2 a^2}-\frac{7b^2 c x^3}{8 \left(b x^2+a\right)^2 a^3}-\frac{5f x^3}{8 \left(b x^2+a\right)^2 b}-\frac{3afx}{8 \left(b x^2+a\right)^2 b^2}+\frac{5dx}{8 \left(b x^2+a\right)^2 a}-\frac{9bcx}{8 \left(b x^2+a\right)^2 a^2}-\frac{ex}{8 \left(b x^2+a\right)^2 b}+\frac{e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, ab}+\frac{3d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^2}-\frac{15bc \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^3}+\frac{3f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, b^2}-\frac{c}{a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -5/8/(b*x^2+a)^2/b*x^3*f+1/8/a/(b*x^2+a)^2*x^3*e+3/8/a^2/(b*x^2+a)^2*b*x^3*
d-7/8/a^3/(b*x^2+a)^2*b^2*x^3*c-3/8*a/(b*x^2+a)^2/b^2*x*f-1/8/(b*x^2+a)^2/b
*x*e+5/8/a/(b*x^2+a)^2*x*d-9/8/a^2/(b*x^2+a)^2*b*x*c+3/8/b^2/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*f+1/8/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e+3
/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-15/8/a^3*b/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*c-c/a^3/x
```

maxima [A] time = 2.99, size = 161, normalized size = 1.05

$$-\frac{8 a^2 b^2 c+\left(15 b^4 c-3 a b^3 d-a^2 b^2 e+5 a^3 b f\right) x^4+\left(25 a b^3 c-5 a^2 b^2 d+a^3 b e+3 a^4 f\right) x^2}{8 \left(a^3 b^4 x^5+2 a^4 b^3 x^3+a^5 b^2 x\right)}-\frac{\left(15 b^3 c-3 a b^2 d-a^2 b e-3 a^3 f\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}\, a^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b^2*c + (15*b^4*c - 3*a*b^3*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (2
5*a*b^3*c - 5*a^2*b^2*d + a^3*b*e + 3*a^4*f)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^3*
x^3 + a^5*b^2*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

mupad [B] time = 1.09, size = 149, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3\,f\,a^3+e\,a^2\,b+3\,d\,a\,b^2-15\,c\,b^3\right)}{8\,a^{7/2}\,b^{5/2}}-\frac{\frac{c}{a}+\frac{x^4\left(5\,f\,a^3-e\,a^2\,b-3\,d\,a\,b^2+15\,c\,b^3\right)}{8\,a^3\,b}+\frac{x^2\left(3\,f\,a^3+e\,a^2\,b-5\,d\,a\,b^2+25\,c\,b^3\right)}{8\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^3*f - 15*b^3*c + 3*a*b^2*d + a^2*b*e))/(8*a
^(7/2)*b^(5/2)) - (c/a + (x^4*(15*b^3*c + 5*a^3*f - 3*a*b^2*d - a^2*b*e))/(
8*a^3*b) + (x^2*(25*b^3*c + 3*a^3*f - 5*a*b^2*d + a^2*b*e))/(8*a^2*b^2))/(a
^2*x + b^2*x^5 + 2*a*b*x^3)
```

sympy [A] time = 26.56, size = 250, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^7b^5}}\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{-8a^2b^2c+x^4\left(-5a^3bf+a^2b^2e+3ab^3d-15b^4c\right)+x^2\left(-3a^4f-a^3be+5a^2b^2d-25ab^3c\right)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-a
**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f + a*
*2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16
 + (-8*a**2*b**2*c + x**4*(-5*a**3*b*f + a**2*b**2*e + 3*a*b**3*d - 15*b**4
*c) + x**2*(-3*a**4*f - a**3*b*e + 5*a**2*b**2*d - 25*a*b**3*c))/(8*a**5*b*
*2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

3.135 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=168

$$\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{x\left(a^3f+3a^2be-7ab^2d+\right.}{8a^4b\left(a+bx^2\right)}$$

Rubi [A] time = 0.24, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1805, 1259, 1261, 205}

$$\frac{x\left(3a^2be+a^3f-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^2be+a^3f-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -c/(3*a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a*f
)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)*x
)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{3b^2c}{a^2}+\frac{3bd}{a}-3e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}-\frac{\int \frac{-8a^2b^2c+8ab^2(2bc-ad)x^2-\cdots}{x^4(\cdots}}{8\cdots}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}-\frac{\int\left(-\frac{8ab^2c}{x^4}+\frac{8b^2(3bc-ad)}{x^2}\cdots\right.}{\cdots}$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}+\cdots$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}+\cdots$$

Mathematica [A] time = 0.16, size = 169, normalized size = 1.01

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{-3a^4fx^4+a^3b\left(3x^2\left(-8d+5ex^2+fx^4\right)-8c\right)+a^2b^2x^2\left(56c-75dx^2+9ex^4\right)+5ab^3x^4\left(35c-9dx^2\right)+105b^4cx^6}{24a^4bx^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2*
(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4))
)/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4 \left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3), x]
```

fricas [A] time = 0.87, size = 570, normalized size = 3.39

$$\left[-\frac{16\,a^4b^2c - 6\left(35\,ab^5c - 15\,a^2b^4d + 3\,a^3b^3e + a^4b^2f\right)x^6 - 2\left(175\,a^2b^4c - 75\,a^3b^3d + 15\,a^4b^2e - 3\,a^5bf\right)x^4 - 16\left(7\,a^3b^3c - 3\,a^4b^2d\right)x^2 + 3\left(\left(35\,b^5c - 15\,ab^4d + 3\,a^2b^3e + a^3b^2f\right)x^7 + 2\left(35\,ab^4c - 15\,a^2b^3d + 3\,a^3b^2e + a^4bf\right)x^5 + \left(35\,a^2b^3c - 15\,a^3b^2d + 3\,a^4be + a^5f\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{48\left(a^5b^4x^7 + 2\,a^6b^3x^5 + a^7b^2x^3\right)}, -\frac{8\,a^4b^2c - 3\left(35\,ab^5c - 15\,a^2b^4d + 3\,a^3b^3e + a^4b^2f\right)x^6 - \left(175\,a^2b^4c - 75\,a^3b^3d + 15\,a^4b^2e - 3\,a^5bf\right)x^4 - 8\left(7\,a^3b^3c - 3\,a^4b^2d\right)x^2 - 3\left(\left(35\,b^5c - 15\,ab^4d + 3\,a^2b^3e + a^3b^2f\right)x^7 + 2\left(35\,ab^4c - 15\,a^2b^3d + 3\,a^3b^2e + a^4bf\right)x^5 + \left(35\,a^2b^3c - 15\,a^3b^2d + 3\,a^4be + a^5f\right)x^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{24\left(a^5b^4x^7 + 2\,a^6b^3x^5 + a^7b^2x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^3*e + a^4*b^2
*f)*x^6 - 2*(175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^4 -
 16*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3
*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)
*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 +
 a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^
3*e + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5
*b*f)*x^4 - 8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*a*b^4*d +
 3*a^2*b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e
+ a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^7*b^2*x^3)]
```

giac [A] time = 0.45, size = 170, normalized size = 1.01

$$\frac{\left(35\,b^3c - 15\,ab^2d + a^3f + 3\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4b} + \frac{11\,b^4cx^3 - 7\,ab^3dx^3 + a^3bfx^3 + 3\,a^2b^2x^3e + 13\,ab^3cx - 9\,a^2b^2dx - a^4fx + 5\,a^3bxe}{8\left(bx^2 + a\right)^2a^4b} + \frac{9\,bcx^2 - 3\,adx^2 - ac}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + a^3*b*f*x^3 + 3*a^2*b^2*
x^3*e + 13*a*b^3*c*x - 9*a^2*b^2*d*x - a^4*f*x + 5*a^3*b*x*e)/((b*x^2 + a)^
2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

maple [A] time = 0.02, size = 264, normalized size = 1.57

$$\frac{f x^3}{8\left(b x^2+a\right)^2 a}+\frac{3be x^3}{8\left(b x^2+a\right)^2 a^2}-\frac{7b^2d x^3}{8\left(b x^2+a\right)^2 a^3}+\frac{11b^3c x^3}{8\left(b x^2+a\right)^2 a^4}+\frac{5ex}{8\left(b x^2+a\right)^2 a}-\frac{9bdx}{8\left(b x^2+a\right)^2 a^2}+\frac{13b^2cx}{8\left(b x^2+a\right)^2 a^3}-\frac{fx}{8\left(b x^2+a\right)^2 b}+\frac{f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{3e\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}-\frac{15bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^3}+\frac{35b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^4}-\frac{d}{a^3x}+\frac{3bc}{a^4x}-\frac{c}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x)
```

```
[Out] 1/8/a/(b*x^2+a)^2*x^3*f+3/8/a^2/(b*x^2+a)^2*x^3*b*e-7/8/a^3/(b*x^2+a)^2*x^3
*b^2*d+11/8/a^4/(b*x^2+a)^2*x^3*b^3*c-1/8/(b*x^2+a)^2/b*x*f+5/8/a/(b*x^2+a)
^2*x*e-9/8/a^2/(b*x^2+a)^2*b*x*d+13/8/a^3/(b*x^2+a)^2*b^2*x*c+1/8/a/b/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)*e-15/8/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+35/8/a^4*b^2/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/3*c/a^3/x^3-1/a^3/x*d+3/a^4/x*b*c
```

maxima [A] time = 3.07, size = 181, normalized size = 1.08

$$\frac{3\left(35\,b^4c-15\,ab^3d+3\,a^2b^2e+a^3bf\right)x^6-8\,a^3bc+\left(175\,ab^3c-75\,a^2b^2d+15\,a^3be-3\,a^4f\right)x^4+8\left(7\,a^2b^2c-3\,a^3bd\right)x^2}{24\left(a^4b^3x^7+2\,a^5b^2x^5+a^6bx^3\right)}+\frac{\left(35\,b^3c-15\,ab^2d+3\,a^2be+a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(35*b^4*c - 15*a*b^3*d + 3*a^2*b^2*e + a^3*b*f)*x^6 - 8*a^3*b*c + (
175*a*b^3*c - 75*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c - 3
*a^3*b*d)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(35*b^3*c -
15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

mupad [B] time = 1.03, size = 166, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^{9/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}-\frac{x^6\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^4}+\frac{x^2\,\left(3\,a\,d-7\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-3\,f\,a^3+15\,e\,a^2\,b-75\,d\,a\,b^2+175\,c\,b^3\right)}{24\,a^3\,b}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b*e))/(8*
a^(9/2)*b^(3/2)) - (c/(3*a) - (x^6*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b
*e))/(8*a^4) + (x^2*(3*a*d - 7*b*c))/(3*a^2) - (x^4*(175*b^3*c - 3*a^3*f -
75*a*b^2*d + 15*a^2*b*e))/(24*a^3*b))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
```

sympy [A] time = 70.49, size = 270, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^9b^3}}\left(a^3f+3a^2be-15ab^2d+35b^3c\right)\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}+\frac{-8a^3bc+x^6\left(3a^3bf+9a^2b^2e-45ab^3d+105b^4c\right)+x^4\left(-3a^4f+15a^3be-75a^2b^2d+175ab^3c\right)+x^2\left(-24a^3bd+56a^2b^2c\right)}{24a^6bx^3+48a^5b^2x^5+24a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2*b*e - 15*a*b**2*d + 35*b**3*c)*log(-
a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + sqrt(-1/(a**9*b**3))*(a**3*f + 3*a**2
*b*e - 15*a*b**2*d + 35*b**3*c)*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + (
-8*a**3*b*c + x**6*(3*a**3*b*f + 9*a**2*b**2*e - 45*a*b**3*d + 105*b**4*c)
+ x**4*(-3*a**4*f + 15*a**3*b*e - 75*a**2*b**2*d + 175*a*b**3*c) + x**2*(-2
4*a**3*b*d + 56*a**2*b**2*c))/(24*a**6*b*x**3 + 48*a**5*b**2*x**5 + 24*a**4
*b**3*x**7)
```

3.136 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=196

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{x\left(-3a^3f+7a^2be-11\right.}{8a^5\left(a+bx\right.}$$

Rubi [A] time = 0.35, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{x\left(7a^2be-3a^3f-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(15a^2be-3a^3f-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -c/(5*a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/(a
^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - ((1
5*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63*b
^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(11/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx=-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{3\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)^2}\,dx}{4a}$$

$$=-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)}+\frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2}{}}{}$$

$$=-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)}+\frac{\int\left(\frac{8c}{ax^6}+\frac{8(-3b}{a^2}\right.}{}$$

$$=-\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\left(15b^3c-1\right.}{}$$

$$=-\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\left(15b^3c-1\right.}{}$$

Mathematica [A] time = 0.12, size = 196, normalized size = 1.00

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}+\frac{a^2(-e)+3abd-6b^2c}{a^5x}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)}{8a^{11/2}\sqrt{b}}+\frac{x\left(3a^3f-7a^2be+11ab^2d-15b^3c\right)}{8a^5\left(a+bx^2\right)}+\frac{x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e)
/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^2)
 + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) +
 ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(11/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3), x]
```

fricas [A] time = 1.16, size = 628, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
48*a^5*b*c + 50*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 16*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 16*(9*a^4*b^2*c - 5
*a^5*b*d)*x^2 - 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^
9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*
b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5),
 -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 8*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 8*(9*a^4*b^2*c - 5*a
^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^9
+ 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*b^
3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

giac [A] time = 0.44, size = 198, normalized size = 1.01

$$-\frac{\left(63\,b^3c-35\,ab^2d-3\,a^3f+15\,a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}-\frac{15\,b^4cx^3-11\,ab^3dx^3-3\,a^3bfx^3+7\,a^2b^2x^3e+17\,ab^3cx-13\,a^2b^2dx-5\,a^4fx+9\,a^3bxe}{8\left(bx^2+a\right)^2a^5}-\frac{90\,b^2cx^4-45\,abdx^4+15\,a^2x^4e-15\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 - 3*a^3*b*f*x^3 + 7*a^2
*b^2*x^3*e + 17*a*b^3*c*x - 13*a^2*b^2*d*x - 5*a^4*f*x + 9*a^3*b*x*e)/((b*x
```

```
^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*x^4*e - 15*a*b*
c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

maple [A] time = 0.02, size = 300, normalized size = 1.53

$$\frac{3bf\,x^3}{8\left(b\,x^2+a\right)^2 a^2}-\frac{7b^2e\,x^3}{8\left(b\,x^2+a\right)^2 a^3}+\frac{11b^3d\,x^3}{8\left(b\,x^2+a\right)^2 a^4}-\frac{15b^4c\,x^3}{8\left(b\,x^2+a\right)^2 a^5}+\frac{5fx}{8\left(b\,x^2+a\right)^2 a}-\frac{9bex}{8\left(b\,x^2+a\right)^2 a^2}+\frac{13b^2dx}{8\left(b\,x^2+a\right)^2 a^3}-\frac{17b^3cx}{8\left(b\,x^2+a\right)^2 a^4}+\frac{3f\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^2}-\frac{15be\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^3}+\frac{35b^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^4}-\frac{63b^3c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^5}-\frac{e}{a^3x}+\frac{3bd}{a^4x}-\frac{6b^2c}{a^5x}-\frac{d}{3a^3x^3}+\frac{bc}{a^4x^3}-\frac{c}{5a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x)
```

```
[Out] 3/8/a^2/(b*x^2+a)^2*x^3*b*f-7/8/a^3/(b*x^2+a)^2*x^3*b^2*e+11/8/a^4/(b*x^2+a
)^2*x^3*b^3*d-15/8/a^5/(b*x^2+a)^2*x^3*b^4*c+5/8/a/(b*x^2+a)^2*f*x-9/8/a^2/
(b*x^2+a)^2*b*e*x+13/8/a^3/(b*x^2+a)^2*b^2*d*x-17/8/a^4/(b*x^2+a)^2*b^3*c*x
+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f-15/8/a^3/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*b*e+35/8/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2
*d-63/8/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^3*c-1/5*c/a^3/x^5-1/3/a
^3/x^3*d+1/a^4/x^3*b*c-1/a^3/x*e+3/a^4/x*b*d-6/a^5/x*b^2*c
```

maxima [A] time = 2.98, size = 202, normalized size = 1.03

$$-\frac{15\left(63\,b^4c-35\,ab^3d+15\,a^2b^2e-3\,a^3bf\right)x^8+25\left(63\,ab^3c-35\,a^2b^2d+15\,a^3be-3\,a^4f\right)x^6+24\,a^4c+8\left(63\,a^2b^2c-35\,a^3bd+15\,a^4e\right)x^4-8\left(9\,a^3bc-5\,a^4d\right)x^2}{120\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}-\frac{\left(63\,b^3c-35\,ab^2d+15\,a^2be-3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(63*b^4*c - 35*a*b^3*d + 15*a^2*b^2*e - 3*a^3*b*f)*x^8 + 25*(63*
a*b^3*c - 35*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2*b
^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)/(a^5*b^2*x
^9 + 2*a^6*b*x^7 + a^7*x^5) - 1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a
^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 1.04, size = 192, normalized size = 0.98

$$-\frac{\frac{c}{5\,a}+\frac{5\,x^6\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{24\,a^4}+\frac{x^2\,(5\,a\,d-9\,b\,c)}{15\,a^2}+\frac{x^4\left(15\,e\,a^2-35\,d\,a\,b+63\,c\,b^2\right)}{15\,a^3}+\frac{b\,x^8\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^{11/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (c/(5*a) + (5*x^6*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(24*a^4
) + (x^2*(5*a*d - 9*b*c))/(15*a^2) + (x^4*(63*b^2*c + 15*a^2*e - 35*a*b*d))
/(15*a^3) + (b*x^8*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^5))
/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(63*b^3*c - 3
*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^(11/2)*b^(1/2))
```

sympy [A] time = 150.75, size = 284, normalized size = 1.45

$$-\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{11}b}}\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16}+\frac{-24a^4c+x^8\left(45a^3bf-225a^2b^2e+525ab^3d-945b^4c\right)+x^6\left(75a^4f-375a^3be+875a^2b^2d-1575ab^3c\right)+x^4\left(-120a^4e+280a^3bd-504a^2b^2c\right)+x^2\left(-40a^4d+72a^3bc\right)}{120a^7x^5+240a^6bx^7+120a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b*e + 35*a*b**2*d - 63*b**3*c)*log(
-a**6*sqrt(-1/(a**11*b)) + x)/16 + sqrt(-1/(a**11*b))*(3*a**3*f - 15*a**2*b
*e + 35*a*b**2*d - 63*b**3*c)*log(a**6*sqrt(-1/(a**11*b)) + x)/16 + (-24*a*
*4*c + x**8*(45*a**3*b*f - 225*a**2*b**2*e + 525*a*b**3*d - 945*b**4*c) + x
**6*(75*a**4*f - 375*a**3*b*e + 875*a**2*b**2*d - 1575*a*b**3*c) + x**4*(-1
20*a**4*e + 280*a**3*b*d - 504*a**2*b**2*c) + x**2*(-40*a**4*d + 72*a**3*b*
c))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5*b**2*x**9)
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=234

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2b\right.}{8a^6\left(a\right.}$$

Rubi [A] time = 0.49, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$\frac{bx\left(11a^2be-7a^3f-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{bx\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{a^6x}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35a^2be-15a^3f-63ab^2d+99b^3c\right)}{8a^{13/2}}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -c/(7*a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/(3
*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15*a
*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*c
- 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{3b(b \cdots}{\cdots}}{x^8\left(a+bx^2\right)^2}}{4a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a} \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \frac{\int \left(\frac{8c}{ax^8} + \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c - \cdots\right.}{\cdots}$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c - \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.15, size = 234, normalized size = 1.00

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2be-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3), x]
```

fricas [A] time = 1.08, size = 678, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 3
50*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 112*(99*a^
2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 240*a^5*c - 16*(99*a^
3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 - 105*
((99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c
 - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), 1/840*(105*(99*
b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175*(99*a*b^4*c -
63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^2*c - 63*a^4*b*
d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^5*c - 63*a*b
^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d + 3
5*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e -
 15*a^5*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9
+ a^8*x^7)]
```

giac [A] time = 0.47, size = 250, normalized size = 1.07

$$\frac{\left(99\,b^4c-63\,ab^3d-15\,a^3bf+35\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}+\frac{19\,b^5cx^3-15\,ab^4dx^3-7\,a^3b^2fx^3+11\,a^2b^3x^3e+21\,ab^4cx-17\,a^2b^3dx-9\,a^4bfx+13\,a^3b^2xe}{8\left(bx^2+a\right)^2a^6}+\frac{1050\,b^3cx^6-630\,ab^2dx^6-105\,a^3fx^6+315\,a^2bx^6e-210\,ab^2cx^4+105\,a^2bdx^4-35\,a^3x^4e+63\,a^2bcx^2-21\,a^3dx^2-15\,a^3c}{105\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 35*a^2*b^2*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 - 7*a^3*b^2*f*x^3 +
```

```
 11*a^2*b^3*x^3*e + 21*a*b^4*c*x - 17*a^2*b^3*d*x - 9*a^4*b*f*x + 13*a^3*b^
2*x*e)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 - 105*
a^3*f*x^6 + 315*a^2*b*x^6*e - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*x^
4*e + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

maple [A] time = 0.02, size = 351, normalized size = 1.50

$$-\frac{7b^2 f x^3}{8\left(b x^2+a\right)^2 a^3}+\frac{11b^3 e x^3}{8\left(b x^2+a\right)^2 a^4}-\frac{15b^4 d x^3}{8\left(b x^2+a\right)^2 a^5}+\frac{19b^5 c x^3}{8\left(b x^2+a\right)^2 a^6}-\frac{9bfx}{8\left(b x^2+a\right)^2 a^2}+\frac{13b^2 ex}{8\left(b x^2+a\right)^2 a^3}-\frac{17b^3 dx}{8\left(b x^2+a\right)^2 a^4}+\frac{21b^4 cx}{8\left(b x^2+a\right)^2 a^5}-\frac{15bf \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^3}+\frac{35b^2 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^4}-\frac{63b^3 d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^5}+\frac{99b^4 c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^6}-\frac{f}{a^3 x}+\frac{3be}{a^4 x}-\frac{6b^2 d}{a^5 x}+\frac{10b^3 c}{a^6 x}-\frac{e}{3a^3x^3}+\frac{bd}{a^4x^3}-\frac{2b^2c}{a^5x^3}-\frac{d}{5a^3x^5}+\frac{3bc}{5a^4x^5}-\frac{c}{7a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x)
```

```
[Out] -7/8/a^3*b^2/(b*x^2+a)^2*x^3*f+11/8/a^4*b^3/(b*x^2+a)^2*x^3*e-15/8/a^5*b^4/
(b*x^2+a)^2*x^3*d+19/8/a^6*b^5/(b*x^2+a)^2*x^3*c-9/8/a^2*b/(b*x^2+a)^2*f*x+
13/8/a^3*b^2/(b*x^2+a)^2*e*x-17/8/a^4*b^3/(b*x^2+a)^2*d*x+21/8/a^5*b^4/(b*x
^2+a)^2*c*x-15/8/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*f+35/8/a^4*b^2
/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*e-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*d+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7
*c/a^3/x^7-1/5/a^3/x^5*d+3/5/a^4/x^5*b*c-1/3/a^3/x^3*e+1/a^4/x^3*b*d-2/a^5/
x^3*b^2*c-1/a^3/x*f+3/a^4/x*b*e-6/a^5/x*b^2*d+10/a^6/x*b^3*c
```

maxima [A] time = 3.03, size = 247, normalized size = 1.06

$$\frac{105\left(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f\right)x^{10}+175\left(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf\right)x^8+56\left(99\,a^2b^3c-63\,a^3b^2d+35\,a^4be-15\,a^5f\right)x^6-120\,a^5c-8\left(99\,a^3b^2c-63\,a^4bd+35\,a^5e\right)x^4+24\left(11\,a^4bc-7\,a^5d\right)x^2}{840\left(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7\right)}+\frac{\left(99\,b^4c-63\,ab^3d+35\,a^2b^2e-15\,a^3bf\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175
*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b
^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^
2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2)/(a^6*b^2*
x^11 + 2*a^7*b*x^9 + a^8*x^7) + 1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^2*e -
 15*a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 1.05, size = 230, normalized size = 0.98

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^{13/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,(7\,a\,d-11\,b\,c)}{35\,a^2}+\frac{x^4\left(35\,e\,a^2-63\,d\,a\,b+99\,c\,b^2\right)}{105\,a^3}-\frac{5\,b\,x^8\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{24\,a^5}-\frac{b^2\,x^{10}\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(99*b^3*c - 15*a^3*f - 63*a*b^2*d + 35*a
^2*b*e))/(8*a^(13/2)) - (c/(7*a) - (x^6*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
```

```
 35*a^2*b*e))/(15*a^4) + (x^2*(7*a*d - 11*b*c))/(35*a^2) + (x^4*(99*b^2*c +
 35*a^2*e - 63*a*b*d))/(105*a^3) - (5*b*x^8*(99*b^3*c - 15*a^3*f - 63*a*b^2
*d + 35*a^2*b*e))/(24*a^5) - (b^2*x^10*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
35*a^2*b*e))/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2*a*b*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=277

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x\left(-11a^3f+1\right.}{8a}$$

Rubi [A] time = 0.60, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1805, 1802, 205}

$$-\frac{b^2x\left(15a^2be-11a^3f-19ab^2d+23b^3c\right)}{8a^7\left(a+bx^2\right)}-\frac{b^2x\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}+\frac{3a^2be+a^3(-f)-6ab^2d+10b^3c}{3a^6x^3}-\frac{b\left(6a^2be-3a^3f-10ab^2d+15b^3c\right)}{a^7x}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(63a^2be-35a^3f-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -c/(9*a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/(5
*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(15
*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*d
+ 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 99*
a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{4b}{}}{x^{10}\left(a+bx^2\right)^2}}{4a}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int \frac{8c-8}{}}{}$$

$$= -\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2\left(23b^3c-19ab^2d+15a^2be-11a^3f\right)x}{8a^7\left(a+bx^2\right)} + \frac{\int \left(\frac{8c}{ax^{10}}\right.}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b\left(15b^3c\right.}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b\left(15b^3c\right.}{}$$

Mathematica [A] time = 0.16, size = 276, normalized size = 1.00

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35a^3f-63a^2be+99ab^2d-143b^3c\right)}{8a^{15/2}}+\frac{b^2x\left(11a^3f-15a^2be+19ab^2d-23b^3c\right)}{8a^7\left(a+bx^2\right)}+\frac{b\left(3a^3f-6a^2be+10ab^2d-15b^3c\right)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}+\frac{b^2x\left(a^3f-a^2be+ab^2d-b^3c\right)}{4a^6\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a
*b^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3
*c + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(15/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^{10} \left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3), x]
```

fricas [A] time = 0.75, size = 772, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 336
*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 560*a^6*c
 - 48*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(143*
a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2 + 31
5*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b
^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c -
99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9), -1
/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 + 52
5*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(14
3*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c - 2
4*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4*b
^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2 + 315*((1
43*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c
- 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 99*a^
3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

giac [A] time = 0.40, size = 301, normalized size = 1.09

$$-\frac{\left(143\,b^5c - 99\,ab^4d - 35\,a^3b^2f + 63\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^7} - \frac{23\,b^6cx^3 - 19\,ab^5dx^3 - 11\,a^3b^3fx^3 + 15\,a^2b^4x^3e + 25\,ab^5cx - 21\,a^2b^4dx - 13\,a^4b^2fx + 17\,a^3b^3xe}{8\left(bx^2 + a\right)^2a^7} - \frac{4725\,b^4cx^8 - 3150\,ab^3dx^8 - 945\,a^3bfx^8 + 1890\,a^2b^2x^8e - 1050\,ab^3cx^6 + 630\,a^2b^2dx^6 + 105\,a^4fx^6 - 315\,a^3bx^6e + 378\,a^2b^2cx^4 - 189\,a^3bdx^4 + 63\,a^4x^4e - 135\,a^3bcx^2 + 45\,a^4dx^2 + 35\,a^4c}{315\,a^7x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^2*f + 63*a^2*b^3*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 - 11*a^3*b^3*f*
x^3 + 15*a^2*b^4*x^3*e + 25*a*b^5*c*x - 21*a^2*b^4*d*x - 13*a^4*b^2*f*x + 1
7*a^3*b^3*x*e)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x
^8 - 945*a^3*b*f*x^8 + 1890*a^2*b^2*x^8*e - 1050*a*b^3*c*x^6 + 630*a^2*b^2*
d*x^6 + 105*a^4*f*x^6 - 315*a^3*b*x^6*e + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x
^4 + 63*a^4*x^4*e - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^9)
```

maple [A] time = 0.02, size = 401, normalized size = 1.45

$$\frac{11b^3 f x^3}{8\left(b x^2+a\right)^2 a^4}-\frac{15b^4 e x^3}{8\left(b x^2+a\right)^2 a^5}+\frac{19b^5 d x^3}{8\left(b x^2+a\right)^2 a^6}-\frac{23b^6 c x^3}{8\left(b x^2+a\right)^2 a^7}+\frac{13b^2 f x}{8\left(b x^2+a\right)^2 a^3}-\frac{17b^3 e x}{8\left(b x^2+a\right)^2 a^4}+\frac{21b^4 d x}{8\left(b x^2+a\right)^2 a^5}-\frac{25b^5 c x}{8\left(b x^2+a\right)^2 a^6}+\frac{35b^2 f \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^4}-\frac{63b^3 e \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^5}+\frac{99b^4 d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^6}-\frac{143b^5 c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^7}+\frac{3bf}{a^4 x}-\frac{6b^2 e}{a^5 x}+\frac{10b^3 d}{a^6 x}-\frac{15b^4 c}{a^7 x}-\frac{f}{3a^3 x^3}+\frac{be}{a^4 x^3}-\frac{2b^2 d}{a^5 x^3}+\frac{10b^3 c}{3a^6 x^3}-\frac{e}{5a^3 x^5}+\frac{3bd}{5a^4 x^5}-\frac{6b^2 c}{5a^5 x^5}-\frac{d}{7a^3 x^7}+\frac{3bc}{7a^4 x^7}-\frac{c}{9a^3 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x)
```

```
[Out] 3/5/a^4/x^5*b*d-6/5/a^5/x^5*b^2*c+1/a^4/x^3*b*e-2/a^5/x^3*b^2*d+10/3/a^6/x^
3*b^3*c+3*b/a^4/x*f-6*b^2/a^5/x*e+10*b^3/a^6/x*d-15*b^4/a^7/x*c+3/7/a^4/x^7
*b*c-1/9*c/a^3/x^9+19/8/a^6*b^5/(b*x^2+a)^2*x^3*d-23/8/a^7*b^6/(b*x^2+a)^2*
x^3*c+13/8/a^3*b^2/(b*x^2+a)^2*f*x-17/8/a^4*b^3/(b*x^2+a)^2*e*x+21/8/a^5*b^
4/(b*x^2+a)^2*d*x-25/8/a^6*b^5/(b*x^2+a)^2*c*x+35/8/a^4*b^2/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*f-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*e+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-143/8/a^7*b^5/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/7/a^3/x^7*d-1/5/a^3/x^5*e-1/3/a^3/x^3*
f+11/8/a^4*b^3/(b*x^2+a)^2*x^3*f-15/8/a^5*b^4/(b*x^2+a)^2*x^3*e
```

maxima [A] time = 3.09, size = 291, normalized size = 1.05

$$-\frac{315\left(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f\right)x^{12}+525\left(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f\right)x^{10}+168\left(143\,a^2b^4c-99\,a^3b^3d+63\,a^4b^2e-35\,a^5bf\right)x^8+280\,a^6c-24\left(143\,a^3b^3c-99\,a^4b^2d+63\,a^5be-35\,a^6f\right)x^6+8\left(143\,a^4b^2c-99\,a^5bd+63\,a^6e\right)x^4-40\left(13\,a^5bc-9\,a^6d\right)x^2}{2520\left(a^7b^2x^{13}+2\,a^8bx^{11}+a^9x^9\right)}-\frac{\left(143\,b^5c-99\,ab^4d+63\,a^2b^3e-35\,a^3b^2f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
525*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(
143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c -
 24*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4
*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2)/(a^7*b
^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) - 1/8*(143*b^5*c - 99*a*b^4*d + 63*a^2*b^
3*e - 35*a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

mupad [B] time = 1.07, size = 268, normalized size = 0.97

$$-\frac{\frac{c}{9\,a}-\frac{x^6\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{105\,a^4}+\frac{x^2\,(9\,a\,d-13\,b\,c)}{63\,a^2}+\frac{x^4\left(63\,e\,a^2-99\,d\,a\,b+143\,c\,b^2\right)}{315\,a^3}+\frac{b\,x^8\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{15\,a^5}+\frac{5\,b^2\,x^{10}\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{24\,a^6}+\frac{b^3\,x^{12}\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^7}}{a^2\,x^9+2\,a\,b\,x^{11}+b^2\,x^{13}}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x)
```

```
[Out] - (c/(9*a) - (x^6*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(105*a^
4) + (x^2*(9*a*d - 13*b*c))/(63*a^2) + (x^4*(143*b^2*c + 63*a^2*e - 99*a*b*
d))/(315*a^3) + (b*x^8*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(1
5*a^5) + (5*b^2*x^10*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(24*
a^6) + (b^3*x^12*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^7))
/(a^2*x^9 + b^2*x^13 + 2*a*b*x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(14
3*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^(15/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.139 $\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be\right.}{3b^6}$$

Rubi [A] time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}}+\frac{a\left(-2b^3c+3ab^2d-4a^2be+5\right.}{b^5}\right.\right. \\
&= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6}-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a\right.}{3b^6}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-2ab^4x^2\left(462c+297dx^2+220ex^4+175fx^6\right)+b^5x^4\left(693c+5\left(99dx^2+77ex^4+63fx^6\right)\right)\right)}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

IntegrateAlgebraic [A] time = 0.11, size = 196, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(-1280a^5f+1408a^4be+640a^4bfx^2-1584a^3b^2d-704a^3b^2ex^2-480a^3b^2fx^4+1848a^2b^3c+792a^2b^3dx^2+528a^2b^3ex^4+400a^2b^3fx^6-924ab^4cx^2-594ab^4dx^4-440ab^4ex^6-350ab^4fx^8+693b^5cx^4+495b^5dx^6+385b^5ex^8+315b^5fx^{10}\right)}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*e - 1280*a^5
*f - 924*a*b^4*c*x^2 + 792*a^2*b^3*d*x^2 - 704*a^3*b^2*e*x^2 + 640*a^4*b*f*
x^2 + 693*b^5*c*x^4 - 594*a*b^4*d*x^4 + 528*a^2*b^3*e*x^4 - 480*a^3*b^2*f*x
^4 + 495*b^5*d*x^6 - 440*a*b^4*e*x^6 + 400*a^2*b^3*f*x^6 + 385*b^5*e*x^8 -
350*a*b^4*f*x^8 + 315*b^5*f*x^10))/(3465*b^6)
```

fricas [A] time = 1.03, size = 177, normalized size = 0.83

$$\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4be-1280\,a^5f+3\left(231\,b^5c-198\,ab^4d+176\,a^2b^3e-160\,a^3b^2f\right)x^4-4\left(231\,ab^4c-198\,a^2b^3d+176\,a^3b^2e-160\,a^4bf\right)x^2\right)\sqrt{bx^2+a}}{3465\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

giac [A] time = 0.45, size = 264, normalized size = 1.23

$$\frac{\left(a^{2} b^{3} c - a^{3} b^{2} d - a^{5} f + a^{4} b e\right) \sqrt{b x^{2} + a}}{b^{6}} + \frac{693 \left(b x^{2} + a\right)^{\frac{5}{2}} b^{3} c - 2310 \left(b x^{2} + a\right)^{\frac{3}{2}} a b^{3} c + 495 \left(b x^{2} + a\right)^{\frac{7}{2}} b^{2} d - 2079 \left(b x^{2} + a\right)^{\frac{5}{2}} a b^{2} d + 3465 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{2} b^{2} d + 315 \left(b x^{2} + a\right)^{\frac{11}{2}} f - 1925 \left(b x^{2} + a\right)^{\frac{9}{2}} a f + 4950 \left(b x^{2} + a\right)^{\frac{7}{2}} a^{2} f - 6930 \left(b x^{2} + a\right)^{\frac{5}{2}} a^{3} f + 5775 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{4} f + 385 \left(b x^{2} + a\right)^{\frac{9}{2}} b e - 1980 \left(b x^{2} + a\right)^{\frac{7}{2}} a b e + 4158 \left(b x^{2} + a\right)^{\frac{5}{2}} a^{2} b e - 4620 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{3} b e}{3465 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

maple [A] time = 0.01, size = 193, normalized size = 0.90

$$-\frac{\sqrt{b \,x^{2}+a}\, \left(-315 f \,x^{10} b^{5}+350 a \,b^{4} f \,x^{8}-385 b^{5} e \,x^{8}-400 a^{2} b^{3} f \,x^{6}+440 a \,b^{4} e \,x^{6}-495 b^{5} d \,x^{6}+480 a^{3} b^{2} f \,x^{4}-528 a^{2} b^{3} e \,x^{4}+594 a \,b^{4} d \,x^{4}-693 b^{5} c \,x^{4}-640 a^{4} b f \,x^{2}+704 a^{3} b^{2} e \,x^{2}-792 a^{2} b^{3} d \,x^{2}+924 a \,b^{4} c \,x^{2}+1280 a^{5} f -1408 a^{4} b e +1584 a^{3} b^{2} d -1848 a^{2} b^{3} c \right)}{3465 b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

maxima [A] time = 1.40, size = 347, normalized size = 1.62

$$\frac{\sqrt{b x^{2} + a} f x^{10}}{11 \, b} + \frac{\sqrt{b x^{2} + a} e x^{8}}{9 \, b} - \frac{10 \, \sqrt{b x^{2} + a} a f x^{8}}{99 \, b^{2}} + \frac{\sqrt{b x^{2} + a} d x^{6}}{7 \, b} - \frac{8 \, \sqrt{b x^{2} + a} a e x^{6}}{63 \, b^{2}} + \frac{80 \, \sqrt{b x^{2} + a} a^{2} f x^{6}}{693 \, b^{3}} + \frac{\sqrt{b x^{2} + a} c x^{4}}{5 \, b} - \frac{6 \, \sqrt{b x^{2} + a} a d x^{4}}{35 \, b^{2}} + \frac{16 \, \sqrt{b x^{2} + a} a^{2} e x^{4}}{105 \, b^{3}} - \frac{32 \, \sqrt{b x^{2} + a} a^{3} f x^{4}}{231 \, b^{4}} - \frac{4 \, \sqrt{b x^{2} + a} a c x^{2}}{15 \, b^{2}} + \frac{8 \, \sqrt{b x^{2} + a} a^{2} d x^{2}}{35 \, b^{3}} - \frac{64 \, \sqrt{b x^{2} + a} a^{3} e x^{2}}{315 \, b^{4}} + \frac{128 \, \sqrt{b x^{2} + a} a^{4} f x^{2}}{693 \, b^{5}} + \frac{8 \, \sqrt{b x^{2} + a} a^{2} c}{15 \, b^{3}} - \frac{16 \, \sqrt{b x^{2} + a} a^{3} d}{35 \, b^{4}} + \frac{128 \, \sqrt{b x^{2} + a} a^{4} e}{315 \, b^{5}} - \frac{256 \, \sqrt{b x^{2} + a} a^{5} f}{693 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
```

```
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

mupad [B] time = 1.19, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

sympy [A] time = 4.68, size = 442, normalized size = 2.07

$$\begin{cases} - \frac{256 a^{5} f \sqrt{a + b x^{2}}}{693 b^{6}} + \frac{128 a^{4} e \sqrt{a + b x^{2}}}{315 b^{5}} + \frac{128 a^{4} f x^{2} \sqrt{a + b x^{2}}}{693 b^{5}} - \frac{16 a^{3} d \sqrt{a + b x^{2}}}{35 b^{4}} - \frac{64 a^{3} e x^{2} \sqrt{a + b x^{2}}}{315 b^{4}} - \frac{32 a^{3} f x^{4} \sqrt{a + b x^{2}}}{231 b^{4}} + \frac{8 a^{2} c \sqrt{a + b x^{2}}}{15 b^{3}} + \frac{8 a^{2} d x^{2} \sqrt{a + b x^{2}}}{35 b^{3}} + \frac{16 a^{2} e x^{4} \sqrt{a + b x^{2}}}{105 b^{3}} + \frac{80 a^{2} f x^{6} \sqrt{a + b x^{2}}}{693 b^{3}} - \frac{4 a c x^{2} \sqrt{a + b x^{2}}}{15 b^{2}} - \frac{6 a d x^{4} \sqrt{a + b x^{2}}}{35 b^{2}} - \frac{8 a e x^{6} \sqrt{a + b x^{2}}}{63 b^{2}} - \frac{10 a f x^{8} \sqrt{a + b x^{2}}}{99 b^{2}} + \frac{c x^{4} \sqrt{a + b x^{2}}}{5 b} + \frac{d x^{6} \sqrt{a + b x^{2}}}{7 b} + \frac{e x^{8} \sqrt{a + b x^{2}}}{9 b} + \frac{f x^{10} \sqrt{a + b x^{2}}}{11 b} & \text{for}\: b \neq 0 \\ \frac{\frac{c x^{6}}{6} + \frac{d x^{8}}{8} + \frac{e x^{10}}{10} + \frac{f x^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.140 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d\right.}{b^5}$$

Rubi [A] time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1799, 1620}

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{7/2}(be-4af)}{7b^5}+\frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\left(c+dx+ex^2+fx^3\right)}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\sqrt{}}{b^4}\right.\right.$$

$$= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+\right.}{3b^5}$$

Mathematica [A] time = 0.12, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(105c+63dx^2+45ex^4+35fx^6\right)\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 148, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(128a^4f-144a^3be-64a^3bfx^2+168a^2b^2d+72a^2b^2ex^2+48a^2b^2fx^4-210ab^3c-84ab^3dx^2-54ab^3ex^4-40ab^3fx^6+105b^4cx^2+63b^4dx^4+45b^4ex^6+35b^4fx^8\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-210*a*b^3*c + 168*a^2*b^2*d - 144*a^3*b*e + 128*a^4*f +
105*b^4*c*x^2 - 84*a*b^3*d*x^2 + 72*a^2*b^2*e*x^2 - 64*a^3*b*f*x^2 + 63*b^4
*d*x^4 - 54*a*b^3*e*x^4 + 48*a^2*b^2*f*x^4 + 45*b^4*e*x^6 - 40*a*b^3*f*x^6
+ 35*b^4*f*x^8))/(315*b^5)
```

fricas [A] time = 1.10, size = 134, normalized size = 0.80

$$\frac{\left(35\,b^4fx^8+5\left(9\,b^4e-8\,ab^3f\right)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\left(21\,b^4d-18\,ab^3e+16\,a^2b^2f\right)x^4+\left(105\,b^4c-84\,ab^3d+72\,a^2b^2e-64\,a^3bf\right)x^2\right)\sqrt{bx^2+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

giac [A] time = 0.51, size = 197, normalized size = 1.18

$$-\frac{\left(ab^3c-a^2b^2d-a^4f+a^3be\right)\sqrt{bx^2+a}}{b^5}+\frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c+63\left(bx^2+a\right)^{\frac{5}{2}}b^2d-210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d+35\left(bx^2+a\right)^{\frac{9}{2}}f-180\left(bx^2+a\right)^{\frac{7}{2}}af+378\left(bx^2+a\right)^{\frac{5}{2}}a^2f-420\left(bx^2+a\right)^{\frac{3}{2}}a^3f+45\left(bx^2+a\right)^{\frac{7}{2}}be-189\left(bx^2+a\right)^{\frac{5}{2}}abe+315\left(bx^2+a\right)^{\frac{3}{2}}a^2be}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

maple [A] time = 0.01, size = 145, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}\,\left(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2+128a^4f-144a^3be+168a^2b^2d-210a\,b^3c\right)}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

maxima [A] time = 1.45, size = 263, normalized size = 1.57

$$\frac{\sqrt{bx^2+a}\,fx^8}{9\,b}+\frac{\sqrt{bx^2+a}\,ex^6}{7\,b}-\frac{8\sqrt{bx^2+a}\,afx^6}{63\,b^2}+\frac{\sqrt{bx^2+a}\,dx^4}{5\,b}-\frac{6\sqrt{bx^2+a}\,aex^4}{35\,b^2}+\frac{16\sqrt{bx^2+a}\,a^2fx^4}{105\,b^3}+\frac{\sqrt{bx^2+a}\,cx^2}{3\,b}-\frac{4\sqrt{bx^2+a}\,adx^2}{15\,b^2}+\frac{8\sqrt{bx^2+a}\,a^2ex^2}{35\,b^3}-\frac{64\sqrt{bx^2+a}\,a^3fx^2}{315\,b^4}-\frac{2\sqrt{bx^2+a}\,ac}{3\,b^2}+\frac{8\sqrt{bx^2+a}\,a^2d}{15\,b^3}-\frac{16\sqrt{bx^2+a}\,a^3e}{35\,b^4}+\frac{128\sqrt{bx^2+a}\,a^4f}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

mupad [B] time = 1.11, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

sympy [A] time = 2.90, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - \frac{6aex^4\sqrt{a+bx^2}}{35b^2} - \frac{8afx^6\sqrt{a+bx^2}}{63b^2} + \frac{cx^2\sqrt{a+bx^2}}{3b} + \frac{dx^4\sqrt{a+bx^2}}{5b} + \frac{ex^6\sqrt{a+bx^2}}{7b} + \frac{fx^8\sqrt{a+bx^2}}{9b} & \text{for}\ b \neq 0 \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

3.141 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}$$

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1799, 1850}

$$\frac{\sqrt{a+bx^2}\,(a^2be+a^3(-f)-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}} + \frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3} + \frac{(be-\cdots}{}\right.\right.$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4} + \frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(be\cdots}{}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b\left(7e+3fx^2\right)-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 102, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+56a^2be+24a^2bfx^2-70ab^2d-28ab^2ex^2-18ab^2fx^4+105b^3c+35b^3dx^2+21b^3ex^4+15b^3fx^6\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 35*b^3*d
*x^2 - 28*a*b^2*e*x^2 + 24*a^2*b*f*x^2 + 21*b^3*e*x^4 - 18*a*b^2*f*x^4 + 15
*b^3*f*x^6))/(105*b^4)
```

fricas [A] time = 0.98, size = 94, normalized size = 0.78

$$\frac{\left(15\,b^3fx^6+3\left(7\,b^3e-6\,ab^2f\right)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+\left(35\,b^3d-28\,ab^2e+24\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

giac [A] time = 0.41, size = 130, normalized size = 1.07

$$\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\sqrt{bx^2+a}}{b^4}+\frac{35\left(bx^2+a\right)^{\frac{3}{2}}b^2d+15\left(bx^2+a\right)^{\frac{7}{2}}f-63\left(bx^2+a\right)^{\frac{5}{2}}af+105\left(bx^2+a\right)^{\frac{3}{2}}a^2f+21\left(bx^2+a\right)^{\frac{5}{2}}be-70\left(bx^2+a\right)^{\frac{3}{2}}abe}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

maple [A] time = 0.00, size = 99, normalized size = 0.82

$$-\frac{\sqrt{b\,x^2+a}\;\left(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c\right)}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

maxima [A] time = 1.35, size = 180, normalized size = 1.49

$$\frac{\sqrt{bx^2+a}\,fx^6}{7\,b}+\frac{\sqrt{bx^2+a}\,ex^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}\,afx^4}{35\,b^2}+\frac{\sqrt{bx^2+a}\,dx^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}\,aex^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2fx^2}{35\,b^3}+\frac{\sqrt{bx^2+a}\,c}{b}-\frac{2\,\sqrt{bx^2+a}\,ad}{3\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2e}{15\,b^3}-\frac{16\,\sqrt{bx^2+a}\,a^3f}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

mupad [B] time = 1.06, size = 103, normalized size = 0.85

$$\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-18\,a\,b^2\,f\right)}{105\,b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

sympy [A] time = 2.19, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4} + \frac{8a^2e\sqrt{a+bx^2}}{15b^3} + \frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3} - \frac{2ad\sqrt{a+bx^2}}{3b^2} - \frac{4aex^2\sqrt{a+bx^2}}{15b^2} - \frac{6afx^4\sqrt{a+bx^2}}{35b^2} + \frac{c\sqrt{a+bx^2}}{b} + \frac{dx^2\sqrt{a+bx^2}}{3b} + \frac{ex^4\sqrt{a+bx^2}}{5b} + \frac{fx^6\sqrt{a+bx^2}}{7b} & \text{for } b \neq 0 \\ \frac{\frac{cx^2}{2} + \frac{dx^4}{4} + \frac{ex^6}{6} + \frac{fx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{a+bx^2}\,\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1799, 1620, 63, 208}

$$\frac{\sqrt{a+bx^2}\,\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^2\sqrt{a+bx}} + \frac{c}{x\sqrt{a+bx}} + \frac{(be-2af)\sqrt{a+bx}}{b^2} + \frac{f(a+bx)^{3/2}}{b^2}\right)\right.$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3} + \frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{f\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{1}{2}c\operatorname{Subs}$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3} + \frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{f\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{c\operatorname{Subst}}{}$$

$$= \frac{\left(b^2d-abe+a^2f\right)\sqrt{a+bx^2}}{b^3} + \frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{f\left(a+bx^2\right)^{5/2}}{5b^3} - \frac{c\tanh^{-1}}{}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(8a^2f-2ab\left(5e+2fx^2\right)+b^2\left(15d+5ex^2+3fx^4\right)\right)}{15b^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 + 3
*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.09, size = 89, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(8a^2f-10abe-4abfx^2+15b^2d+5b^2ex^2+3b^2fx^4\right)}{15b^3} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(15*b^2*d - 10*a*b*e + 8*a^2*f + 5*b^2*e*x^2 - 4*a*b*f*x^2
 + 3*b^2*f*x^4))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.03, size = 205, normalized size = 1.99

$$\left[\frac{15\,\sqrt{a}\,b^3c\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,ab^2fx^4+15\,ab^2d-10\,a^2be+8\,a^3f+\left(5\,ab^2e-4\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{30\,ab^3},\ \frac{15\,\sqrt{-a}\,b^3c\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,ab^2fx^4+15\,ab^2d-10\,a^2be+8\,a^3f+\left(5\,ab^2e-4\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{15\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f + (5*a*b^2*e - 4*a^
2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f +
(5*a*b^2*e - 4*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

giac [A] time = 0.40, size = 127, normalized size = 1.23

$$\frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\,\sqrt{bx^2+a}\,b^{14}d+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15\,\sqrt{bx^2+a}\,a^2b^{12}f+5\left(bx^2+a\right)^{\frac{3}{2}}b^{13}e-15\,\sqrt{bx^2+a}\,ab^{13}e}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^14
*d + 3*(b*x^2 + a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b
*x^2 + a)*a^2*b^12*f + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^
13*e)/b^15
```

maple [A] time = 0.01, size = 134, normalized size = 1.30

$$\frac{\sqrt{b\,x^2+a}\;f\,x^4}{5b}-\frac{4\sqrt{b\,x^2+a}\;af\,x^2}{15b^2}+\frac{\sqrt{b\,x^2+a}\;e\,x^2}{3b}-\frac{c\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{8\sqrt{b\,x^2+a}\;a^2f}{15b^3}-\frac{2\sqrt{b\,x^2+a}\;ae}{3b^2}+\frac{\sqrt{b\,x^2+a}\;d}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*f*x^4/b*(b*x^2+a)^(1/2)-4/15*f*a/b^2*x^2*(b*x^2+a)^(1/2)+8/15*f*a^2/b^3
*(b*x^2+a)^(1/2)+1/3*e*x^2/b*(b*x^2+a)^(1/2)-2/3*e*a/b^2*(b*x^2+a)^(1/2)+d/
b*(b*x^2+a)^(1/2)-c/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 122, normalized size = 1.18

$$\frac{\sqrt{bx^2+a}\,fx^4}{5\,b}+\frac{\sqrt{bx^2+a}\,ex^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}\,afx^2}{15\,b^2}-\frac{c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,d}{b}-\frac{2\,\sqrt{bx^2+a}\,ae}{3\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2f}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*f*x^4/b + 1/3*sqrt(b*x^2 + a)*e*x^2/b - 4/15*sqrt(b*x^2
 + a)*a*f*x^2/b^2 - c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 +
a)*d/b - 2/3*sqrt(b*x^2 + a)*a*e/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*f/b^3
```

mupad [B] time = 1.81, size = 99, normalized size = 0.96

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2\,f}{15\,b^3}+\frac{f\,x^4}{5\,b}-\frac{4\,a\,f\,x^2}{15\,b^2}\right)+\frac{d\,\sqrt{b\,x^2+a}}{b}-\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{e\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2*f)/(15*b^3) + (f*x^4)/(5*b) - (4*a*f*x^2)/(15*b^2
)) + (d*(a + b*x^2)^(1/2))/b - (c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
 - (e*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 37.87, size = 102, normalized size = 0.99

$$\frac{f\left(a+bx^2\right)^{\frac{5}{2}}}{5b^3}-\frac{\left(a+bx^2\right)^{\frac{3}{2}}\left(2af-be\right)}{3b^3}+\frac{\sqrt{a+bx^2}\left(a^2f-abe+b^2d\right)}{b^3}+\frac{c\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f - b*e)/(3*b**3)
 + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3 + c*atan(1/(sqrt(-1/a)*s
qrt(a + b*x**2)))/(a*sqrt(-1/a))
```

3.143 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\,(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1799, 1621, 897, 1153, 208}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\,(be-af)}{b^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-aex-afx^2}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(bc-2ad)+a^2be-a^3f}{b^2}-\frac{\left(abe-2a^2f\right)x^2}{b^2}-\frac{afx^4}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \left(-a\left(e-\frac{af}{b}\right)-\frac{afx^2}{b}+\frac{bc-2ad}{2\left(-\frac{a}{b}+\frac{x^2}{b}\right)}\right)dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f\left(a+bx^2\right)^{3/2}}{3b^2} - \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{2ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.41, size = 131, normalized size = 1.31

$$\frac{3b^3cx^2\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)-\left(a+bx^2\right)\left(4a^2fx^2-2abx^2\left(3e+fx^2\right)+3b^2c\right)}{6ab^2x^2\sqrt{a+bx^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -((d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) + (-((a + b*x^2)*(3*b^2*c +
 4*a^2*f*x^2 - 2*a*b*x^2*(3*e + f*x^2))) + 3*b^3*c*x^2*Sqrt[1 + (b*x^2)/a]*
ArcTanh[Sqrt[1 + (b*x^2)/a]])/(6*a*b^2*x^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.19, size = 92, normalized size = 0.92

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\left(-4a^2fx^2+6abex^2+2abfx^4-3b^2c\right)}{6ab^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*b^2*c + 6*a*b*e*x^2 - 4*a^2*f*x^2 + 2*a*b*f*x^4))/(6*a
*b^2*x^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

fricas [A] time = 0.93, size = 210, normalized size = 2.10

$$\left[-\frac{3\left(b^3c-2ab^2d\right)\sqrt{a}x^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(2a^2bfx^4-3ab^2c+2\left(3a^2be-2a^3f\right)x^2\right)\sqrt{bx^2+a}}{12a^2b^2x^2},\ -\frac{3\left(b^3c-2ab^2d\right)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2a^2bfx^4-3ab^2c+2\left(3a^2be-2a^3f\right)x^2\right)\sqrt{bx^2+a}}{6a^2b^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^2*b*e - 2*a^3*f)
*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-a)*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^
2*b*e - 2*a^3*f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

giac [A] time = 0.55, size = 114, normalized size = 1.14

$$-\frac{\frac{3\left(b^2c-2abd\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\sqrt{bx^2+a}\,bc}{ax^2}-\frac{2\left(\left(bx^2+a\right)^{\frac{3}{2}}b^2f-3\sqrt{bx^2+a}\,ab^2f+3\sqrt{bx^2+a}\,b^3e\right)}{b^3}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 3
*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*((b*x^2 + a)^(3/2)*b^2*f - 3*sqrt(b*x^2 +
a)*a*b^2*f + 3*sqrt(b*x^2 + a)*b^3*e)/b^3)/b
```

maple [A] time = 0.01, size = 127, normalized size = 1.27

$$\frac{\sqrt{b\,x^2+a}\;f\,x^2}{3b}-\frac{d\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bc\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{2\sqrt{b\,x^2+a}\;af}{3b^2}+\frac{\sqrt{b\,x^2+a}\;e}{b}-\frac{\sqrt{b\,x^2+a}\;c}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*f*x^2/b*(b*x^2+a)^(1/2)-2/3*f*a/b^2*(b*x^2+a)^(1/2)+e/b*(b*x^2+a)^(1/2)
-1/2*c*(b*x^2+a)^(1/2)/a/x^2+1/2*c*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-d/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.33, size = 104, normalized size = 1.04

$$\frac{\sqrt{bx^2+a}\,f x^2}{3\,b}+\frac{bc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,e}{b}-\frac{2\,\sqrt{bx^2+a}\,af}{3\,b^2}-\frac{\sqrt{bx^2+a}\,c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*f*x^2/b + 1/2*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2)
 - d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*e/b - 2/3*sqrt
(b*x^2 + a)*a*f/b^2 - 1/2*sqrt(b*x^2 + a)*c/(a*x^2)
```

mupad [B] time = 1.95, size = 99, normalized size = 0.99

$$\frac{e\,\sqrt{b\,x^2+a}}{b}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{f\,\sqrt{b\,x^2+a}\;\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (e*(a + b*x^2)^(1/2))/b - (d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (c
*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a
^(3/2)) - (f*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 127.43, size = 138, normalized size = 1.38

$$e\left(\begin{cases} \frac{x^2}{2\sqrt{a}} & \text{for } b = 0 \\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise} \end{cases}\right) + f\left(\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}\right) - \frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + f*P
iecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(
b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x)
- d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

3.144 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a+bx^2}\,(3bc-4ad)}{8a^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Rubi [A] time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 388, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2e-4abd+3b^2c\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}\,(3bc-4ad)}{8a^2x^2}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc-4ad)-2aex-2afx^2}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(3bc-4ad)+2a^2be-2a^3f}{b^2} - \frac{(2abe-4a^2f)x^2}{b^2} - \frac{2afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{2ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3bc+4ad-\frac{8a^2e}{b}+\frac{8a^3f}{b^2}\right)-\frac{4a^2fx^2}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{4a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3bc-4ad+\frac{8a^2e}{b}\right)\text{Subst}\left(\int\right.}{8a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{(3b^2c-4abd+8a^2e)\tanh^{-1}\left(\right.}{8a^{5/2}}$$

Mathematica [C] time = 0.37, size = 141, normalized size = 1.24

$$-\frac{b^2c\sqrt{a+bx^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3} - \frac{bd\sqrt{a+bx^2}\left(\frac{a}{bx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2} - \frac{e\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (e*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] - (b*d
*Sqrt[a + b*x^2]*(a/(b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)
/a]))/(2*a^2) - (b^2*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (
b*x^2)/a])/a^3
```

IntegrateAlgebraic [A] time = 0.19, size = 102, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(8a^2fx^4-2abc-4abdx^2+3b^2cx^2\right)}{8a^2bx^4} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-8a^2e+4abd-3b^2c\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 3*b^2*c*x^2 - 4*a*b*d*x^2 + 8*a^2*f*x^4))/(8*a
^2*b*x^4) + ((-3*b^2*c + 4*a*b*d - 8*a^2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(8*a^(5/2))
```

fricas [A] time = 1.08, size = 221, normalized size = 1.94

$$\left[\frac{\left(3\,b^{3}c-4\,ab^{2}d+8\,a^{2}be\right)\sqrt{a}\,x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(8\,a^{3}fx^{4}-2\,a^{2}bc+\left(3\,ab^{2}c-4\,a^{2}bd\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,a^{3}bx^{4}},\ \frac{\left(3\,b^{3}c-4\,ab^{2}d+8\,a^{2}be\right)\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(8\,a^{3}fx^{4}-2\,a^{2}bc+\left(3\,ab^{2}c-4\,a^{2}bd\right)x^{2}\right)\sqrt{bx^{2}+a}}{8\,a^{3}bx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2*c - 4
*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((3*b^3*c - 4*a*b^2*d + 8*
a^2*b*e)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3*f*x^4 - 2*a
^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

giac [A] time = 0.40, size = 141, normalized size = 1.24

$$\frac{8\,\sqrt{bx^{2}+a}\,f+\frac{\left(3\,b^{3}c-4\,ab^{2}d+8\,a^{2}be\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}c-5\,\sqrt{bx^{2}+a}\,ab^{3}c-4\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}d+4\,\sqrt{bx^{2}+a}\,a^{2}b^{2}d}{a^{2}b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b*
x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b*x
^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^2*d
)/(a^2*b^2*x^4))/b
```

maple [A] time = 0.01, size = 162, normalized size = 1.42

$$-\frac{e\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bd\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^{2}c\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{2}+a}\;f}{b}-\frac{\sqrt{b\,x^{2}+a}\;d}{2a\,x^{2}}+\frac{3\sqrt{b\,x^{2}+a}\;bc}{8a^{2}x^{2}}-\frac{\sqrt{b\,x^{2}+a}\;c}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] f*(b*x^2+a)^(1/2)/b-1/2*d/a/x^2*(b*x^2+a)^(1/2)+1/2*d*b/a^(3/2)*ln((2*a+2*(
b*x^2+a)^(1/2)*a^(1/2))/x)-1/4*c*(b*x^2+a)^(1/2)/a/x^4+3/8*c/a^2*b/x^2*(b*x
^2+a)^(1/2)-3/8*c/a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-e/a^(1/
2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 128, normalized size = 1.12

$$-\frac{3\,b^{2}c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bd\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{e\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^{2}+a}\,f}{b}+\frac{3\,\sqrt{bx^{2}+a}\,bc}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}\,d}{2\,ax^{2}}-\frac{\sqrt{bx^{2}+a}\,c}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*d*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(3/2) - e*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^
2 + a)*f/b + 3/8*sqrt(b*x^2 + a)*b*c/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*d/(a*x
^2) - 1/4*sqrt(b*x^2 + a)*c/(a*x^4)
```

mupad [B] time = 2.19, size = 133, normalized size = 1.17

$$\frac{f\,\sqrt{b\,x^2+a}}{b}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,c\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{d\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*(a + b*x^2)^(1/2))/b - (e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5
*c*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*c*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (d*
(a + b*x^2)^(1/2))/(2*a*x^2) + (b*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^
(3/2)) - (3*b^2*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{a+bx^2}\,(5bc-6ad)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\,\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{c\sqrt{a\cdots}}{6\cdots}$$

Rubi [A] time = 0.28, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1799, 1621, 897, 1157, 385, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2be-16a^3f-6ab^2d+5b^3c\right)}{16a^{7/2}}-\frac{\sqrt{a+bx^2}\,\left(8a^2e-6abd+5b^2c\right)}{16a^3x^2}+\frac{\sqrt{a+bx^2}\,(5bc-6ad)}{24a^2x^4}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5*b
^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(16*a^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc-6ad)-3aex-3afx^2}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(5bc-6ad)+3a^2be-3a^3f}{b^2}-\frac{(3abe-6a^2f)x^2}{b^2}-\frac{3afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+bx^2}\right)}{3ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)-\frac{12a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \right.}{12a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(b^2\left(\frac{12a^3f}{b^3}\right.\right.}{}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + \frac{(5b^3c-6}{}$$

Mathematica [C] time = 1.02, size = 162, normalized size = 1.11

$$\frac{b^3c\sqrt{a+bx^2}\;{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^4} - \frac{b^2d\sqrt{a+bx^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3} - \frac{be\sqrt{a+bx^2}\left(\frac{a}{bx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a^2} - \frac{f\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -((f*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]) - (b*e*Sqrt[a + b*x^2]*(a/(
b*x^2) - ArcTanh[Sqrt[1 + (b*x^2)/a]]/Sqrt[1 + (b*x^2)/a]))/(2*a^2) - (b^2*
d*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3 + (b^3
*c*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a])/a^4
```

IntegrateAlgebraic [A] time = 0.25, size = 126, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(-8a^2c-12a^2dx^2-24a^2ex^4+10abcx^2+18abdx^4-15b^2cx^4\right)}{48a^3x^6} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*c + 10*a*b*c*x^2 - 12*a^2*d*x^2 - 15*b^2*c*x^4 + 1
8*a*b*d*x^4 - 24*a^2*e*x^4))/(48*a^3*x^6) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b
*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

fricas [A]   time = 1.15, size = 261, normalized size = 1.79

$$\left[-\frac{3\left(5\,b^{3}c-6\,ab^{2}d+8\,a^{2}be-16\,a^{3}f\right)\sqrt{a}\,x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(3\left(5\,ab^{2}c-6\,a^{2}bd+8\,a^{3}e\right)x^{4}+8\,a^{3}c-2\left(5\,a^{2}bc-6\,a^{3}d\right)x^{2}\right)\sqrt{bx^{2}+a}}{96\,a^{4}x^{6}},\ -\frac{3\left(5\,b^{3}c-6\,ab^{2}d+8\,a^{2}be-16\,a^{3}f\right)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3\left(5\,ab^{2}c-6\,a^{2}bd+8\,a^{3}e\right)x^{4}+8\,a^{3}c-2\left(5\,a^{2}bc-6\,a^{3}d\right)x^{2}\right)\sqrt{bx^{2}+a}}{48\,a^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(a)*x^6*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(5*a*b^2*c - 6*a^2*b*d +
 8*a^3*e)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^
4*x^6), -1/48*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(-a)*x^6*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(5*a*b^2*c - 6*a^2*b*d + 8*a^3*e)*x^4
 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

giac [A]   time = 0.39, size = 232, normalized size = 1.59

$$-\frac{\frac{3\left(5\,b^{4}c-6\,ab^{3}d-16\,a^{3}bf+8\,a^{2}b^{2}e\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}+\frac{15\left(bx^{2}+a\right)^{\frac{5}{2}}b^{4}c-40\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{4}c+33\,\sqrt{bx^{2}+a}\,a^{2}b^{4}c-18\left(bx^{2}+a\right)^{\frac{5}{2}}ab^{3}d+48\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b^{3}d-30\,\sqrt{bx^{2}+a}\,a^{3}b^{3}d+24\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}b^{2}e-48\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}b^{2}e+24\,\sqrt{bx^{2}+a}\,a^{4}b^{2}e}{a^{3}b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d - 16*a^3*b*f + 8*a^2*b^2*e)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2 +
a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a*b^
3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b
*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2
+ a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

maple [A]   time = 0.01, size = 238, normalized size = 1.63

$$-\frac{f\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{be\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^{2}d\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^{3}c\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{7}{2}}}-\frac{\sqrt{b\,x^{2}+a}\,e}{2a\,x^{2}}+\frac{3\sqrt{b\,x^{2}+a}\,bd}{8a^{2}x^{2}}-\frac{5\sqrt{b\,x^{2}+a}\,b^{2}c}{16a^{3}x^{2}}-\frac{\sqrt{b\,x^{2}+a}\,d}{4a\,x^{4}}+\frac{5\sqrt{b\,x^{2}+a}\,bc}{24a^{2}x^{4}}-\frac{\sqrt{b\,x^{2}+a}\,c}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*e/a/x^2*(b*x^2+a)^(1/2)+1/2*e*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+5/24*c/a^2*b/x^4*(b*x^2+a)^(1/2)-5/16*c
/a^3*b^2/x^2*(b*x^2+a)^(1/2)+5/16*c/a^(7/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a
^(1/2))/x)-1/4*d/a/x^4*(b*x^2+a)^(1/2)+3/8*d/a^2*b/x^2*(b*x^2+a)^(1/2)-3/8*
d/a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-f/a^(1/2)*ln((2*a+2*(b*
x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.39, size = 193, normalized size = 1.32

$$\frac{5\,b^3c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{3\,b^2d\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{be\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{f\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}} - \frac{5\,\sqrt{bx^2+a}\,b^2c}{16\,a^3x^2} + \frac{3\,\sqrt{bx^2+a}\,bd}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}\,e}{2\,ax^2} + \frac{5\,\sqrt{bx^2+a}\,bc}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}\,d}{4\,ax^4} - \frac{\sqrt{bx^2+a}\,c}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*d*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(5/2) + 1/2*b*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - f
*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - 5/16*sqrt(b*x^2 + a)*b^2*c/(a^3*x^
2) + 3/8*sqrt(b*x^2 + a)*b*d/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*e/(a*x^2) + 5/
24*sqrt(b*x^2 + a)*b*c/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*d/(a*x^4) - 1/6*sqrt
(b*x^2 + a)*c/(a*x^6)
```

mupad [B] time = 2.54, size = 199, normalized size = 1.36

$$\frac{5\,c\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6} - \frac{11\,c\,\sqrt{b\,x^2+a}}{16\,a\,x^6} - \frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{5\,c\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6} - \frac{5\,d\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,d\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4} - \frac{e\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{3\,b^2\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} - \frac{b^3\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^7*(a + b*x^2)^(1/2)),x)
```

```
[Out] (5*c*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (11*c*(a + b*x^2)^(1/2))/(16*a*x^6) -
 (f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5*c*(a + b*x^2)^(5/2))/(16
*a^3*x^6) - (5*d*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*d*(a + b*x^2)^(3/2))/(8*
a^2*x^4) - (e*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*e*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/(2*a^(3/2)) - (b^3*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*
a^(7/2)) - (3*b^2*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{a+bx^2}\,(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\,\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-64a^3f+48a^2be-40ab^2d+35\right.}{128a^{9/2}}$$

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1799, 1621, 897, 1157, 385, 199, 208}

$$\frac{\sqrt{a+bx^2}\,\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^4x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(48a^2be-64a^3f-40ab^2d+35b^3c\right)}{128a^{9/2}}-\frac{\sqrt{a+bx^2}\,\left(48a^2e-40abd+35b^2c\right)}{192a^3x^4}+\frac{\sqrt{a+bx^2}\,(7bc-8ad)}{48a^2x^6}-\frac{c\sqrt{a+bx^2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(8*a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^2*
x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) + (
(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4*x
^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(7bc-8ad)-4aex-4afx^2}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(7bc-8ad)+4a^2be-4a^3f}{b^2}-\frac{(4abe-8a^2f)x^2}{b^2}-\frac{4afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^4}\,dx, x, \sqrt{a+bx^2}\right)}{4ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)-\frac{24a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx \ldots}{24a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \ldots$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \ldots$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} + \ldots$$

Mathematica [C] time = 0.34, size = 140, normalized size = 0.72

$$\frac{b\sqrt{a+bx^2}\left(-\frac{a^4f}{bx^2}+\frac{a^3f\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}-2a^2be\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)-2b^3c\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{bx^2}{a}+1\right)+2ab^2d\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx^2}{a}+1\right)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]

[Out] (b*Sqrt[a + b*x^2]*(-((a^4*f)/(b*x^2)) + (a^3*f*ArcTanh[Sqrt[1 + (b*x^2)/a]
])/Sqrt[1 + (b*x^2)/a] - 2*a^2*b*e*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^
2)/a] + 2*a*b^2*d*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^2)/a] - 2*b^3*c*H
ypergeometric2F1[1/2, 5, 3/2, 1 + (b*x^2)/a]))/(2*a^5)
```

IntegrateAlgebraic [A] time = 0.40, size = 172, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(64a^3bf-48a^2b^2e+40ab^3d-35b^4c\right)}{128a^{9/2}}+\frac{\sqrt{a+bx^2}\left(-48a^3c-64a^3dx^2-96a^3ex^4-192a^3fx^6+56a^2bcx^2+80a^2bdx^4+144a^2bex^6-70ab^2cx^4-120ab^2dx^6+105b^3cx^6\right)}{384a^4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*c + 56*a^2*b*c*x^2 - 64*a^3*d*x^2 - 70*a*b^2*c*x^
4 + 80*a^2*b*d*x^4 - 96*a^3*e*x^4 + 105*b^3*c*x^6 - 120*a*b^2*d*x^6 + 144*a
^2*b*e*x^6 - 192*a^3*f*x^6))/(384*a^4*x^8) + ((-35*b^4*c + 40*a*b^3*d - 48*
a^2*b^2*e + 64*a^3*b*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(9/2))
```

fricas [A] time = 1.40, size = 341, normalized size = 1.75

$$\left[-\frac{3\left(35b^4c-40ab^3d+48a^2b^2e-64a^3bf\right)\sqrt{a}x^8\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3\left(35ab^3c-40a^2b^2d+48a^3be-64a^4f\right)x^6-48a^4c-2\left(35a^2b^2c-40a^3bd+48a^4e\right)x^4+8\left(7a^3bc-8a^4d\right)x^2\right)\sqrt{bx^2+a}}{768a^5x^8},\ \frac{3\left(35b^4c-40ab^3d+48a^2b^2e-64a^3bf\right)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\left(35ab^3c-40a^2b^2d+48a^3be-64a^4f\right)x^6-48a^4c-2\left(35a^2b^2c-40a^3bd+48a^4e\right)x^4+8\left(7a^3bc-8a^4d\right)x^2\right)\sqrt{bx^2+a}}{384a^5x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(a)*x^8*
log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(35*a*b^3*c - 40
*a^2*b^2*d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a
^3*b*d + 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5
*x^8), 1/384*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(-a
)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c - 40*a^2*b^2*d + 48
*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a^3*b*d + 48*a^4
*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8)]
```

giac [B] time = 0.40, size = 361, normalized size = 1.85

$$\frac{\frac{3\left(35b^5c-40ab^4d-64a^3b^2f+48a^2b^3e\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\left(bx^2+a\right)^{\frac{7}{2}}b^5c-385\left(bx^2+a\right)^{\frac{5}{2}}ab^5c+511\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5c-279\sqrt{bx^2+a}a^3b^5c-120\left(bx^2+a\right)^{\frac{7}{2}}ab^4d+440\left(bx^2+a\right)^{\frac{5}{2}}a^2b^4d-584\left(bx^2+a\right)^{\frac{3}{2}}a^3b^4d+264\sqrt{bx^2+a}a^4b^4d-192\left(bx^2+a\right)^{\frac{7}{2}}a^3b^2f+576\left(bx^2+a\right)^{\frac{5}{2}}a^4b^2f-576\left(bx^2+a\right)^{\frac{3}{2}}a^5b^2f+192\sqrt{bx^2+a}a^6b^2f+144\left(bx^2+a\right)^{\frac{7}{2}}a^2b^3e-528\left(bx^2+a\right)^{\frac{5}{2}}a^3b^3e+624\left(bx^2+a\right)^{\frac{3}{2}}a^4b^3e-240\sqrt{bx^2+a}a^5b^3e}{a^4b^4x^8}}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d - 64*a^3*b^2*f + 48*a^2*b^3*e)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*(b
*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x^2
+ a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*
b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d - 1
92*(b*x^2 + a)^(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x
^2 + a)^(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f + 144*(b*x^2 + a)^(
```

```
7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x^2 + a)^(3/2)*a^
4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e)/(a^4*b^4*x^8))/b
```

maple [A] time = 0.02, size = 320, normalized size = 1.64

$$\frac{bf\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3d\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{7}{2}}}-\frac{35b^4c\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{9}{2}}}-\frac{\sqrt{bx^2+a}\,f}{2ax^2}+\frac{3\sqrt{bx^2+a}\,be}{8a^2x^2}-\frac{5\sqrt{bx^2+a}\,b^2d}{16a^3x^2}+\frac{35\sqrt{bx^2+a}\,b^3c}{128a^4x^2}-\frac{\sqrt{bx^2+a}\,e}{4ax^4}+\frac{5\sqrt{bx^2+a}\,bd}{24a^2x^4}-\frac{35\sqrt{bx^2+a}\,b^2c}{192a^3x^4}-\frac{\sqrt{bx^2+a}\,d}{6ax^6}+\frac{7\sqrt{bx^2+a}\,bc}{48a^2x^6}-\frac{\sqrt{bx^2+a}\,c}{8ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/8*c*(b*x^2+a)^(1/2)/a/x^8+7/48*c/a^2*b/x^6*(b*x^2+a)^(1/2)-35/192*c/a^3*
b^2/x^4*(b*x^2+a)^(1/2)+35/128*c/a^4*b^3/x^2*(b*x^2+a)^(1/2)-35/128*c/a^(9/
2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/2*f/a/x^2*(b*x^2+a)^(1/2)+1/
2*f*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/6*d/a/x^6*(b*x^2+a)^(
1/2)+5/24*d/a^2*b/x^4*(b*x^2+a)^(1/2)-5/16*d/a^3*b^2/x^2*(b*x^2+a)^(1/2)+5/
16*d/a^(7/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/4*e/a/x^4*(b*x^2+a
)^(1/2)+3/8*e/a^2*b/x^2*(b*x^2+a)^(1/2)-3/8*e/a^(5/2)*b^2*ln((2*a+2*(b*x^2+
a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 275, normalized size = 1.41

$$-\frac{35\,b^4c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{9}{2}}}+\frac{5\,b^3d\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}-\frac{3\,b^2e\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bf\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}+\frac{35\,\sqrt{bx^2+a}\,b^3c}{128\,a^4x^2}-\frac{5\,\sqrt{bx^2+a}\,b^2d}{16\,a^3x^2}+\frac{3\,\sqrt{bx^2+a}\,be}{8\,a^2x^2}-\frac{\sqrt{bx^2+a}\,f}{2\,ax^2}-\frac{35\,\sqrt{bx^2+a}\,b^2c}{192\,a^3x^4}+\frac{5\,\sqrt{bx^2+a}\,bd}{24\,a^2x^4}-\frac{\sqrt{bx^2+a}\,e}{4\,ax^4}+\frac{7\,\sqrt{bx^2+a}\,bc}{48\,a^2x^6}-\frac{\sqrt{bx^2+a}\,d}{6\,ax^6}-\frac{\sqrt{bx^2+a}\,c}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -35/128*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 5/16*b^3*d*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/
2) + 1/2*b*f*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 35/128*sqrt(b*x^2 + a)
*b^3*c/(a^4*x^2) - 5/16*sqrt(b*x^2 + a)*b^2*d/(a^3*x^2) + 3/8*sqrt(b*x^2 +
a)*b*e/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*f/(a*x^2) - 35/192*sqrt(b*x^2 + a)*b
^2*c/(a^3*x^4) + 5/24*sqrt(b*x^2 + a)*b*d/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*e
/(a*x^4) + 7/48*sqrt(b*x^2 + a)*b*c/(a^2*x^6) - 1/6*sqrt(b*x^2 + a)*d/(a*x^
6) - 1/8*sqrt(b*x^2 + a)*c/(a*x^8)
```

mupad [B] time = 2.91, size = 277, normalized size = 1.42

$$\frac{511\,c\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a^2\,x^8}-\frac{93\,c\,\sqrt{b\,x^2+a}}{128\,a\,x^8}-\frac{385\,c\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^3\,x^8}+\frac{35\,c\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^4\,x^8}-\frac{11\,d\,\sqrt{b\,x^2+a}}{16\,a\,x^6}+\frac{5\,d\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{5\,d\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,e\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,e\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{f\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}+\frac{b^4\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{128\,a^{9/2}}-\frac{b^3\,d\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^9*(a + b*x^2)^(1/2)),x)
```

```
[Out] (511*c*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (93*c*(a + b*x^2)^(1/2))/(128*a*x
^8) - (385*c*(a + b*x^2)^(5/2))/(384*a^3*x^8) + (35*c*(a + b*x^2)^(7/2))/(1
```

```
28*a^4*x^8) - (11*d*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*d*(a + b*x^2)^(3/2))
/(6*a^2*x^6) - (5*d*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*e*(a + b*x^2)^(1/2
))/(8*a*x^4) + (3*e*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (f*(a + b*x^2)^(1/2))/
(2*a*x^2) + (b*f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) + (b^4*c*ata
n(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (b^3*d*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*e*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/(8*a^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

3.147 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{480b^3}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)}{256b^{11/2}}-\frac{ax\sqrt{a+bx^2}}{}$$

Rubi [A] time = 0.26, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x^3\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{384b^4}-\frac{ax\sqrt{a+bx^2}\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{256b^5}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(70a^2be-63a^3f-80ab^2d+96b^3c\right)}{256b^{11/2}}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{480b^3}+\frac{x^7\sqrt{a+bx^2}\,(10be-9af)}{80b^2}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2])/(256
*b^5) + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(10bc+10bdx^2+(10be-9af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{10b}$$

$$= \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(80b^2c+\left(80b^2d-70abe+63a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,dx}{80b^2}$$

$$= \frac{\left(80b^2d-70abe+63a^2f\right)x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+}}{10b}$$

$$= \frac{\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x^3\sqrt{a+bx^2}}{384b^4} + \frac{\left(80b^2d-70abe+63a^2f\right)}{480b^3}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a^2\right.}{38}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a^2\right.}{38}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + \frac{\left(96b^3c-80ab^2d+70a^2\right.}{38}$$

Mathematica [A] time = 0.24, size = 184, normalized size = 0.75

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(945a^4f-210a^3b\left(5e+3fx^2\right)+4a^2b^2\left(300d+175ex^2+126fx^4\right)-16ab^3\left(90c+50dx^2+35ex^4+27fx^6\right)+32b^4x^2\left(30c+20dx^2+15ex^4+12fx^6\right)\right)-15a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(63a^3f-70a^2be+80ab^2d-96b^3c\right)}{3840b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^4*f - 210*a^3*b*(5*e + 3*f*x^2) + 4*a^2*b
^2*(300*d + 175*e*x^2 + 126*f*x^4) + 32*b^4*x^2*(30*c + 20*d*x^2 + 15*e*x^4
 + 12*f*x^6) - 16*a*b^3*(90*c + 50*d*x^2 + 35*e*x^4 + 27*f*x^6)) - 15*a^2*(
-96*b^3*c + 80*a*b^2*d - 70*a^2*b*e + 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(3840*b^(11/2))
```

IntegrateAlgebraic [A] time = 0.44, size = 215, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(945a^4fx-1050a^3bex-630a^3bfx^3+1200a^2b^2dx+700a^2b^2ex^3+504a^2b^2fx^5-1440ab^3cx-800ab^3dx^3-560ab^3ex^5-432ab^3fx^7+960b^4cx^3+640b^4dx^5+480b^4ex^7+384b^4fx^9\right)}{3840b^5}+\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)\left(63a^5f-70a^4be+80a^3b^2d-96a^2b^3c\right)}{256b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1440*a*b^3*c*x + 1200*a^2*b^2*d*x - 1050*a^3*b*e*x + 945
*a^4*f*x + 960*b^4*c*x^3 - 800*a*b^3*d*x^3 + 700*a^2*b^2*e*x^3 - 630*a^3*b*
f*x^3 + 640*b^4*d*x^5 - 560*a*b^3*e*x^5 + 504*a^2*b^2*f*x^5 + 480*b^4*e*x^7
 - 432*a*b^3*f*x^7 + 384*b^4*f*x^9))/(3840*b^5) + ((-96*a^2*b^3*c + 80*a^3*
b^2*d - 70*a^4*b*e + 63*a^5*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(256*b^
(11/2))
```

fricas [A] time = 1.44, size = 414, normalized size = 1.69

$$\left[-\frac{15\left(96a^2b^3c-80a^3b^2d+70a^4be-63a^5f\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(384b^5fx^9+48\left(10b^5e-9ab^4f\right)x^7+8\left(80b^5d-70ab^4e+63a^2b^3f\right)x^5+10\left(96b^5c-80ab^4d+70a^2b^3e-63a^3b^2f\right)x^3-15\left(96ab^4c-80a^2b^3d+70a^3b^2e-63a^4bf\right)x\right)\sqrt{bx^2+a}}{7680b^6}, -\frac{15\left(96a^2b^3c-80a^3b^2d+70a^4be-63a^5f\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(384b^5fx^9+48\left(10b^5e-9ab^4f\right)x^7+8\left(80b^5d-70ab^4e+63a^2b^3f\right)x^5+10\left(96b^5c-80ab^4d+70a^2b^3e-63a^3b^2f\right)x^3-15\left(96ab^4c-80a^2b^3d+70a^3b^2e-63a^4bf\right)x\right)\sqrt{bx^2+a}}{3840b^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*b^5*f*x^9 + 48*(10
*b^5*e - 9*a*b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10
*(96*b^5*c - 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c
 - 80*a^2*b^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6, -1/38
40*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 + 48*(10*b^5*e - 9*a*b^4*f)*
x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^
4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d + 70
*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6]
```

giac [A] time = 0.53, size = 224, normalized size = 0.91

$$\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8fx^2}{b}-\frac{9ab^7f-10b^8e}{b^9}\right)x^2+\frac{80b^8d+63a^2b^6f-70ab^7e}{b^9}\right)x^2+\frac{5\left(96b^8c-80ab^7d-63a^3b^5f+70a^2b^6e\right)}{b^9}\right)x^2-\frac{15\left(96ab^7c-80a^2b^6d-63a^4b^4f+70a^3b^5e\right)}{b^9}\right)\sqrt{bx^2+a}\,x-\frac{\left(96a^2b^3c-80a^3b^2d-63a^5f+70a^4be\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b - (9*a*b^7*f - 10*b^8*e)/b^9)*x^2 + (80*b^8*d +
63*a^2*b^6*f - 70*a*b^7*e)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d - 63*a^3*b^5
*f + 70*a^2*b^6*e)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d - 63*a^4*b^4*f
+ 70*a^3*b^5*e)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d
 - 63*a^5*f + 70*a^4*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

maple [A] time = 0.02, size = 368, normalized size = 1.50

$$\frac{\sqrt{bx^2+a}\,fx^9}{10b}-\frac{9\sqrt{bx^2+a}\,afx^7}{80b^2}+\frac{\sqrt{bx^2+a}\,ex^7}{8b}+\frac{21\sqrt{bx^2+a}\,a^2fx^5}{160b^3}-\frac{7\sqrt{bx^2+a}\,aex^5}{48b^2}+\frac{\sqrt{bx^2+a}\,dx^5}{6b}-\frac{21\sqrt{bx^2+a}\,a^3fx^3}{128b^4}+\frac{35\sqrt{bx^2+a}\,a^2ex^3}{192b^3}-\frac{5\sqrt{bx^2+a}\,adx^3}{24b^2}+\frac{\sqrt{bx^2+a}\,cx^3}{4b}-\frac{63a^5f\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{256b^{\frac{11}{2}}}+\frac{35a^4e\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3d\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2c\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{8b^{\frac{5}{2}}}+\frac{63\sqrt{bx^2+a}\,a^4fx}{256b^5}-\frac{35\sqrt{bx^2+a}\,a^3ex}{128b^4}+\frac{5\sqrt{bx^2+a}\,a^2dx}{16b^3}-\frac{3\sqrt{bx^2+a}\,acx}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*f*x^9*(b*x^2+a)^(1/2)/b-9/80*f*a/b^2*x^7*(b*x^2+a)^(1/2)+21/160*f*a^2/
b^3*x^5*(b*x^2+a)^(1/2)-21/128*f*a^3/b^4*x^3*(b*x^2+a)^(1/2)+63/256*f*a^4/b
^5*x*(b*x^2+a)^(1/2)-63/256*f*a^5/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/
8*e*x^7/b*(b*x^2+a)^(1/2)-7/48*e*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*e*a^2/b^3
*x^3*(b*x^2+a)^(1/2)-35/128*e*a^3/b^4*x*(b*x^2+a)^(1/2)+35/128*e*a^4/b^(9/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/6*d*x^5/b*(b*x^2+a)^(1/2)-5/24*d*a/b^2*x^
3*(b*x^2+a)^(1/2)+5/16*d*a^2/b^3*x*(b*x^2+a)^(1/2)-5/16*d*a^3/b^(7/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))+1/4*c*x^3/b*(b*x^2+a)^(1/2)-3/8*c*a/b^2*x*(b*x^2+a
)^(1/2)+3/8*c*a^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 339, normalized size = 1.38

$$\frac{\sqrt{bx^2+a}\,fx^9}{10\,b}+\frac{\sqrt{bx^2+a}\,ex^7}{8\,b}-\frac{9\,\sqrt{bx^2+a}\,afx^7}{80\,b^2}+\frac{\sqrt{bx^2+a}\,dx^5}{6\,b}-\frac{7\,\sqrt{bx^2+a}\,aex^5}{48\,b^2}+\frac{21\,\sqrt{bx^2+a}\,a^2fx^5}{160\,b^3}+\frac{\sqrt{bx^2+a}\,cx^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,adx^3}{24\,b^2}+\frac{35\,\sqrt{bx^2+a}\,a^2ex^3}{192\,b^3}-\frac{21\,\sqrt{bx^2+a}\,a^3fx^3}{128\,b^4}-\frac{3\,\sqrt{bx^2+a}\,acx}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2dx}{16\,b^3}-\frac{35\,\sqrt{bx^2+a}\,a^3ex}{128\,b^4}+\frac{63\,\sqrt{bx^2+a}\,a^4fx}{256\,b^5}+\frac{3\,a^2c\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}+\frac{35\,a^4e\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}}-\frac{63\,a^5f\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(b*x^2 + a)*f*x^9/b + 1/8*sqrt(b*x^2 + a)*e*x^7/b - 9/80*sqrt(b*x^
2 + a)*a*f*x^7/b^2 + 1/6*sqrt(b*x^2 + a)*d*x^5/b - 7/48*sqrt(b*x^2 + a)*a*e
*x^5/b^2 + 21/160*sqrt(b*x^2 + a)*a^2*f*x^5/b^3 + 1/4*sqrt(b*x^2 + a)*c*x^3
/b - 5/24*sqrt(b*x^2 + a)*a*d*x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*e*x^3/b^
3 - 21/128*sqrt(b*x^2 + a)*a^3*f*x^3/b^4 - 3/8*sqrt(b*x^2 + a)*a*c*x/b^2 +
5/16*sqrt(b*x^2 + a)*a^2*d*x/b^3 - 35/128*sqrt(b*x^2 + a)*a^3*e*x/b^4 + 63/
256*sqrt(b*x^2 + a)*a^4*f*x/b^5 + 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/b^(5/2)
- 5/16*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 35/128*a^4*e*arcsinh(b*x/sqrt
(a*b))/b^(9/2) - 63/256*a^5*f*arcsinh(b*x/sqrt(a*b))/b^(11/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

sympy [B] time = 42.12, size = 586, normalized size = 2.39

$$\frac{63a^{\frac{9}{2}}fx}{256b^5\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{7}{2}}ex}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{21a^{\frac{7}{2}}fx^3}{256b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}dx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}ex^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^{\frac{5}{2}}fx^5}{640b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}cx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}dx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}ex^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{3}{2}}fx^7}{160b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,cx^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,dx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,ex^7}{48b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^9}{80b\sqrt{1+\frac{bx^2}{a}}}-\frac{63a^5f\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{11}{2}}}+\frac{35a^4e\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{cx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{dx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{ex^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 63*a**(9/2)*f*x/(256*b**5*sqrt(1 + b*x**2/a)) - 35*a**(7/2)*e*x/(128*b**4*s
qrt(1 + b*x**2/a)) + 21*a**(7/2)*f*x**3/(256*b**4*sqrt(1 + b*x**2/a)) + 5*a
**(5/2)*d*x/(16*b**3*sqrt(1 + b*x**2/a)) - 35*a**(5/2)*e*x**3/(384*b**3*sqr
t(1 + b*x**2/a)) - 21*a**(5/2)*f*x**5/(640*b**3*sqrt(1 + b*x**2/a)) - 3*a**
(3/2)*c*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d*x**3/(48*b**2*sqrt(1 +
 b*x**2/a)) + 7*a**(3/2)*e*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + 3*a**(3/2)*
f*x**7/(160*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*c*x**3/(8*b*sqrt(1 + b*x**2/
a)) - sqrt(a)*d*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*e*x**7/(48*b*sqrt(
1 + b*x**2/a)) - sqrt(a)*f*x**9/(80*b*sqrt(1 + b*x**2/a)) - 63*a**5*f*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(11/2)) + 35*a**4*e*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(9/2)) - 5*a**3*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*c*a
sinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + c*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)
) + d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + e*x**9/(8*sqrt(a)*sqrt(1 + b*x*
*2/a)) + f*x**11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.148 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=194

$$\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x\sqrt{a+bx^2}\left(-\right.}{}$$

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1809, 1267, 459, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^4}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(40a^2be-35a^3f-48ab^2d+64b^3c\right)}{128b^{9/2}}+\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{192b^3}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx = \frac{fx^7\sqrt{a+bx^2}}{8b} + \frac{\int \frac{x^2\left(8bc+8bdx^2+(8be-7af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2} + \frac{fx^7\sqrt{a+bx^2}}{8b} + \frac{\int \frac{x^2\left(48b^2c+\left(48b^2d-40abe+35a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,dx}{48b^2}$$

$$= \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3} + \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2} + \frac{fx^7\sqrt{a+}}{8b}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b} + \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx}}{192b^3}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b} + \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx}}{192b^3}$$

$$= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b} + \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx}}{192b^3}$$

Mathematica [A] time = 0.17, size = 149, normalized size = 0.77

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(35a^3f-40a^2be+48ab^2d-64b^3c\right)+\sqrt{b}x\sqrt{a+bx^2}\left(-105a^3f+10a^2b\left(12e+7fx^2\right)-8ab^2\left(18d+10ex^2+7fx^4\right)+16b^3\left(12c+6dx^2+4ex^4+3fx^6\right)\right)}{384b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b^
2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^6)
) + 3*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(384*b^(9/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 167, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(-105a^3fx+120a^2bex+70a^2bfx^3-144ab^2dx-80ab^2ex^3-56ab^2fx^5+192b^3cx+96b^3dx^3+64b^3ex^5+48b^3fx^7\right)}{384b^4}+\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)\left(-35a^4f+40a^3be-48a^2b^2d+64ab^3c\right)}{128b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(192*b^3*c*x - 144*a*b^2*d*x + 120*a^2*b*e*x - 105*a^3*f*x
 + 96*b^3*d*x^3 - 80*a*b^2*e*x^3 + 70*a^2*b*f*x^3 + 64*b^3*e*x^5 - 56*a*b^2
```

```
*f*x^5 + 48*b^3*f*x^7))/(384*b^4) + ((64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*
e - 35*a^4*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(9/2))
```

fricas [A] time = 1.07, size = 329, normalized size = 1.70

$$\left[-\frac{3\left(64\,ab^{3}c-48\,a^{2}b^{2}d+40\,a^{3}be-35\,a^{4}f\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(48\,b^{4}fx^{7}+8\left(8\,b^{4}e-7\,ab^{3}f\right)x^{5}+2\left(48\,b^{4}d-40\,ab^{3}e+35\,a^{2}b^{2}f\right)x^{3}+3\left(64\,b^{4}c-48\,ab^{3}d+40\,a^{2}b^{2}e-35\,a^{3}bf\right)x\right)\sqrt{bx^{2}+a}}{768\,b^{5}},\ \frac{3\left(64\,ab^{3}c-48\,a^{2}b^{2}d+40\,a^{3}be-35\,a^{4}f\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(48\,b^{4}fx^{7}+8\left(8\,b^{4}e-7\,ab^{3}f\right)x^{5}+2\left(48\,b^{4}d-40\,ab^{3}e+35\,a^{2}b^{2}f\right)x^{3}+3\left(64\,b^{4}c-48\,ab^{3}d+40\,a^{2}b^{2}e-35\,a^{3}bf\right)x\right)\sqrt{bx^{2}+a}}{384\,b^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*f*x^7 + 8*(8*b^4*e
- 7*a*b^3*f)*x^5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4
*c - 48*a*b^3*d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5, 1/384
*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 + 8*(8*b^4*e - 7*a*b^3*f)*x^5 + 2*
(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d + 40*
a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5]
```

giac [A] time = 0.54, size = 175, normalized size = 0.90

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,fx^{2}}{b}-\frac{7\,ab^{5}f-8\,b^{6}e}{b^{7}}\right)x^{2}+\frac{48\,b^{6}d+35\,a^{2}b^{4}f-40\,ab^{5}e}{b^{7}}\right)x^{2}+\frac{3\left(64\,b^{6}c-48\,ab^{5}d-35\,a^{3}b^{3}f+40\,a^{2}b^{4}e\right)}{b^{7}}\right)\sqrt{bx^{2}+a}\,x+\frac{\left(64\,ab^{3}c-48\,a^{2}b^{2}d-35\,a^{4}f+40\,a^{3}be\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b - (7*a*b^5*f - 8*b^6*e)/b^7)*x^2 + (48*b^6*d + 35*a^
2*b^4*f - 40*a*b^5*e)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d - 35*a^3*b^3*f +
40*a^2*b^4*e)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d - 3
5*a^4*f + 40*a^3*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.01, size = 284, normalized size = 1.46

$$\frac{\sqrt{b\,x^{2}+a}\,f\,x^{7}}{8b}-\frac{7\sqrt{b\,x^{2}+a}\,af\,x^{5}}{48b^{2}}+\frac{\sqrt{b\,x^{2}+a}\,e\,x^{5}}{6b}+\frac{35\sqrt{b\,x^{2}+a}\,a^{2}f\,x^{3}}{192b^{3}}-\frac{5\sqrt{b\,x^{2}+a}\,ae\,x^{3}}{24b^{2}}+\frac{\sqrt{b\,x^{2}+a}\,d\,x^{3}}{4b}+\frac{35a^{4}f\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{9}{2}}}-\frac{5a^{3}e\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{7}{2}}}+\frac{3a^{2}d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}-\frac{ac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}-\frac{35\sqrt{b\,x^{2}+a}\,a^{3}fx}{128b^{4}}+\frac{5\sqrt{b\,x^{2}+a}\,a^{2}ex}{16b^{3}}-\frac{3\sqrt{b\,x^{2}+a}\,adx}{8b^{2}}+\frac{\sqrt{b\,x^{2}+a}\,cx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*f*x^7*(b*x^2+a)^(1/2)/b-7/48*f*a/b^2*x^5*(b*x^2+a)^(1/2)+35/192*f*a^2/b
^3*x^3*(b*x^2+a)^(1/2)-35/128*f*a^3/b^4*x*(b*x^2+a)^(1/2)+35/128*f*a^4/b^(9
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/6*e*x^5/b*(b*x^2+a)^(1/2)-5/24*e*a/b^2*
x^3*(b*x^2+a)^(1/2)+5/16*e*a^2/b^3*x*(b*x^2+a)^(1/2)-5/16*e*a^3/b^(7/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*d*x^3/b*(b*x^2+a)^(1/2)-3/8*d*a/b^2*x*(b*x^2
```

```
+a)^(1/2)+3/8*d*a^2/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*c*x/b*(b*x^2+
a)^(1/2)-1/2*c*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.39, size = 255, normalized size = 1.31

$$\frac{\sqrt{bx^2+a}\,fx^7}{8\,b}+\frac{\sqrt{bx^2+a}\,ex^5}{6\,b}-\frac{7\,\sqrt{bx^2+a}\,afx^5}{48\,b^2}+\frac{\sqrt{bx^2+a}\,dx^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,aex^3}{24\,b^2}+\frac{35\,\sqrt{bx^2+a}\,a^2fx^3}{192\,b^3}+\frac{\sqrt{bx^2+a}\,cx}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,adx}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2ex}{16\,b^3}-\frac{35\,\sqrt{bx^2+a}\,a^3fx}{128\,b^4}-\frac{ac\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3e\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}+\frac{35\,a^4f\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*f*x^7/b + 1/6*sqrt(b*x^2 + a)*e*x^5/b - 7/48*sqrt(b*x^2
 + a)*a*f*x^5/b^2 + 1/4*sqrt(b*x^2 + a)*d*x^3/b - 5/24*sqrt(b*x^2 + a)*a*e*
x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*f*x^3/b^3 + 1/2*sqrt(b*x^2 + a)*c*x/b
- 3/8*sqrt(b*x^2 + a)*a*d*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*e*x/b^3 - 35/128
*sqrt(b*x^2 + a)*a^3*f*x/b^4 - 1/2*a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8
*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*e*arcsinh(b*x/sqrt(a*b))/b
^(7/2) + 35/128*a^4*f*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

sympy [B] time = 32.56, size = 444, normalized size = 2.29

$$-\frac{35a^{\frac{7}{2}}fx}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}ex}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}fx^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}dx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}ex^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}fx^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,dx^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,ex^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^7}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^4f\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3e\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+\frac{dx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{ex^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**(7/2)*f*x/(128*b**4*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*e*x/(16*b**3*sq
rt(1 + b*x**2/a)) - 35*a**(5/2)*f*x**3/(384*b**3*sqrt(1 + b*x**2/a)) - 3*a*
*(3/2)*d*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*e*x**3/(48*b**2*sqrt(1
+ b*x**2/a)) + 7*a**(3/2)*f*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*d*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*e*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**7/(48*b*sqrt(1 + b*x**2
/a)) + 35*a**4*f*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*e*asinh(s
qrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(
5/2)) - a*c*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*x**5/(4*sqrt(a)*sqrt(
1 + b*x**2/a)) + e*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**9/(8*sqrt(a)*
sqrt(1 + b*x**2/a))
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=145

$$\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}\,(6be-5af)}{24b^2}+$$

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1815, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6a^2be-5a^3f-8ab^2d+16b^3c\right)}{16b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{16b^3}+\frac{x^3\sqrt{a+bx^2}\,(6be-5af)}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
```

```
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{6bc+6bdx^2+(6be-5af)x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{24b^2c+3\left(8b^2d-6abe+5a^2f\right)x^2}{\sqrt{a+bx^2}}\,dx}{24b^2}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{16}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{16}$$

$$= \frac{\left(8b^2d-6abe+5a^2f\right)x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{(1}{}$$

Mathematica [A] time = 0.11, size = 118, normalized size = 0.81

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^2f-2ab\left(9e+5fx^2\right)+4b^2\left(6d+3ex^2+2fx^4\right)\right)+3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*f - 2*a*b*(9*e + 5*f*x^2) + 4*b^2*(6*d +
 3*e*x^2 + 2*f*x^4)) + 3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(48*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 123, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,\left(15a^2fx-18abex-10abfx^3+24b^2dx+12b^2ex^3+8b^2fx^5\right)}{48b^3}+\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)\left(5a^3f-6a^2be+8ab^2d-16b^3c\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(24*b^2*d*x - 18*a*b*e*x + 15*a^2*f*x + 12*b^2*e*x^3 - 10*
a*b*f*x^3 + 8*b^2*f*x^5))/(48*b^3) + ((-16*b^3*c + 8*a*b^2*d - 6*a^2*b*e +
5*a^3*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(7/2))
```

fricas [A] time = 1.01, size = 250, normalized size = 1.72

$$\left[-\frac{3\left(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}x-a\right)-2\left(8\,b^3fx^5+2\left(6\,b^3e-5\,ab^2f\right)x^3+3\left(8\,b^3d-6\,ab^2e+5\,a^2bf\right)x\right)\sqrt{bx^2+a}}{96\,b^4},-\frac{3\left(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8\,b^3fx^5+2\left(6\,b^3e-5\,ab^2f\right)x^3+3\left(8\,b^3d-6\,ab^2e+5\,a^2bf\right)x\right)\sqrt{bx^2+a}}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2
*f)*x^3 + 3*(8*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4, -1/4
8*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2*f)*x^3 + 3*(8*b^3*
d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.56, size = 129, normalized size = 0.89

$$\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}-\frac{5\,ab^3f-6\,b^4e}{b^5}\right)x^2+\frac{3\left(8\,b^4d+5\,a^2b^2f-6\,ab^3e\right)}{b^5}\right)\sqrt{bx^2+a}\,x-\frac{\left(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b - (5*a*b^3*f - 6*b^4*e)/b^5)*x^2 + 3*(8*b^4*d + 5*a^2*b^
2*f - 6*a*b^3*e)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d - 5*a^
3*f + 6*a^2*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 203, normalized size = 1.40

$$\frac{\sqrt{b\,x^2+a}\,f\,x^5}{6b}-\frac{5\sqrt{b\,x^2+a}\,af\,x^3}{24b^2}+\frac{\sqrt{b\,x^2+a}\,e\,x^3}{4b}-\frac{5a^3f\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2e\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{ad\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,a^2fx}{16b^3}-\frac{3\sqrt{b\,x^2+a}\,aex}{8b^2}+\frac{\sqrt{b\,x^2+a}\,dx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*x^5*(b*x^2+a)^(1/2)/b-5/24*f*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*f*a^2/b^3
*x*(b*x^2+a)^(1/2)-5/16*f*a^3/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*e*x
^3/b*(b*x^2+a)^(1/2)-3/8*e*a/b^2*x*(b*x^2+a)^(1/2)+3/8*e*a^2/b^(5/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))+1/2*d*x/b*(b*x^2+a)^(1/2)-1/2*d*a/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))+c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.35, size = 174, normalized size = 1.20

$$\frac{\sqrt{bx^2+a}\,fx^5}{6\,b}+\frac{\sqrt{bx^2+a}\,ex^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,afx^3}{24\,b^2}+\frac{\sqrt{bx^2+a}\,dx}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,aex}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2fx}{16\,b^3}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*f*x^5/b + 1/4*sqrt(b*x^2 + a)*e*x^3/b - 5/24*sqrt(b*x^2
 + a)*a*f*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*d*x/b - 3/8*sqrt(b*x^2 + a)*a*e*x/b
^2 + 5/16*sqrt(b*x^2 + a)*a^2*f*x/b^3 + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) -
1/2*a*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*e*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 5/16*a^3*f*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 13.71, size = 362, normalized size = 2.50

$$\frac{5a^{\frac{5}{2}}fx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}ex}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}fx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,ex^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3f\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2e\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)+\frac{ex^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*f*x/(16*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*e*x/(8*b**2*sqrt(1
 + b*x**2/a)) + 5*a**(3/2)*f*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*d*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*e*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
```

```
(a)*f*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*f*asinh(sqrt(b)*x/sqrt(a))/(1
6*b**(7/2)) + 3*a**2*e*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*d*asinh(sq
rt(b)*x/sqrt(a))/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/
sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0)
 & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
+ e*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
/a))
```

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Rubi [A] time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1159, 388, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
```

```
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{ax}-\frac{\int \frac{-adx-aex^3-afx^5}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}-\frac{\int \frac{-ad-aex^2-afx^4}{\sqrt{a+bx^2}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}-\frac{\int \frac{-4abd-a(4be-3af)x^2}{\sqrt{a+bx^2}}\,dx}{4ab}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{\left(8ab^2d-a^2(4be-3af)\right)\int}{8ab^2}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{\left(8ab^2d-a^2(4be-3af)\right)S}{8}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{\left(8b^2d-4abe+3a^2f\right)\tan}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 103, normalized size = 0.88

$$\frac{\frac{\sqrt{b}\sqrt{a+bx^2}\left(-3a^2fx^2+2abx^2\left(2e+fx^2\right)-8b^2c\right)}{ax}+\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(3a^2f-4abe+8b^2d\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[a + b*x^2]*(-8*b^2*c - 3*a^2*f*x^2 + 2*a*b*x^2*(2*e + f*x^2)
))/(a*x) + (8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 105, normalized size = 0.90

$$\frac{\sqrt{a+bx^2}\left(-3a^2fx^2+4abex^2+2abfx^4-8b^2c\right)}{8ab^2x}+\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)\left(-3a^2f+4abe-8b^2d\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*b^2*c + 4*a*b*e*x^2 - 3*a^2*f*x^2 + 2*a*b*f*x^4))/(8*a
*b^2*x) + ((-8*b^2*d + 4*a*b*e - 3*a^2*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

fricas [A] time = 0.88, size = 216, normalized size = 1.85

$$\left[\frac{\left(8\,ab^2d-4\,a^2be+3\,a^3f\right)\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,ab^2fx^4-8\,b^3c+\left(4\,ab^2e-3\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{16\,ab^3x},-\frac{\left(8\,ab^2d-4\,a^2be+3\,a^3f\right)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,ab^2fx^4-8\,b^3c+\left(4\,ab^2e-3\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{8\,ab^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2*b
*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f
)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^2*f*x^4 - 8*b^3*c
+ (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

giac [A] time = 0.52, size = 121, normalized size = 1.03

$$\frac{1}{8}\,\sqrt{bx^2+a}\left(\frac{2\,fx^2}{b}-\frac{3\,abf-4\,b^2e}{b^3}\right)x+\frac{2\,\sqrt{b}\,c}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(8\,b^{\frac{5}{2}}d+3\,a^2\sqrt{b}\,f-4\,ab^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b - (3*a*b*f - 4*b^2*e)/b^3)*x + 2*sqrt(b)*c/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^(5/2)*d + 3*a^2*sqrt(b)*f
- 4*a*b^(3/2)*e)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^3
```

maple [A] time = 0.01, size = 140, normalized size = 1.20

$$\frac{\sqrt{b\,x^2+a}\;f\,x^3}{4b}+\frac{3a^2 f\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{ae\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{d\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{3\sqrt{b\,x^2+a}\;afx}{8b^2}+\frac{\sqrt{b\,x^2+a}\;ex}{2b}-\frac{\sqrt{b\,x^2+a}\;c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x^3*(b*x^2+a)^(1/2)/b-3/8*f*a/b^2*x*(b*x^2+a)^(1/2)+3/8*f*a^2/b^(5/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*e*x/b*(b*x^2+a)^(1/2)-1/2*e*a/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))+d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-c*(b*x^
2+a)^(1/2)/a/x
```

maxima [A] time = 1.26, size = 118, normalized size = 1.01

$$\frac{\sqrt{bx^2+a}\,fx^3}{4\,b}+\frac{\sqrt{bx^2+a}\,ex}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,afx}{8\,b^2}+\frac{d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ae\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2 f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{\sqrt{bx^2+a}\,c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*f*x^3/b + 1/2*sqrt(b*x^2 + a)*e*x/b - 3/8*sqrt(b*x^2 +
a)*a*f*x/b^2 + d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*e*arcsinh(b*x/sqrt(
a*b))/b^(3/2) + 3/8*a^2*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - sqrt(b*x^2 + a)*
c/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)), x)
```

sympy [A] time = 9.05, size = 250, normalized size = 2.14

$$-\frac{3a^{\frac{3}{2}}fx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,ex\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,fx^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{fx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*e*x*sqrt(1 + b*x**2/a
)/(2*b) - sqrt(a)*f*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*f*asinh(sqrt(b)*
x/sqrt(a))/(8*b**(5/2)) - a*e*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*Pie
cewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/
b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt
(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a +
f*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.151 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{a+bx^2}\,(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Rubi [A] time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1265, 388, 217, 206}

$$\frac{\sqrt{a+bx^2}\,(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2*x
) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
```

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{(2bc-3ad)x-3aex^3-3afx^5}{x^3\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{2bc-3ad-3aex^2-3afx^4}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{\int \frac{3a^2e+3a^2fx^2}{\sqrt{a+bx^2}}\,dx}{3a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,d\right.}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right.}{2b^{3/2}}$$

Mathematica [A] time = 0.11, size = 93, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(3a^2fx^4-2ab\left(c+3dx^2\right)+4b^2cx^2\right)}{6a^2bx^3}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*b^2*c*x^2 + 3*a^2*f*x^4 - 2*a*b*(c + 3*d*x^2)))/(6*a^2*
b*x^3) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 95, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(3a^2fx^4-2abc-6abdx^2+4b^2cx^2\right)}{6a^2bx^3}+\frac{(af-2be)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 4*b^2*c*x^2 - 6*a*b*d*x^2 + 3*a^2*f*x^4))/(6*a
^2*b*x^3) + ((-2*b*e + a*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2)
)
```

fricas [A] time = 0.74, size = 210, normalized size = 1.91

$$\left[-\frac{3\left(2a^2be-a^3f\right)\sqrt{b}\,x^3\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}\,x-a\right)-2\left(3a^2bfx^4-2ab^2c+2\left(2b^3c-3ab^2d\right)x^2\right)\sqrt{bx^2+a}}{12a^2b^2x^3},-\frac{3\left(2a^2be-a^3f\right)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(3a^2bfx^4-2ab^2c+2\left(2b^3c-3ab^2d\right)x^2\right)\sqrt{bx^2+a}}{6a^2b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*b*e - a^3*f)*sqrt(b)*x^3*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)*x^2
)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), -1/6*(3*(2*a^2*b*e - a^3*f)*sqrt(-b)*x^3*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*
c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

giac [A] time = 0.56, size = 176, normalized size = 1.60

$$\frac{\sqrt{bx^2+a}\,fx}{2b}+\frac{\left(a\sqrt{b}\,f-2b^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4b^2}+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4\sqrt{b}\,d+6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a\sqrt{b}\,d-2ab^{\frac{3}{2}}c+3a^2\sqrt{b}\,d\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + 1/4*(a*sqrt(b)*f - 2*b^(3/2)*e)*log((sqrt(b)*x
- sqrt(b*x^2 + a))^2)/b^2 + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*
d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 117, normalized size = 1.06

$$-\frac{af\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{e\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,fx}{2b}-\frac{\sqrt{b\,x^2+a}\,d}{ax}+\frac{2\sqrt{b\,x^2+a}\,bc}{3a^2x}-\frac{\sqrt{b\,x^2+a}\,c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(b*x^2+a)^(1/2)/b-1/2*f*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+e*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/3*c*(b*x^2+a)^(1/2)/a/x^3+2/3*c*b/a^
2/x*(b*x^2+a)^(1/2)-d/a/x*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 102, normalized size = 0.93

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b}+\frac{e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{af\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{2\,\sqrt{bx^2+a}\,bc}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,d}{ax}-\frac{\sqrt{bx^2+a}\,c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + e*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*f*arcs
inh(b*x/sqrt(a*b))/b^(3/2) + 2/3*sqrt(b*x^2 + a)*b*c/(a^2*x) - sqrt(b*x^2 +
 a)*d/(a*x) - 1/3*sqrt(b*x^2 + a)*c/(a*x^3)
```

mupad [B] time = 2.20, size = 143, normalized size = 1.30

$$\begin{cases} -\frac{-f\,x^6-3\,e\,x^4+3\,d\,x^2+c}{3\,\sqrt{a}\,x^3} & \text{if } b=0\\ \frac{e\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{d\,\sqrt{b\,x^2+a}}{a\,x}-\frac{a\,f\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{f\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{c\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] piecewise(b == 0, -(c + 3*d*x^2 - 3*e*x^4 - f*x^6)/(3*a^(1/2)*x^3), b ~= 0,
 (e*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (d*(a + b*x^2)^(1/2))/(a*
x) - (a*f*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (f*x*(a + b
*x^2)^(1/2))/(2*b) - (c*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3))
```

sympy [A] time = 4.73, size = 197, normalized size = 1.79

$$\frac{\sqrt{a}\,fx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+e\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + b*x**2/a)/(2*b) - a*f*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + e*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x*
*2) + 1)/(3*a*x**2) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(
a/(b*x**2) + 1)/(3*a**2)
```

3.152 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{a+bx^2}\,(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}\,\left(15a^2e-10abd+8b^2c\right)}{15a^3x}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1807, 1585, 1265, 451, 217, 206}

$$-\frac{\sqrt{a+bx^2}\,\left(15a^2e-10abd+8b^2c\right)}{15a^3x}+\frac{\sqrt{a+bx^2}\,(4bc-5ad)}{15a^2x^3}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{(4bc-5ad)x-5aex^3-5afx^5}{x^5\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{4bc-5ad-5aex^2-5afx^4}{x^4\sqrt{a+bx^2}}\,dx}{5a}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} + \frac{\int \frac{8b^2c-10abd+15a^2e+15a^2fx^2}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} + f\int -$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} + f\,\mathrm{Sub}$$

$$= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} + \frac{f\tanh}{}$$

Mathematica [A] time = 0.11, size = 95, normalized size = 0.81

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{\sqrt{a+bx^2}\left(a^2\left(3c+5dx^2+15ex^4\right)-2abx^2\left(2c+5dx^2\right)+8b^2cx^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/15*(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c
+ 5*d*x^2 + 15*e*x^4)))/(a^3*x^5) + (f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.28, size = 101, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(-3a^2c-5a^2dx^2-15a^2ex^4+4abcx^2+10abdx^4-8b^2cx^4\right)}{15a^3x^5} - \frac{f\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*c + 4*a*b*c*x^2 - 5*a^2*d*x^2 - 8*b^2*c*x^4 + 10*a
*b*d*x^4 - 15*a^2*e*x^4))/(15*a^3*x^5) - (f*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/Sqrt[b]
```

fricas [A] time = 1.02, size = 221, normalized size = 1.87

$$\left[\frac{15\,a^{3}\sqrt{b}\,fx^{5}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,\left(\left(8\,b^{3}c-10\,ab^{2}d+15\,a^{2}be\right)x^{4}+3\,a^{2}bc-\left(4\,ab^{2}c-5\,a^{2}bd\right)x^{2}\right)\sqrt{bx^{2}+a}}{30\,a^{3}bx^{5}},\ -\frac{15\,a^{3}\sqrt{-b}\,fx^{5}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(\left(8\,b^{3}c-10\,ab^{2}d+15\,a^{2}be\right)x^{4}+3\,a^{2}bc-\left(4\,ab^{2}c-5\,a^{2}bd\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\,a^{3}bx^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*
a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x^5*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4
 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5)]
```

giac [B] time = 0.60, size = 324, normalized size = 2.75

$$-\frac{f\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)}{2\,\sqrt{b}}+\frac{2\,\left(15\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}\sqrt{b}e+30\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}b^{\frac{3}{2}}d-60\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}a\sqrt{b}e+80\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}b^{\frac{5}{2}}c-70\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}ab^{\frac{3}{2}}d+90\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{2}\sqrt{b}e-40\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}ab^{\frac{5}{2}}c+50\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{2}b^{\frac{3}{2}}d-60\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{3}\sqrt{b}e+8\,a^{2}b^{\frac{5}{2}}c-10\,a^{3}b^{\frac{3}{2}}d+15\,a^{4}\sqrt{b}e\right)}{15\,\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*
d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 9
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10*a^3
*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 136, normalized size = 1.15

$$\frac{f\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{\sqrt{b\,x^{2}+a}\;e}{ax}+\frac{2\sqrt{b\,x^{2}+a}\;bd}{3a^{2}x}-\frac{8\sqrt{b\,x^{2}+a}\;b^{2}c}{15a^{3}x}-\frac{\sqrt{b\,x^{2}+a}\;d}{3a\,x^{3}}+\frac{4\sqrt{b\,x^{2}+a}\;bc}{15a^{2}x^{3}}-\frac{\sqrt{b\,x^{2}+a}\;c}{5a\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] f*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+4/15*c/
a^2*b/x^3*(b*x^2+a)^(1/2)-8/15*c/a^3*b^2/x*(b*x^2+a)^(1/2)-1/3*d/a/x^3*(b*x
^2+a)^(1/2)+2/3*d*b/a^2/x*(b*x^2+a)^(1/2)-e/a/x*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 128, normalized size = 1.08

$$\frac{f \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{8\sqrt{bx^2+a}\,b^2c}{15\,a^3x} + \frac{2\sqrt{bx^2+a}\,bd}{3\,a^2x} - \frac{\sqrt{bx^2+a}\,e}{ax} + \frac{4\sqrt{bx^2+a}\,bc}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}\,d}{3\,ax^3} - \frac{\sqrt{bx^2+a}\,c}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] f*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 8/15*sqrt(b*x^2 + a)*b^2*c/(a^3*x) + 2/3
*sqrt(b*x^2 + a)*b*d/(a^2*x) - sqrt(b*x^2 + a)*e/(a*x) + 4/15*sqrt(b*x^2 +
a)*b*c/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*d/(a*x^3) - 1/5*sqrt(b*x^2 + a)*c/(a
*x^5)
```

mupad [B] time = 1.72, size = 105, normalized size = 0.89

$$\frac{f\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{e\sqrt{b\,x^2+a}}{a\,x} - \frac{d\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} - \frac{c\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (e*(a + b*x^2)^(1/2))/(a*x
) - (d*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (c*(a + b*x^2)^(1/2)*
(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

sympy [A] time = 6.21, size = 456, normalized size = 3.86

$$-\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12ab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8b^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + f\left(\begin{cases} \frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a > 0 \wedge b < 0 \\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a > 0 \wedge b > 0 \\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b > 0 \wedge a < 0 \end{cases}\right) - \frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13/2)
*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + f*
Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt
```

```
(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*s
qrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(
3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*d*sqrt(a/(b*x**2)
 + 1)/(3*a**2)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{a+bx^2}\,(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}\,\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)}{105a^4x}-$$

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1803, 12, 264}

$$\frac{\sqrt{a+bx^2}\,\left(70a^2be-105a^3f-56ab^2d+48b^3c\right)}{105a^4x}-\frac{\sqrt{a+bx^2}\,\left(35a^2e-28abd+24b^2c\right)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(6bc-7ad)}{35a^2x^5}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (
(48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^4*
x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{7ax^7} - \frac{\int \frac{6bc-7a\left(d+ex^2+fx^4\right)}{x^6\sqrt{a+bx^2}}\,dx}{7a}$$
$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} + \frac{\int \frac{4b(6bc-7ad)-5a\left(-7ae-7afx^2\right)}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}$$
$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - \frac{\int \frac{2b(2}{}}{}$$
$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} - \frac{(48b^3}{}$$
$$= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3} + \frac{(48b^3}{}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-a^3\left(15c+21dx^2+35x^4\left(e+3fx^2\right)\right)+2a^2bx^2\left(9c+14dx^2+35ex^4\right)-8ab^2x^4\left(3c+7dx^2\right)+48b^3cx^6\right)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2*
(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)))
)/(105*a^4*x^7)
```

IntegrateAlgebraic [A] time = 0.26, size = 114, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\left(-15a^3c-21a^3dx^2-35a^3ex^4-105a^3fx^6+18a^2bcx^2+28a^2bdx^4+70a^2bex^6-24ab^2cx^4-56ab^2dx^6+48b^3cx^6\right)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3*c + 18*a^2*b*c*x^2 - 21*a^3*d*x^2 - 24*a*b^2*c*x^
4 + 28*a^2*b*d*x^4 - 35*a^3*e*x^4 + 48*b^3*c*x^6 - 56*a*b^2*d*x^6 + 70*a^2*
b*e*x^6 - 105*a^3*f*x^6))/(105*a^4*x^7)
```

fricas [A] time = 1.21, size = 100, normalized size = 0.71

$$\frac{\left(\left(48\,b^3c-56\,ab^2d+70\,a^2be-105\,a^3f\right)x^6-\left(24\,ab^2c-28\,a^2bd+35\,a^3e\right)x^4-15\,a^3c+3\left(6\,a^2bc-7\,a^3d\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24*a*b^2*c -
 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2)*sqrt(
b*x^2 + a)/(a^4*x^7)
```

giac [B] time = 0.57, size = 554, normalized size = 3.96

$$\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}f-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}f+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e+560\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}b^{\frac{5}{2}}d+1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2\sqrt{b}f-910\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{3}{2}}e+1680\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}b^{\frac{7}{2}}c-1400\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}ab^{\frac{5}{2}}d-2100\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^3\sqrt{b}f+1540\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{3}{2}}e-1008\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}ab^{\frac{7}{2}}c+1176\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^2b^{\frac{5}{2}}d+1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^4\sqrt{b}f-1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{3}{2}}e+336\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^2b^{\frac{7}{2}}c-392\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^3b^{\frac{5}{2}}d-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^5\sqrt{b}f+490\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{3}{2}}e-48a^3b^{\frac{7}{2}}c+56a^4b^{\frac{5}{2}}d+105a^6\sqrt{b}f-70a^5b^{\frac{3}{2}}e\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f - 630*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a*sqrt(b)*f + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(3/2
)*e + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^2*sqrt(b)*f - 910*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(3/
2)*e + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(5/2)*d - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sq
rt(b)*f + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*e - 1008*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f - 1260*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*e + 336*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d
- 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*f + 490*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^4*b^(3/2)*e - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d + 105*a^
6*sqrt(b)*f - 70*a^5*b^(3/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 111, normalized size = 0.79

$$-\frac{\sqrt{b\,x^2+a}\,\left(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15c\,a^3\right)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(105*a^3*f*x^6-70*a^2*b*e*x^6+56*a*b^2*d*x^6-48*b^3*
c*x^6+35*a^3*e*x^4-28*a^2*b*d*x^4+24*a*b^2*c*x^4+21*a^3*d*x^2-18*a^2*b*c*x^
2+15*a^3*c)/x^7/a^4
```

maxima [A] time = 1.40, size = 193, normalized size = 1.38

$$\frac{16\sqrt{bx^2+a}\,b^3c}{35\,a^4x}-\frac{8\sqrt{bx^2+a}\,b^2d}{15\,a^3x}+\frac{2\sqrt{bx^2+a}\,be}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,f}{ax}-\frac{8\sqrt{bx^2+a}\,b^2c}{35\,a^3x^3}+\frac{4\sqrt{bx^2+a}\,bd}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,e}{3\,ax^3}+\frac{6\sqrt{bx^2+a}\,bc}{35\,a^2x^5}-\frac{\sqrt{bx^2+a}\,d}{5\,ax^5}-\frac{\sqrt{bx^2+a}\,c}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(b*x^2 + a)*b^3*c/(a^4*x) - 8/15*sqrt(b*x^2 + a)*b^2*d/(a^3*x) +
2/3*sqrt(b*x^2 + a)*b*e/(a^2*x) - sqrt(b*x^2 + a)*f/(a*x) - 8/35*sqrt(b*x^2
 + a)*b^2*c/(a^3*x^3) + 4/15*sqrt(b*x^2 + a)*b*d/(a^2*x^3) - 1/3*sqrt(b*x^2
 + a)*e/(a*x^3) + 6/35*sqrt(b*x^2 + a)*b*c/(a^2*x^5) - 1/5*sqrt(b*x^2 + a)*
d/(a*x^5) - 1/7*sqrt(b*x^2 + a)*c/(a*x^7)
```

mupad [B] time = 1.28, size = 124, normalized size = 0.89

$$\frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+70\,e\,a^2\,b-56\,d\,a\,b^2+48\,c\,b^3\right)}{105\,a^4\,x}-\frac{\sqrt{b\,x^2+a}\,\left(7\,a\,d-6\,b\,c\right)}{35\,a^2\,x^5}-\frac{\sqrt{b\,x^2+a}\,\left(35\,e\,a^2-28\,d\,a\,b+24\,c\,b^2\right)}{105\,a^3\,x^3}-\frac{c\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(48*b^3*c - 105*a^3*f - 56*a*b^2*d + 70*a^2*b*e))/(105*a
^4*x) - ((a + b*x^2)^(1/2)*(7*a*d - 6*b*c))/(35*a^2*x^5) - ((a + b*x^2)^(1/
2)*(24*b^2*c + 35*a^2*e - 28*a*b*d))/(105*a^3*x^3) - (c*(a + b*x^2)^(1/2))/
(7*a*x^7)
```

sympy [B] time = 6.71, size = 891, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4
*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x**6*sq
rt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**1
1*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 30*a**2*b*
*(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(1
```

```
5/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9
*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a+bx^2}\,(8bc-9ad)}{63a^2x^7}-\frac{\sqrt{a+bx^2}\,\left(21a^2e-18abd+16b^2c\right)}{105a^3x^5}-\frac{2b\sqrt{a+bx^2}\,\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{315a^5x}+$$

Rubi [A] time = 0.25, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1803, 12, 271, 264}

$$-\frac{2b\sqrt{a+bx^2}\,\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^5x}+\frac{\sqrt{a+bx^2}\,\left(84a^2be-105a^3f-72ab^2d+64b^3c\right)}{315a^4x^3}-\frac{\sqrt{a+bx^2}\,\left(21a^2e-18abd+16b^2c\right)}{105a^3x^5}+\frac{\sqrt{a+bx^2}\,(8bc-9ad)}{63a^2x^7}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -(c*Sqrt[a + b*x^2])/(9*a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^2*
x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) + (
(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^4*
x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2
])/(315*a^5*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
```

```
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{9ax^9} - \frac{\int \frac{8bc-9a\left(d+ex^2+fx^4\right)}{x^8\sqrt{a+bx^2}}\,dx}{9a}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} + \frac{\int \frac{6b(8bc-9ad)-7a\left(-9ae-9afx^2\right)}{x^6\sqrt{a+bx^2}}\,dx}{63a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \int \frac{4b(\cdots}{\cdots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} - \frac{(64b\cdots}{\cdots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64b\cdots}{\cdots}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{\left(16b^2c-18abd+21a^2e\right)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64b\cdots}{\cdots}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.71

$$-\frac{\sqrt{a+bx^2}\left(a^4\left(35c+45dx^2+63ex^4+105fx^6\right)-2a^3bx^2\left(20c+27dx^2+42ex^4+105fx^6\right)+24a^2b^2x^4\left(2c+3dx^2+7ex^4\right)-16ab^3x^6\left(4c+9dx^2\right)+128b^4cx^8\right)}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/315*(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24*
a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42*e
*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(a^5*x^9
)
```

IntegrateAlgebraic [A] time = 0.39, size = 160, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(-35a^4c-45a^4dx^2-63a^4ex^4-105a^4fx^6+40a^3bcx^2+54a^3bdx^4+84a^3bex^6+210a^3bfx^8-48a^2b^2cx^4-72a^2b^2dx^6-168a^2b^2ex^8+64ab^3cx^6+144ab^3dx^8-128b^4cx^8\right)}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4*c + 40*a^3*b*c*x^2 - 45*a^4*d*x^2 - 48*a^2*b^2*c*
x^4 + 54*a^3*b*d*x^4 - 63*a^4*e*x^4 + 64*a*b^3*c*x^6 - 72*a^2*b^2*d*x^6 + 8
4*a^3*b*e*x^6 - 105*a^4*f*x^6 - 128*b^4*c*x^8 + 144*a*b^3*d*x^8 - 168*a^2*b
^2*e*x^8 + 210*a^3*b*f*x^8))/(315*a^5*x^9)
```

fricas [A] time = 1.60, size = 141, normalized size = 0.75

$$-\frac{\left(2\left(64\,b^{4}c-72\,ab^{3}d+84\,a^{2}b^{2}e-105\,a^{3}bf\right)x^{8}-\left(64\,ab^{3}c-72\,a^{2}b^{2}d+84\,a^{3}be-105\,a^{4}f\right)x^{6}+35\,a^{4}c+3\left(16\,a^{2}b^{2}c-18\,a^{3}bd+21\,a^{4}e\right)x^{4}-5\left(8\,a^{3}bc-9\,a^{4}d\right)x^{2}\right)\sqrt{bx^{2}+a}}{315\,a^{5}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^8 - (64*a*
b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35*a^4*c + 3*(16*a^2*b
^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*b*c - 9*a^4*d)*x^2)*sqrt(b*x^2
 + a)/(a^5*x^9)
```

giac [B] time = 0.60, size = 667, normalized size = 3.53

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{3}{2}}f-1995\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{3}{2}}f+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{5}{2}}e+2520\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}d+5355\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^{2}b^{\frac{3}{2}}f-3780\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{5}{2}}e+8064\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}b^{\frac{9}{2}}c-6552\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}d-7875\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^{3}b^{\frac{3}{2}}f+6804\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^{2}b^{\frac{5}{2}}e-5376\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}ab^{\frac{9}{2}}c+6048\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{2}b^{\frac{7}{2}}d+6825\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{4}b^{\frac{3}{2}}f-6216\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{3}b^{\frac{5}{2}}e+2304\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{2}b^{\frac{9}{2}}c-2592\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{3}b^{\frac{7}{2}}d-3465\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{5}b^{\frac{3}{2}}f+3024\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{4}b^{\frac{5}{2}}e-576\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{3}b^{\frac{9}{2}}c+648\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{4}b^{\frac{7}{2}}d+945\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{6}b^{\frac{3}{2}}f-756\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{5}b^{\frac{5}{2}}e+64\,a^{4}b^{\frac{9}{2}}c-72\,a^{5}b^{\frac{7}{2}}d-105\,a^{7}b^{\frac{3}{2}}f+84\,a^{6}b^{\frac{5}{2}}e\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f - 1995*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*a*b^(3/2)*f + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(5/
2)*e + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d + 5355*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^2*b^(3/2)*f - 3780*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(5/2)*e + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^3*b^(3/2)*f + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*e - 5376*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*d + 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f -
 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*e + 2304*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(7/2)*d - 3465*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f + 3024*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*d + 945*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f - 756*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^5*b^(5/2)*e + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d - 105*a^7*b^(3/
2)*f + 84*a^6*b^(5/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 157, normalized size = 0.83

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-210 a^{3} b f\,x^{8}+168 a^{2} b^{2} e\,x^{8}-144 a\,b^{3} d\,x^{8}+128 b^{4} c\,x^{8}+105 a^{4} f\,x^{6}-84 a^{3} b e\,x^{6}+72 a^{2} b^{2} d\,x^{6}-64 a\,b^{3} c\,x^{6}+63 a^{4} e\,x^{4}-54 a^{3} b d\,x^{4}+48 a^{2} b^{2} c\,x^{4}+45 a^{4} d\,x^{2}-40 a^{3} b c\,x^{2}+35 c\,a^{4}\right)}{315 a^{5} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(1/2)*(-210*a^3*b*f*x^8+168*a^2*b^2*e*x^8-144*a*b^3*d*x^8+
128*b^4*c*x^8+105*a^4*f*x^6-84*a^3*b*e*x^6+72*a^2*b^2*d*x^6-64*a*b^3*c*x^6+
63*a^4*e*x^4-54*a^3*b*d*x^4+48*a^2*b^2*c*x^4+45*a^4*d*x^2-40*a^3*b*c*x^2+35
*a^4*c)/x^9/a^5
```

maxima [A] time = 1.39, size = 275, normalized size = 1.46

$$-\frac{128\,\sqrt{bx^2+a}\,b^4 c}{315\,a^5 x}+\frac{16\,\sqrt{bx^2+a}\,b^3 d}{35\,a^4 x}-\frac{8\,\sqrt{bx^2+a}\,b^2 e}{15\,a^3 x}+\frac{2\,\sqrt{bx^2+a}\,b f}{3\,a^2 x}+\frac{64\,\sqrt{bx^2+a}\,b^3 c}{315\,a^4 x^3}-\frac{8\,\sqrt{bx^2+a}\,b^2 d}{35\,a^3 x^3}+\frac{4\,\sqrt{bx^2+a}\,b e}{15\,a^2 x^3}-\frac{\sqrt{bx^2+a}\,f}{3\,a x^3}-\frac{16\,\sqrt{bx^2+a}\,b^2 c}{105\,a^3 x^5}+\frac{6\,\sqrt{bx^2+a}\,b d}{35\,a^2 x^5}-\frac{\sqrt{bx^2+a}\,e}{5\,a x^5}+\frac{8\,\sqrt{bx^2+a}\,b c}{63\,a^2 x^7}-\frac{\sqrt{bx^2+a}\,d}{7\,a x^7}-\frac{\sqrt{bx^2+a}\,c}{9\,a x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -128/315*sqrt(b*x^2 + a)*b^4*c/(a^5*x) + 16/35*sqrt(b*x^2 + a)*b^3*d/(a^4*x
) - 8/15*sqrt(b*x^2 + a)*b^2*e/(a^3*x) + 2/3*sqrt(b*x^2 + a)*b*f/(a^2*x) +
64/315*sqrt(b*x^2 + a)*b^3*c/(a^4*x^3) - 8/35*sqrt(b*x^2 + a)*b^2*d/(a^3*x^
3) + 4/15*sqrt(b*x^2 + a)*b*e/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*f/(a*x^3) - 1
6/105*sqrt(b*x^2 + a)*b^2*c/(a^3*x^5) + 6/35*sqrt(b*x^2 + a)*b*d/(a^2*x^5)
- 1/5*sqrt(b*x^2 + a)*e/(a*x^5) + 8/63*sqrt(b*x^2 + a)*b*c/(a^2*x^7) - 1/7*
sqrt(b*x^2 + a)*d/(a*x^7) - 1/9*sqrt(b*x^2 + a)*c/(a*x^9)
```

mupad [B] time = 1.28, size = 171, normalized size = 0.90

$$\frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+84\,e\,a^2\,b-72\,d\,a\,b^2+64\,c\,b^3\right)}{315\,a^4\,x^3}-\frac{\sqrt{b\,x^2+a}\,\left(9\,a\,d-8\,b\,c\right)}{63\,a^2\,x^7}-\frac{\sqrt{b\,x^2+a}\,\left(21\,e\,a^2-18\,d\,a\,b+16\,c\,b^2\right)}{105\,a^3\,x^5}-\frac{\sqrt{b\,x^2+a}\,\left(-210\,f\,a^3\,b+168\,e\,a^2\,b^2-144\,d\,a\,b^3+128\,c\,b^4\right)}{315\,a^5\,x}-\frac{c\,\sqrt{b\,x^2+a}}{9\,a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(64*b^3*c - 105*a^3*f - 72*a*b^2*d + 84*a^2*b*e))/(315*a
^4*x^3) - ((a + b*x^2)^(1/2)*(9*a*d - 8*b*c))/(63*a^2*x^7) - ((a + b*x^2)^(
1/2)*(16*b^2*c + 21*a^2*e - 18*a*b*d))/(105*a^3*x^5) - ((a + b*x^2)^(1/2)*(
128*b^4*c + 168*a^2*b^2*e - 144*a*b^3*d - 210*a^3*b*f))/(315*a^5*x) - (c*(a
 + b*x^2)^(1/2))/(9*a*x^9)
```

sympy [B] time = 7.69, size = 1642, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)

[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*
b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**2
0*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(3
15*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a
**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b*x**2
) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(315*a**
9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**
19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqr
t(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b**(25
/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 +
105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*e*x**2*sqrt(
a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - 560*a**2*b**(45/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1
260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*
a**5*b**20*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
2) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b**(47/2)*c*x**14*sqrt(a/(b*x**2
) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12
 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**
11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2) + 1
)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 128*b**(49/
2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) +
 16*b**(31/2)*d*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b*
*10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8
*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f*sqrt(a/(b
*x**2) + 1)/(3*a**2)
```

3.155 $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=381

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x\sqrt{a+bx^2}\,\left(16Ab^3-3a\left(143a^2D-66abC\right.\right.}{16ab^7}$$

Rubi [A] time = 0.66, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}-\frac{x^5\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{30a^2b^5\sqrt{a+bx^2}}+\frac{x^3\sqrt{a+bx^2}\,\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{24a^2b^6}-\frac{x\sqrt{a+bx^2}\,\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16ab^7}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(198a^2bC-429a^3D-72ab^2B+16Ab^3\right)}{16b^{15/2}}+\frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^7\left(\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^8\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^7\left(\left(8\ldots\right.\right.}{\ldots}}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^8\left(8\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{D\ldots}{6b^3\left(a\ldots\right.}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.58, size = 273, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16\sqrt{a}b^{15/2}\sqrt{\frac{bx^2}{a}+1}}+\frac{x\left(45045a^6D-2310a^5b\left(9C-65Dx^2\right)+42a^4b^2\left(180B-1650Cx^2+4147Dx^4\right)-12a^3b^3\left(140A-2100Bx^2+6699Cx^4-6292Dx^6\right)+a^2b^4x^2\left(-5600A+29232Bx^2-34848Cx^4+5005Dx^6\right)-2ab^5x^4\left(3248A-6336Bx^2+1155Cx^4+455Dx^6\right)+4b^6x^6\left(35\left(6Bx^2+3Cx^4+2Dx^6\right)-704A\right)\right)}{1680b^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C*
x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*x^
6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^4*x
^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A +
 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((16*A*
b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[
b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(15/2)*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 1.40, size = 306, normalized size = 0.80

$$\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)\left(429a^3D-198a^2bC+72ab^2B-16Ab^3\right)}{16b^{15/2}}+\frac{45045a^6Dx-20790a^5bCx+150150a^5bDx^3+7560a^4b^2Bx-69300a^4b^2Cx^3+174174a^4b^2Dx^5-1680a^3Ab^3x+25200a^3b^3Bx^3-80388a^3b^3Cx^5+75504a^3b^3Dx^7-5600a^2Ab^4x^3+29232a^2b^4Bx^5-34848a^2b^4Cx^7+5005a^2b^4Dx^9-6496aAb^5x^5+12672ab^5Bx^7-2310ab^5Cx^9-910ab^5Dx^{11}-2816Ab^6x^7+840b^6Bx^9+420b^6Cx^{11}+280b^6Dx^{13}}{1680b^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-1680*a^3*A*b^3*x + 7560*a^4*b^2*B*x - 20790*a^5*b*C*x + 45045*a^6*D*x - 5
600*a^2*A*b^4*x^3 + 25200*a^3*b^3*B*x^3 - 69300*a^4*b^2*C*x^3 + 150150*a^5*
b*D*x^3 - 6496*a*A*b^5*x^5 + 29232*a^2*b^4*B*x^5 - 80388*a^3*b^3*C*x^5 + 17
4174*a^4*b^2*D*x^5 - 2816*A*b^6*x^7 + 12672*a*b^5*B*x^7 - 34848*a^2*b^4*C*x
^7 + 75504*a^3*b^3*D*x^7 + 840*b^6*B*x^9 - 2310*a*b^5*C*x^9 + 5005*a^2*b^4*
D*x^9 + 420*b^6*C*x^11 - 910*a*b^5*D*x^11 + 280*b^6*D*x^13)/(1680*b^7*(a +
b*x^2)^(7/2)) + ((-16*A*b^3 + 72*a*b^2*B - 198*a^2*b*C + 429*a^3*D)*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(15/2))
```

fricas [A] time = 1.77, size = 987, normalized size = 2.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 +
 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 -
 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*
a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2
 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(280*D*b^7*x^13 - 70*(13*D*a*b^6 - 6*C*b^7)*x^11 + 35
```

```
*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*D*a^3*b^4 - 198*C*a
^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3 - 198*C*a^3*b^4 +
72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*
a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b
^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b^11*x^6 + 6*a^2*b^1
0*x^4 + 4*a^3*b^9*x^2 + a^4*b^8), 1/1680*(105*((429*D*a^3*b^4 - 198*C*a^2*b
^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 -
16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6
)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4
 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (280*D*b^7*x^13 - 70*(13*D*a*b^6 -
 6*C*b^7)*x^11 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*
D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3
 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 19
8*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a
^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b
^11*x^6 + 6*a^2*b^10*x^4 + 4*a^3*b^9*x^2 + a^4*b^8)]
```

giac [A] time = 0.64, size = 342, normalized size = 0.90

$$\frac{\left(\left(\left(\left(35\left(2\left(\frac{4Dx^2}{b}-\frac{13Da^4b^{11}-6Ca^3b^{12}}{a^3b^{13}}\right)x^2+\frac{143Da^5b^{10}-66Ca^4b^{11}+24Ba^3b^{12}}{a^3b^{13}}\right)x^2+\frac{176\left(429Da^6b^9-198Ca^5b^{10}+72Ba^4b^{11}-16Aa^3b^{12}\right)}{a^3b^{13}}\right)x^2+\frac{406\left(429Da^7b^8-198Ca^6b^9+72Ba^5b^{10}-16Aa^4b^{11}\right)}{a^3b^{13}}\right)x^2+\frac{350\left(429Da^8b^7-198Ca^7b^8+72Ba^6b^9-16Aa^5b^{10}\right)}{a^3b^{13}}\right)x^2+\frac{105\left(429Da^9b^6-198Ca^8b^7+72Ba^7b^8-16Aa^6b^9\right)}{a^3b^{13}}\right)x}{1680\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{\left(429Da^3-198Ca^2b+72Bab^2-16Ab^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x^
2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 176*
(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^13)
)*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)
/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A
*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*
b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3 - 198
*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(15/2)
```

maple [A] time = 0.30, size = 517, normalized size = 1.36

$$\frac{Dx^{13}}{6\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{Cx^{11}}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{13Dax^{11}}{24\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{Bx^9}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{11Cax^9}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{143Da^2x^9}{48\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{Ax^7}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{9Bax^7}{14\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{99Ca^2x^7}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{429Da^3x^7}{112\left(bx^2+a\right)^{\frac{7}{2}}b^4}-\frac{Ax^5}{5\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{9Bax^5}{10\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{99Ca^2x^5}{40\left(bx^2+a\right)^{\frac{5}{2}}b^4}+\frac{429Da^3x^5}{80\left(bx^2+a\right)^{\frac{5}{2}}b^5}-\frac{Ax^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{3Bax^3}{2\left(bx^2+a\right)^{\frac{3}{2}}b^4}-\frac{33Ca^2x^3}{8\left(bx^2+a\right)^{\frac{3}{2}}b^5}+\frac{143Da^3x^3}{16\left(bx^2+a\right)^{\frac{3}{2}}b^6}-\frac{Ax}{\sqrt{bx^2+a}\,b^4}+\frac{9Bax}{2\sqrt{bx^2+a}\,b^5}-\frac{99Ca^2x}{8\sqrt{bx^2+a}\,b^6}+\frac{429Da^3x}{16\sqrt{bx^2+a}\,b^7}+\frac{A\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{b^{\frac{9}{2}}}-\frac{9Ba\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{2b^{\frac{11}{2}}}+\frac{99Ca^2\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{8b^{\frac{13}{2}}}-\frac{429Da^3\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{16b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/4*C*x^11/b/(b*x^2+a)^(7/2)+99/8*C*a^2/b^(13/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2))+1/2*B*x^9/b/(b*x^2+a)^(7/2)-9/2*B*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
```

```
2))+1/6*D*x^13/b/(b*x^2+a)^(7/2)-429/16*D*a^3/b^(15/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))-1/7*A*x^7/b/(b*x^2+a)^(7/2)-1/5*A/b^2*x^5/(b*x^2+a)^(5/2)-1/3*A/b
^3*x^3/(b*x^2+a)^(3/2)-A/b^4*x/(b*x^2+a)^(1/2)+9/10*B*a/b^3*x^5/(b*x^2+a)^(
5/2)+3/2*B*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*B*a/b^5*x/(b*x^2+a)^(1/2)-13/24*D*
a/b^2*x^11/(b*x^2+a)^(7/2)+143/48*D*a^2/b^3*x^9/(b*x^2+a)^(7/2)+429/112*D*a
^3/b^4*x^7/(b*x^2+a)^(7/2)+429/80*D*a^3/b^5*x^5/(b*x^2+a)^(5/2)+143/16*D*a^
3/b^6*x^3/(b*x^2+a)^(3/2)+429/16*D*a^3/b^7*x/(b*x^2+a)^(1/2)-11/8*C*a/b^2*x
^9/(b*x^2+a)^(7/2)-99/56*C*a^2/b^3*x^7/(b*x^2+a)^(7/2)-99/40*C*a^2/b^4*x^5/
(b*x^2+a)^(5/2)-33/8*C*a^2/b^5*x^3/(b*x^2+a)^(3/2)+A/b^(9/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))-99/8*C*a^2/b^6*x/(b*x^2+a)^(1/2)+9/14*B*a/b^2*x^7/(b*x^2+a)
^(7/2)
```

maxima [B] time = 1.89, size = 1221, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")

[Out] 1/6*D*x^13/((b*x^2 + a)^(7/2)*b) - 13/24*D*a*x^11/((b*x^2 + a)^(7/2)*b^2) +
 1/4*C*x^11/((b*x^2 + a)^(7/2)*b) + 143/48*D*a^2*x^9/((b*x^2 + a)^(7/2)*b^3
) - 11/8*C*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1/2*B*x^9/((b*x^2 + a)^(7/2)*b)
- 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*A*x + 4
29/560*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*a^3*x
/b^3 - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b
^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*
C*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7
/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b
^4))*B*a*x/b + 143/80*D*a^3*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*
x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^4 - 33/40*C*a^2*x*(1
5*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*
x^2 + a)^(5/2)*b^3))/b^3 + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*
x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*A*x
*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/(
(b*x^2 + a)^(5/2)*b^3))/b + 143/16*D*a^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b^5 - 33/8*C*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b
) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 429/16*D*a^4*x^3/((b*x^2 + a)^(5/2)*b
^6) - 99/8*C*a^3*x^3/((b*x^2 + a)^(5/2)*b^5) + 9/2*B*a^2*x^3/((b*x^2 + a)^(
5/2)*b^4) - A*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 19877/560*D*a^3*x/(sqrt(b*x^2
 + a)*b^7) - 2431/560*D*a^4*x/((b*x^2 + a)^(3/2)*b^7) + 12441/560*D*a^5*x/(
(b*x^2 + a)^(5/2)*b^7) + 4587/280*C*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*C
*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*C*a^4*x/((b*x^2 + a)^(5/2)*b^6) -
```

```
 417/70*B*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*B*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*B*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*A*x/(sqrt(b*x^2 + a)*b^
4) + 17/105*A*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*A*a^2*x/((b*x^2 + a)^(5/2
)*b^4) - 429/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(15/2) + 99/8*C*a^2*arcsinh(
b*x/sqrt(a*b))/b^(13/2) - 9/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + A*arcsi
nh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.156 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=279

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(99a^2D-36abC+8b^2B\right)}{8b^{13/2}}-\frac{x\sqrt{a+bx^2}\,\left(99a^2D-36abC+8b^2B\right)}{8ab^6}+\frac{x^3}{}$$

Rubi [A] time = 0.45, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 459, 288, 321, 217, 206}

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(3a^2D-2abC+b^2B\right)}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(99a^2D-36abC+8b^2B\right)}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(99a^2D-36abC+8b^2B\right)}{12ab^5\sqrt{a+bx^2}}-\frac{x\sqrt{a+bx^2}\,\left(99a^2D-36abC+8b^2B\right)}{8ab^6}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(99a^2D-36abC+8b^2B\right)}{8b^{13/2}}+\frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^5\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^5\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)x \cdots\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b} \cdots$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^6\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right) \cdots\right.}{\left(a+bx^2\right)^{5/2}}}{35a^2b} \cdots$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(8b \cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/\cdots}}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8b^2B-36abC+99a^2 \cdots\right.}{60ab^4\left(a+bx^2\right)^{3/\cdots}}$$

Mathematica [A] time = 0.48, size = 229, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99a^2D-36abC+8b^2B\right)}{8\sqrt{a}\,b^{13/2}\sqrt{\frac{bx^2}{a}+1}}-\frac{x\left(10395a^6D-630a^5b\left(6C-55Dx^2\right)+42a^4b^2\left(20B-300Cx^2+957Dx^4\right)+8a^3b^3x^2\left(350B-1827Cx^2+2178Dx^4\right)+a^2b^4x^4\left(3248B-6336Cx^2+1155Dx^4\right)+2ab^5x^6\left(704B-105\left(2Cx^2+Dx^4\right)\right)-120Ab^6x^6\right)}{840ab^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/840*(x*(10395*a^6*D - 120*A*b^6*x^6 - 630*a^5*b*(6*C - 55*D*x^2) + 42*a^
4*b^2*(20*B - 300*C*x^2 + 957*D*x^4) + a^2*b^4*x^4*(3248*B - 6336*C*x^2 + 1
155*D*x^4) + 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*
(704*B - 105*(2*C*x^2 + D*x^4))))/(a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B - 3
6*a*b*C + 99*a^2*D)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a
]*b^(13/2)*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 1.12, size = 241, normalized size = 0.86

$$\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)\left(-99a^2D+36abC-8b^2B\right)}{8b^{13/2}}+\frac{-10395a^6Dx+3780a^5bCx-34650a^5bDx^3-840a^4b^2Bx+12600a^4b^2Cx^3-40194a^4b^2Dx^5-2800a^3b^3Bx^3+14616a^3b^3Cx^5-17424a^3b^3Dx^7-3248a^2b^4Bx^5+6336a^2b^4Cx^7-1155a^2b^4Dx^9-1408ab^5Bx^7+420ab^5Cx^9+210ab^5Dx^{11}+120Ab^6x^7}{840ab^6\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-840*a^4*b^2*B*x + 3780*a^5*b*C*x - 10395*a^6*D*x - 2800*a^3*b^3*B*x^3 + 1
2600*a^4*b^2*C*x^3 - 34650*a^5*b*D*x^3 - 3248*a^2*b^4*B*x^5 + 14616*a^3*b^3
*C*x^5 - 40194*a^4*b^2*D*x^5 + 120*A*b^6*x^7 - 1408*a*b^5*B*x^7 + 6336*a^2*
b^4*C*x^7 - 17424*a^3*b^3*D*x^7 + 420*a*b^5*C*x^9 - 1155*a^2*b^4*D*x^9 + 21
0*a*b^5*D*x^11)/(840*a*b^6*(a + b*x^2)^(7/2)) + ((-8*b^2*B + 36*a*b*C - 99*
a^2*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(13/2))
```

fricas [A] time = 1.55, size = 816, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/1680*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 36
*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6
+ 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36*C*
a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(21
78*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*
b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 +
8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*
```

```
x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^
5*b^7), -1/840*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a
^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^
5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b
- 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) - (210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(2178*D*a^
3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*b^3 - 3
6*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3
*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*x^2 + a
))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^5*b^7)]
```

giac [A] time = 0.59, size = 265, normalized size = 0.95

$$\frac{\left(\left(\left(\left(105\left(\frac{2Dx^2}{b}-\frac{11Da^4b^9-4Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8\left(2178Da^5b^8-792Ca^4b^9+176Ba^3b^{10}-15Aa^2b^{11}\right)}{a^3b^{11}}\right)x^2-\frac{406\left(99Da^6b^7-36Ca^5b^8+8Ba^4b^9\right)}{a^3b^{11}}\right)x^2-\frac{350\left(99Da^7b^6-36Ca^6b^7+8Ba^5b^8\right)}{a^3b^{11}}\right)x^2-\frac{105\left(99Da^8b^5-36Ca^7b^6+8Ba^6b^7\right)}{a^3b^{11}}\right)x}{840\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\left(99Da^2-36Cab+8Bb^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*b^
11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x^2
- 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99
*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) -
1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(13/2)
```

maple [A] time = 0.01, size = 460, normalized size = 1.65

$$\frac{Dx^{11}}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{Cx^9}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{11Dax^9}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Bx^7}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{9Cax^7}{14\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{99Da^2x^7}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{Ax^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Bx^5}{5\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{9Cax^5}{10\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{99Da^2x^5}{40\left(bx^2+a\right)^{\frac{5}{2}}b^4}-\frac{5Aax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{15Aa^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{Bx^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{3Cax^3}{2\left(bx^2+a\right)^{\frac{3}{2}}b^4}-\frac{33Da^2x^3}{8\left(bx^2+a\right)^{\frac{3}{2}}b^5}+\frac{3Aax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}+\frac{Ax}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Ax}{7\sqrt{bx^2+a}\,ab^3}-\frac{Bx}{\sqrt{bx^2+a}\,b^4}+\frac{9Cax}{2\sqrt{bx^2+a}\,b^5}-\frac{99Da^2x}{8\sqrt{bx^2+a}\,b^6}+\frac{B\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{b^{\frac{9}{2}}}-\frac{9Ca\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{2b^{\frac{11}{2}}}+\frac{99Da^2\ln\left(\sqrt{b}x+\sqrt{bx^2+a}\right)}{8b^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/4*D*x^11/b/(b*x^2+a)^(7/2)-11/8*D*a/b^2*x^9/(b*x^2+a)^(7/2)-99/56*D*a^2/b
^3*x^7/(b*x^2+a)^(7/2)-99/40*D*a^2/b^4*x^5/(b*x^2+a)^(5/2)-33/8*D*a^2/b^5*x
^3/(b*x^2+a)^(3/2)-99/8*D*a^2/b^6*x/(b*x^2+a)^(1/2)+99/8*D*a^2/b^(13/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*C*x^9/b/(b*x^2+a)^(7/2)+9/14*C*a/b^2*x^7/(b*
x^2+a)^(7/2)+9/10*C*a/b^3*x^5/(b*x^2+a)^(5/2)+3/2*C*a/b^4*x^3/(b*x^2+a)^(3/
2)+9/2*C*a/b^5*x/(b*x^2+a)^(1/2)-9/2*C*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))-1/7/(b*x^2+a)^(7/2)*B/b*x^7-1/5/(b*x^2+a)^(5/2)*B/b^2*x^5-1/3/(b*x^2+a
)^(3/2)*B/b^3*x^3-1/(b*x^2+a)^(1/2)*B/b^4*x+B/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))-1/2/(b*x^2+a)^(7/2)*A/b*x^5-5/8/(b*x^2+a)^(7/2)*A*a/b^2*x^3-15/56/
(b*x^2+a)^(7/2)*A*a^2/b^3*x+3/56/(b*x^2+a)^(5/2)*A*a/b^3*x+1/14/(b*x^2+a)^(
3/2)*A/b^3*x+1/7/(b*x^2+a)^(1/2)*A/a/b^3*x
```

maxima [B] time = 1.79, size = 986, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/4*D*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*D*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1
/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*
x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/(
(b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*D*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 7
0*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*D*a^2*x*(15*x^4/((b*x^2 + a)^(5
/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/
b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2
)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/((b*x^2 + a)
^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3
))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*D*a^2*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2/((b*x^2 + a)^
(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*D*a^3*x^3/((b*x^2 + a)^(5
/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3/((b*x^2 + a)^(5/
2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*D*a^2*x/(sqrt(b*x^
2 + a)*b^6) + 561/280*D*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*D*a^4*x/((
b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*C*a^2*x/
((b*x^2 + a)^(3/2)*b^5) + 261/70*C*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*
B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*
a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/
(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*
x/((b*x^2 + a)^(7/2)*b^3) + 99/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(13/2) - 9/
2*C*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + B*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.157 $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=210

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(4bC-15a}{3b^5\sqrt{a+bx}}$$

Rubi [A] time = 0.39, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1804, 1585, 1263, 1584, 455, 1157, 388, 217, 206}

$$\frac{x^5\left(a\left(19a^2D-12abC+5b^2B\right)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}+\frac{Dx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^3\left(-\left(\left(2Ab+\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}\right)x\right)-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^4\left(-2Ab-\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{\int \frac{x^3\left(35\cdots\right.}{\cdots}}{\cdots}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{\int \frac{x^4\left(35\cdots\right.}{\cdots}}{\cdots}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

$$=\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a\left(5b^2B-12abC+19a^2D\right)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC\cdots}{3b^5\left(a\cdots\right.}$$

Mathematica [A] time = 0.50, size = 194, normalized size = 0.92

$$\frac{105a^{5/2}\sqrt{\frac{bx^2}{a}+1}\,(a+bx^2)^3(2bC-9aD)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(945a^6D-210a^5b\left(C-15Dx^2\right)+14a^4b^2x^2\left(261Dx^2-50C\right)+4a^3b^3x^4\left(396Dx^2-203C\right)+a^2b^4x^6\left(105Dx^2-352C\right)+6ab^5x^4\left(7A+5Bx^2\right)+12Ab^6x^6\right)}{210a^2b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^
5*b*(C - 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50
*C + 261*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)) + 105*a^(5/2)*(2*b*C
- 9*a*D)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2
10*a^2*b^(11/2)*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.76, size = 188, normalized size = 0.90

$$\frac{945a^6Dx-210a^5bCx+3150a^5bDx^3-700a^4b^2Cx^3+3654a^4b^2Dx^5-812a^3b^3Cx^5+1584a^3b^3Dx^7-352a^2b^4Cx^7+105a^2b^4Dx^9+42aAb^5x^5+30ab^5Bx^7+12Ab^6x^7}{210a^2b^5\left(a+bx^2\right)^{7/2}}+\frac{(9aD-2bC)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-210*a^5*b*C*x + 945*a^6*D*x - 700*a^4*b^2*C*x^3 + 3150*a^5*b*D*x^3 + 42*a
*A*b^5*x^5 - 812*a^3*b^3*C*x^5 + 3654*a^4*b^2*D*x^5 + 12*A*b^6*x^7 + 30*a*b
^5*B*x^7 - 352*a^2*b^4*C*x^7 + 1584*a^3*b^3*D*x^7 + 105*a^2*b^4*D*x^9)/(210
*a^2*b^5*(a + b*x^2)^(7/2)) + ((-2*b*C + 9*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(2*b^(11/2))
```

fricas [A] time = 1.09, size = 653, normalized size = 3.11

$$\left[\frac{105\left(\left(9Da^3b^4-2Ca^2b^5\right)x^8+9Da^7-2Ca^6b+4\left(9Da^4b^3-2Ca^3b^4\right)x^6+6\left(9Da^5b^2-2Ca^4b^3\right)x^4+4\left(9Da^6b-2Ca^5b^2\right)x^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(105Da^2b^5x^9+2\left(792Da^3b^4-176Ca^2b^5+15Bab^6+6Ab^7\right)x^7+14\left(261Da^4b^3-58Ca^3b^4+3Aab^6\right)x^5+350\left(9Da^5b^2-2Ca^4b^3\right)x^3+105\left(9Da^6b-2Ca^5b^2\right)x\right)\sqrt{bx^2+a}}{420\left(a^2b^{10}x^8+4a^3b^9x^6+6a^4b^8x^4+4a^5b^7x^2+a^6b^6\right)},\ \frac{105\left(\left(9Da^3b^4-2Ca^2b^5\right)x^8+9Da^7-2Ca^6b+4\left(9Da^4b^3-2Ca^3b^4\right)x^6+6\left(9Da^5b^2-2Ca^4b^3\right)x^4+4\left(9Da^6b-2Ca^5b^2\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(105Da^2b^5x^9+2\left(792Da^3b^4-176Ca^2b^5+15Bab^6+6Ab^7\right)x^7+14\left(261Da^4b^3-58Ca^3b^4+3Aab^6\right)x^5+350\left(9Da^5b^2-2Ca^4b^3\right)x^3+105\left(9Da^6b-2Ca^5b^2\right)x\right)\sqrt{bx^2+a}}{210\left(a^2b^{10}x^8+4a^3b^9x^6+6a^4b^8x^4+4a^5b^7x^2+a^6b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D
*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*a^
6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6
 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(
9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^
2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6
*b^6), 1/210*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b +
4*(9*D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(
9*D*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
```

```
(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6 + 6*A*b
^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(9*D*a^5*
b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^2 + a))/
(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6*b^6)]
```

giac [A] time = 0.60, size = 203, normalized size = 0.97

$$\frac{\left(\left(\left(\left(\frac{105\,Dx^2}{b}+\frac{2\left(792\,Da^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10}\right)}{a^3b^9}\right)x^2+\frac{14\left(261\,Da^5b^6-58\,Ca^4b^7+3\,Aa^2b^9\right)}{a^3b^9}\right)x^2+\frac{350\left(9\,Da^6b^5-2\,Ca^5b^6\right)}{a^3b^9}\right)x^2+\frac{105\left(9\,Da^7b^4-2\,Ca^6b^5\right)}{a^3b^9}\right)x}{210\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{(9\,Da-2\,Cb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 + 6
*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^9)
/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(9*D
*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

maple [B] time = 0.01, size = 405, normalized size = 1.93

$$\frac{Dx^9}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Cx^7}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{9Dax^7}{14\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Bx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{Cx^5}{5\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{9Dax^5}{10\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{Ax^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5Bax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{3Aax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{15Ba^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{Cx^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{3Dax^3}{2\left(bx^2+a\right)^{\frac{3}{2}}b^4}+\frac{3Ax}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{3Bax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}+\frac{Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}+\frac{Bx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{2Ax}{35\sqrt{bx^2+a}\,a^2b^2}+\frac{Bx}{7\sqrt{bx^2+a}\,ab^3}-\frac{Cx}{\sqrt{bx^2+a}\,b^4}+\frac{9Dax}{2\sqrt{bx^2+a}\,b^5}+\frac{C\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{b^{\frac{9}{2}}}-\frac{9Da\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*D*x^9/b/(b*x^2+a)^(7/2)+9/14*D*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*D*a/b^3*x
^5/(b*x^2+a)^(5/2)+3/2*D*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*D*a/b^5*x/(b*x^2+a)^
(1/2)-9/2*D*a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/7/(b*x^2+a)^(7/2)*C/
b*x^7-1/5/(b*x^2+a)^(5/2)*C/b^2*x^5-1/3/(b*x^2+a)^(3/2)*C/b^3*x^3-1/(b*x^2+
a)^(1/2)*C/b^4*x+C/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/2*B*x^5/b/(b*x^2
+a)^(7/2)-5/8*B*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*B*a^2/b^3*x/(b*x^2+a)^(7/2)
+3/56*B*a/b^3*x/(b*x^2+a)^(5/2)+1/14*B/b^3*x/(b*x^2+a)^(3/2)+1/7*B/a/b^3*x/
(b*x^2+a)^(1/2)-1/4*A*x^3/b/(b*x^2+a)^(7/2)-3/28*A*a/b^2*x/(b*x^2+a)^(7/2)+
3/140*A/b^2*x/(b*x^2+a)^(5/2)+1/35*A/a/b^2*x/(b*x^2+a)^(3/2)+2/35*A/a^2/b^2
*x/(b*x^2+a)^(1/2)
```

maxima [B] time = 1.69, size = 753, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*D*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
```

```
((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*D*a*x/b + 3/10*D*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*D*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*C*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*D*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*B*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*D*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*D*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*D*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)*b^2) - 9/2*D*a*arc
sinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.158 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x^3\left(Ab^3-10a^3D\right)}{3ab^3\left(a+bx^2\right)^{7/2}}-\frac{a^3Dx}{b^4\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(-176a^3D+15a^2bC+6ab^2B+8Ab^3\right)}{105a^3b\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(-58a^3D+3ab^2B+4Ab^3\right)}{15a^2b^2\left(a+bx^2\right)^{7/2}}+\cdots$$

Rubi [A] time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1804, 1585, 1263, 1584, 452, 288, 217, 206}

$$\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{105a^3b^3\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2D-10abC+3b^2B\right)+4Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{Dx}{b^4\sqrt{a+bx^2}}+\frac{D\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + ((4*A
*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(105*a^3*b^3*(a +
b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a + b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(\left(4Ab+\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}\right)x\right)-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x\left(\left(8A \cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^2\left(8A \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 \cdots\right.}{\cdots}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a\left(3b^2B-10abC+17a^2D\right)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 \cdots\right.}{\cdots}$$

Mathematica [A]   time = 0.46, size = 168, normalized size = 0.94

$$\frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5-176a^3b^3Dx^7+a^2b^4x^3\left(35A+21Bx^2+15Cx^4\right)+2ab^5x^5\left(14A+3Bx^2\right)+8Ab^6x^7}{105a^3b^4\left(a+bx^2\right)^{7/2}}+\frac{\sqrt{a}\,D\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^3
*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2 +
15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*D*Sqrt[1 + (b*x^2)/a]
*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.65, size = 158, normalized size = 0.88

$$\frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5-176a^3b^3Dx^7+35a^2Ab^4x^3+21a^2b^4Bx^5+15a^2b^4Cx^7+28aAb^5x^5+6ab^5Bx^7+8Ab^6x^7}{105a^3b^4\left(a+bx^2\right)^{7/2}}-\frac{D\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x + 35*a^2*A*b^4*x^3 - 350*a^5*b*D*x^3 + 28*a*A*b^5*x^5 + 21*a^
2*b^4*B*x^5 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 + 6*a*b^5*B*x^7 + 15*a^2*b^4*
C*x^7 - 176*a^3*b^3*D*x^7)/(105*a^3*b^4*(a + b*x^2)^(7/2)) - (D*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

fricas [A] time = 1.43, size = 491, normalized size = 2.74

$$\left[\frac{105\left(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(105\,Da^6bx+\left(176\,Da^3b^4-15\,Ca^2b^5-6\,Bab^6-8\,Ab^7\right)x^7+7\left(58\,Da^4b^3-3\,Ba^2b^5-4\,Aab^6\right)x^5+35\left(10\,Da^5b^2-Aa^2b^5\right)x^3\right)\sqrt{bx^2+a}}{210\left(a^3b^9x^8+4\,a^4b^8x^6+6\,a^5b^7x^4+4\,a^6b^6x^2+a^7b^5\right)},-\frac{105\left(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(105\,Da^6bx+\left(176\,Da^3b^4-15\,Ca^2b^5-6\,Bab^6-8\,Ab^7\right)x^7+7\left(58\,Da^4b^3-3\,Ba^2b^5-4\,Aab^6\right)x^5+35\left(10\,Da^5b^2-Aa^2b^5\right)x^3\right)\sqrt{bx^2+a}}{105\left(a^3b^9x^8+4\,a^4b^8x^6+6\,a^5b^7x^4+4\,a^6b^6x^2+a^7b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b*
x^2 + D*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7 + 7
*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 - A*a^2*b^
5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4 + 4*a
^6*b^6*x^2 + a^7*b^5), -1/105*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a
^5*b^2*x^4 + 4*D*a^6*b*x^2 + D*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7
)*x^7 + 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 -
 A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*
x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

giac [A] time = 0.55, size = 160, normalized size = 0.89

$$-\frac{\left(\left(x^2\left(\frac{\left(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9\right)x^2}{a^3b^7}+\frac{7\left(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8\right)}{a^3b^7}\right)+\frac{35\left(10\,Da^5b^4-Aa^2b^7\right)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{D\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^3
*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*a^
5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) - D*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 363, normalized size = 2.03

$$-\frac{Dx^{7}}{7\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{Cx^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{Dx^{5}}{5\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}-\frac{Bx^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{5Cax^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Ax}{7\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{3Bax}{28\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{15Ca^{2}x}{56\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}-\frac{Dx^{3}}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}+\frac{Ax}{35\left(bx^{2}+a\right)^{\frac{5}{2}}ab}+\frac{3Bx}{140\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{3Cax}{56\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}+\frac{4Ax}{105\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b}+\frac{Bx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}}+\frac{Cx}{14\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}+\frac{8Ax}{105\sqrt{bx^{2}+a}\,a^{3}b}+\frac{2Bx}{35\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Cx}{7\sqrt{bx^{2}+a}\,ab^{3}}-\frac{Dx}{\sqrt{bx^{2}+a}\,b^{4}}+\frac{D\ln\left(\sqrt{b}\,x+\sqrt{bx^{2}+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*D*x^7/b/(b*x^2+a)^(7/2)-1/5*D/b^2*x^5/(b*x^2+a)^(5/2)-1/3*D/b^3*x^3/(b
*x^2+a)^(3/2)-D*x/b^4/(b*x^2+a)^(1/2)+D/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-1/2*C*x^5/b/(b*x^2+a)^(7/2)-5/8*C*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*C*a^2/
b^3*x/(b*x^2+a)^(7/2)+3/56*C*a/b^3*x/(b*x^2+a)^(5/2)+1/14*C/b^3*x/(b*x^2+a)
^(3/2)+1/7*C/a/b^3*x/(b*x^2+a)^(1/2)-1/4*B*x^3/b/(b*x^2+a)^(7/2)-3/28*B*a/b
^2*x/(b*x^2+a)^(7/2)+3/140*B/b^2*x/(b*x^2+a)^(5/2)+1/35*B/a/b^2*x/(b*x^2+a)
^(3/2)+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)-1/7*A/b*x/(b*x^2+a)^(7/2)+1/35*A/a/
b*x/(b*x^2+a)^(5/2)+4/105*A/a^2/b*x/(b*x^2+a)^(3/2)+8/105*A/a^3/b*x/(b*x^2+
a)^(1/2)
```

maxima [B] time = 1.66, size = 533, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35x^{6}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{70ax^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+\frac{56a^{2}x^{2}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}+\frac{16a^{3}}{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{4}}\right)Dx-\frac{Dx\left(\frac{15x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b}+\frac{20ax^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{8a^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}\right)}{15b}-\frac{Cx^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{Dx\left(\frac{3x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)}{3b^{2}}-\frac{Dax^{3}}{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}-\frac{5Cax^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Bx^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{139Dx}{105\sqrt{bx^{2}+a}\,b^{4}}+\frac{17Dax}{105\left(bx^{2}+a\right)^{\frac{3}{2}}b^{4}}-\frac{29Da^{2}x}{35\left(bx^{2}+a\right)^{\frac{5}{2}}b^{4}}+\frac{Cx}{14\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}+\frac{Cx}{7\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3Cax}{56\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}-\frac{15Ca^{2}x}{56\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}+\frac{3Bx}{140\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{2Bx}{35\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Bx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}}-\frac{3Bax}{28\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Ax}{7\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{8Ax}{105\sqrt{bx^{2}+a}\,a^{3}b}+\frac{4Ax}{105\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b}+\frac{Ax}{35\left(bx^{2}+a\right)^{\frac{5}{2}}ab}+\frac{D\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*x - 1/
15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - D*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/(
(b*x^2 + a)^(7/2)*b) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D*a*x/((b
*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*C*x/((b
*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*x/((b*x^
2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 +
 a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a
```

```
)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/
2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + D*arcsinh(b*x/sqrt(a*b))/b^(
9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(A + B x^2 + C x^4 + x^6 D\right)}{\left(b x^2 + a\right)^{9/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
[Out] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.159 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=134

$$\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.21, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1813, 1803, 12, 264}

$$\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^2\left(6Ab+a\left(B+Cx^2+Dx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^4\left(4b(6Ab+aB)+3a\left(aC+aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{\left(2b\left(24Ab^2+4a \ldots\right.\right.}{(\ldots}}{\ldots}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\left(\ldots\right.\right.}{\ldots}$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a\left(\ldots\right.\right.}{10\ldots}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.73

$$\frac{a^3\left(105Ax+35Bx^3+21Cx^5+15Dx^7\right)+2a^2bx^3\left(105A+14Bx^2+3Cx^4\right)+8ab^2x^5\left(21A+Bx^2\right)+48Ab^3x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
+ 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a +
b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.46, size = 112, normalized size = 0.84

$$\frac{105a^3Ax+35a^3Bx^3+21a^3Cx^5+15a^3Dx^7+210a^2Abx^3+28a^2bBx^5+6a^2bCx^7+168aAb^2x^5+8ab^2Bx^7+48Ab^3x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (105*a^3*A*x + 210*a^2*A*b*x^3 + 35*a^3*B*x^3 + 168*a*A*b^2*x^5 + 28*a^2*b*
B*x^5 + 21*a^3*C*x^5 + 48*A*b^3*x^7 + 8*a*b^2*B*x^7 + 6*a^2*b*C*x^7 + 15*a^
3*D*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.01, size = 141, normalized size = 1.05

$$\frac{\left(\left(15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3\right)x^7+7\left(3\,Ca^3+4\,Ba^2b+24\,Aab^2\right)x^5+105\,Aa^3x+35\left(Ba^3+6\,Aa^2b\right)x^3\right)\sqrt{bx^2+a}}{105\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*B
*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b
*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 0.53, size = 131, normalized size = 0.98

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6\right)x^2}{a^4b^3}+\frac{7\left(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5\right)}{a^4b^3}\right)+\frac{35\left(Ba^3b^3+6\,Aa^2b^4\right)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*b
^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3
 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 109, normalized size = 0.81

$$\frac{\left(48A\,b^3x^6+8Ba\,b^2x^6+6a^2bC\,x^6+15Da^3x^6+168Aa\,b^2x^4+28B\,a^2b\,x^4+21a^3C\,x^4+210A\,a^2b\,x^2+35B\,a^3x^2+105A\,a^3\right)x}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x*(48*A*b^3*x^6+8*B*a*b^2*x^6+6*C*a^2*b*x^6+15*D*a^3*x^6+168*A*a*b^2*
x^4+28*B*a^2*b*x^4+21*C*a^3*x^4+210*A*a^2*b*x^2+35*B*a^3*x^2+105*A*a^3)/(b*
x^2+a)^(7/2)/a^4
```

maxima [B] time = 1.42, size = 335, normalized size = 2.50

$$-\frac{Dx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,Dax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Cx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Ax}{7\left(bx^2+a\right)^{\frac{7}{2}}a}+\frac{Dx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Dx}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Dax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,Da^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{3\,Cx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Cx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Bx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^
2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7\left(4b\left(48Ab^2-a(aC+6bB)\right)-3a^3D\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(4b\left(48Ab^2-a(aC\right.\right.}{15a^4\left(a+bx\right.}$$

Rubi [A] time = 0.25, antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$-\frac{2bx^7\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(19
2*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{\left(4b\left(48Ab^2-6a\right.\right.}{\left(a+\right.}}{}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4a\right.}{}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4a\right.}{15a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-4a\right.}{15a}$$

Mathematica [A] time = 0.18, size = 133, normalized size = 0.72

$$\frac{-7a^4\left(15A-15Bx^2-5Cx^4-3Dx^6\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-210A+21Bx^2+Cx^4\right)+48ab^3x^6\left(Bx^2-28A\right)-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x^2
*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.51, size = 160, normalized size = 0.86

$$\frac{-105a^4A+105a^4Bx^2+35a^4Cx^4+21a^4Dx^6-840a^3Abx^2+210a^3bBx^4+28a^3bCx^6+6a^3bDx^8-1680a^2Ab^2x^4+168a^2b^2Bx^6+8a^2b^2Cx^8-1344aAb^3x^6+48ab^3Bx^8-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-105*a^4*A - 840*a^3*A*b*x^2 + 105*a^4*B*x^2 - 1680*a^2*A*b^2*x^4 + 210*a^
3*b*B*x^4 + 35*a^4*C*x^4 - 1344*a*A*b^3*x^6 + 168*a^2*b^2*B*x^6 + 28*a^3*b*
C*x^6 + 21*a^4*D*x^6 - 384*A*b^4*x^8 + 48*a*b^3*B*x^8 + 8*a^2*b^2*C*x^8 + 6
*a^3*b*D*x^8)/(105*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.93, size = 182, normalized size = 0.98

$$\frac{\left(2\left(3\,Da^3b+4\,Ca^2b^2+24\,Bab^3-192\,Ab^4\right)x^8+7\left(3\,Da^4+4\,Ca^3b+24\,Ba^2b^2-192\,Aab^3\right)x^6-105\,Aa^4+35\left(Ca^4+6\,Ba^3b-48\,Aa^2b^2\right)x^4+105\left(Ba^4-8\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*D*a^
4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 + 6
*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)
/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

giac [A] time = 0.58, size = 211, normalized size = 1.14

$$\frac{\left(\left(x^2\left(\frac{\left(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right)x^2+\frac{105\left(Ba^{13}b^3-4\,Aa^{12}b^4\right)}{a^{14}b^3}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)*
x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^1
0*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b
^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 157, normalized size = 0.85

$$-\frac{384 A\,b^{4} x^{8}-48 B a\,b^{3} x^{8}-8 C\,a^{2} b^{2} x^{8}-6 D a^{3} b\,x^{8}+1344 A a\,b^{3} x^{6}-168 B\,a^{2} b^{2} x^{6}-28 C\,a^{3} b\,x^{6}-21 D a^{4} x^{6}+1680 A\,a^{2} b^{2} x^{4}-210 B\,a^{3} b\,x^{4}-35 C\,a^{4} x^{4}+840 A\,a^{3} b\,x^{2}-105 B\,a^{4} x^{2}+105 A\,a^{4}}{105 \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{5} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8+1344*A*a
*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*a^2*b^2*x^4-2
10*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*A*a^4)/(b*x^2
+a)^(7/2)/x/a^5
```

maxima [A] time = 1.47, size = 313, normalized size = 1.69

$$-\frac{Dx^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{16\,Bx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Bx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{6\,Bx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{Bx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}+\frac{3\,Dx}{140\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{2\,Dx}{35\,\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Dx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}}-\frac{3\,Dax}{28\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}-\frac{Cx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^{2}+a}\,a^{3}b}+\frac{4\,Cx}{105\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b}+\frac{Cx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}ab}-\frac{128\,Abx}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{64\,Abx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{4}}-\frac{48\,Abx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{3}}-\frac{8\,Abx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}}-\frac{A}{\left(bx^{2}+a\right)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B
*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x
^2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a
)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2
)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2
)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)
*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.161 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=242

$$\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-\right.}{1}$$

Rubi [A] time = 0.32, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 1813, 12, 271, 264}

$$\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{3a^4\left(a+bx^2\right)^{7/2}}+\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7/2
)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160*A
*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (4*
b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(7/
2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10Ab-3a\left(B+Cx^2+Dx^4\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{8b(10Ab-3aB)-a\left(-3aC-3aDx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{(6b(80A\cdots}{\cdots}}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{(160Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{(160Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{(160Ab^3\cdots}{\cdots}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{(160Ab^3\cdots}{\cdots}$$

Mathematica [A] time = 0.13, size = 165, normalized size = 0.68

$$\frac{-35a^5\left(A+3Bx^2-3Cx^4-Dx^6\right)+14a^4bx^2\left(25A-60Bx^2+15Cx^4+2Dx^6\right)+8a^3b^2x^4\left(350A-210Bx^2+21Cx^4+Dx^6\right)+16a^2b^3x^6\left(350A-84Bx^2+3Cx^4\right)+128ab^4x^8\left(35A-3Bx^2\right)+1280Ab^5x^{10}}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A -
 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2*x
^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^2 +
 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.57, size = 208, normalized size = 0.86

$$\frac{-35a^5A-105a^5Bx^2+105a^5Cx^4+35a^5Dx^6+350a^4Abx^2-840a^4bBx^4+210a^4bCx^6+28a^4bDx^8+2800a^3Ab^2x^4-1680a^3b^2Bx^6+168a^3b^2Cx^8+8a^3b^2Dx^{10}+5600a^2Ab^3x^6-1344a^2b^3Bx^8+48a^2b^3Cx^{10}+4480aAb^4x^8-384ab^4Bx^{10}+1280Ab^5x^{10}}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^5*A + 350*a^4*A*b*x^2 - 105*a^5*B*x^2 + 2800*a^3*A*b^2*x^4 - 840*a^4
*b*B*x^4 + 105*a^5*C*x^4 + 5600*a^2*A*b^3*x^6 - 1680*a^3*b^2*B*x^6 + 210*a^
4*b*C*x^6 + 35*a^5*D*x^6 + 4480*a*A*b^4*x^8 - 1344*a^2*b^3*B*x^8 + 168*a^3*
b^2*C*x^8 + 28*a^4*b*D*x^8 + 1280*A*b^5*x^10 - 384*a*b^4*B*x^10 + 48*a^2*b^
3*C*x^10 + 8*a^3*b^2*D*x^10)/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.47, size = 225, normalized size = 0.93

$$\frac{\left(8\left(Da^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5\right)x^{10}+28\left(Da^4b+6\,Ca^3b^2-48\,Ba^2b^3+160\,Aab^4\right)x^8+35\left(Da^5+6\,Ca^4b-48\,Ba^3b^2+160\,Aa^2b^3\right)x^6-35\,Aa^5+35\left(3\,Ca^5-24\,Ba^4b+80\,Aa^3b^2\right)x^4-35\left(3\,Ba^5-10\,Aa^4b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(D*a^
4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(D*a^5 + 6*C*a^4*b
 - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4*b
+ 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b
^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

giac [A] time = 0.56, size = 349, normalized size = 1.44

$$\frac{\left(\left(x^2\left(\frac{\left(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8\right)x^2}{a^{18}b^3}+\frac{7\left(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6+365\,Aa^{13}b^7\right)}{a^{18}b^3}\right)+\frac{35\left(Da^{17}b^3+6\,Ca^{16}b^4-30\,Ba^{15}b^5+80\,Aa^{14}b^6\right)}{a^{18}b^3}\right)x^2+\frac{105\left(Ca^{17}b^3-4\,Ba^{16}b^4+10\,Aa^{15}b^5\right)}{a^{18}b^3}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}+3\,Ba^3\sqrt{b}-14\,Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b^
8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 365*
A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5 + 8
0*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*
b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3*a^5)
```

maple [A] time = 0.01, size = 205, normalized size = 0.85

$$-\frac{-1280 A b^5 x^{10}+384 B a b^4 x^{10}-48 C a^2 b^3 x^{10}-8 D a^3 b^2 x^{10}-4480 A a b^4 x^8+1344 B a^2 b^3 x^8-168 C a^3 b^2 x^8-28 D a^4 b x^8-5600 A a^2 b^3 x^6+1680 B a^3 b^2 x^6-210 C a^4 b x^6-35 D a^5 x^6-2800 A a^3 b^2 x^4+840 B a^4 b x^4-105 C a^5 x^4-350 A a^4 b x^2+105 B a^5 x^2+35 A a^5}{105 \left(b x^2+a\right)^{\frac{7}{2}} a^6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-1280*A*b^5*x^10+384*B*a*b^4*x^10-48*C*a^2*b^3*x^10-8*D*a^3*b^2*x^1
0-4480*A*a*b^4*x^8+1344*B*a^2*b^3*x^8-168*C*a^3*b^2*x^8-28*D*a^4*b*x^8-5600
*A*a^2*b^3*x^6+1680*B*a^3*b^2*x^6-210*C*a^4*b*x^6-35*D*a^5*x^6-2800*A*a^3*b
^2*x^4+840*B*a^4*b*x^4-105*C*a^5*x^4-350*A*a^4*b*x^2+105*B*a^5*x^2+35*A*a^5
)/(b*x^2+a)^(7/2)/x^3/a^6
```

maxima [A] time = 1.47, size = 337, normalized size = 1.39

$$\frac{16 C x}{35 \sqrt{b x^2+a}\, a^4}+\frac{8 C x}{35 \left(b x^2+a\right)^{\frac{3}{2}} a^3}+\frac{6 C x}{35 \left(b x^2+a\right)^{\frac{5}{2}} a^2}+\frac{C x}{7 \left(b x^2+a\right)^{\frac{7}{2}} a}-\frac{D x}{7 \left(b x^2+a\right)^{\frac{7}{2}} b}+\frac{8 D x}{105 \sqrt{b x^2+a}\, a^3 b}+\frac{4 D x}{105 \left(b x^2+a\right)^{\frac{3}{2}} a^2 b}+\frac{D x}{35 \left(b x^2+a\right)^{\frac{5}{2}} a b}-\frac{128 B b x}{35 \sqrt{b x^2+a}\, a^5}-\frac{64 B b x}{35 \left(b x^2+a\right)^{\frac{3}{2}} a^4}-\frac{48 B b x}{35 \left(b x^2+a\right)^{\frac{5}{2}} a^3}-\frac{8 B b x}{7 \left(b x^2+a\right)^{\frac{7}{2}} a^2}+\frac{256 A b^2 x}{21 \sqrt{b x^2+a}\, a^6}+\frac{128 A b^2 x}{21 \left(b x^2+a\right)^{\frac{3}{2}} a^5}+\frac{32 A b^2 x}{7 \left(b x^2+a\right)^{\frac{5}{2}} a^4}+\frac{80 A b^2 x}{21 \left(b x^2+a\right)^{\frac{7}{2}} a^3}-\frac{B}{\left(b x^2+a\right)^{\frac{7}{2}} a x}+\frac{10 A b}{3 \left(b x^2+a\right)^{\frac{7}{2}} a^2 x}-\frac{A}{3 \left(b x^2+a\right)^{\frac{7}{2}} a x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*D*x/((b*x^
2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*D*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(b*x^
2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^2 + a
)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sqrt(b*x
^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x/((b*x^
2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x^2 + a)^
(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 + a)^(7/2)*
a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.162 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3\right.\right.}{105a^{6}}$$

Rubi [A] time = 0.43, antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1803, 12, 192, 191}

$$-\frac{16x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{35a^5\left(a+bx^2\right)^{5/2}}-\frac{x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{21a^4\left(a+bx^2\right)^{7/2}}-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(
7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((192
*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(21*a^4*(a + b*x^2)^(7/2)) -
(2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^(5
/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a +
b*x^2)^(3/2)) - (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(105*
a^7*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12Ab-5a\left(B+Cx^2+Dx^4\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{10b(12Ab-5aB)-3a\left(-5aC-5aDx^2\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8b\left(120Ab^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3 - \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.16, size = 202, normalized size = 0.72

$$\frac{-7a^6\left(3A+5x^2\left(B+3Cx^2-3Dx^4\right)\right)+14a^5bx^2\left(6A+25Bx^2-60Cx^4+15Dx^6\right)+56a^4b^2x^4\left(-15A+50Bx^2-30Cx^4+3Dx^6\right)+16a^3b^3x^6\left(-420A+350Bx^2-84Cx^4+3Dx^6\right)-128a^2b^4x^8\left(105A-35Bx^2+3Cx^4\right)+256ab^5x^{10}\left(5Bx^2-42A\right)-3072Ab^6x^{12}}{105a^7x^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(105
*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4 +
3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a^5*
b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*
x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.64, size = 256, normalized size = 0.91

$$\frac{-21a^6A-35a^6Bx^2-105a^6Cx^4+105a^6Dx^6+84a^5Abx^2+350a^5bBx^4-840a^5bCx^6+210a^5bDx^8-840a^4Ab^2x^4+2800a^4b^2Bx^6-1680a^4b^2Cx^8+168a^4b^2Dx^{10}-6720a^3Ab^3x^6+5600a^3b^3Bx^8-1344a^3b^3Cx^{10}+48a^3b^3Dx^{12}-13440a^2Ab^4x^8+4480a^2b^4Bx^{10}-384a^2b^4Cx^{12}-10752aAb^5x^{10}+1280ab^5Bx^{12}-3072Ab^6x^{12}}{105a^7x^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-21*a^6*A + 84*a^5*A*b*x^2 - 35*a^6*B*x^2 - 840*a^4*A*b^2*x^4 + 350*a^5*b*
B*x^4 - 105*a^6*C*x^4 - 6720*a^3*A*b^3*x^6 + 2800*a^4*b^2*B*x^6 - 840*a^5*b
*C*x^6 + 105*a^6*D*x^6 - 13440*a^2*A*b^4*x^8 + 5600*a^3*b^3*B*x^8 - 1680*a^
4*b^2*C*x^8 + 210*a^5*b*D*x^8 - 10752*a*A*b^5*x^10 + 4480*a^2*b^4*B*x^10 -
1344*a^3*b^3*C*x^10 + 168*a^4*b^2*D*x^10 - 3072*A*b^6*x^12 + 1280*a*b^5*B*x
^12 - 384*a^2*b^4*C*x^12 + 48*a^3*b^3*D*x^12)/(105*a^7*x^5*(a + b*x^2)^(7/2
))
```

fricas [A] time = 1.83, size = 270, normalized size = 0.96

$$\frac{\left(16\left(3Da^3b^3-24Ca^2b^4+80Bab^5-192Ab^6\right)x^{12}+56\left(3Da^4b^2-24Ca^3b^3+80Ba^2b^4-192Aab^5\right)x^{10}+70\left(3Da^5b-24Ca^4b^2+80Ba^3b^3-192Aa^2b^4\right)x^8-21Aa^6+35\left(3Da^6-24Ca^5b+80Ba^4b^2-192Aa^3b^3\right)x^6-35\left(3Ca^6-10Ba^5b+24Aa^4b^2\right)x^4-7\left(5Ba^6-12Aa^5b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^7b^4x^{13}+4a^8b^3x^{11}+6a^9b^2x^9+4a^{10}bx^7+a^{11}x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*(
3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*D*a^5
*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3*D*
a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a
^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a
^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

giac [B] time = 0.63, size = 592, normalized size = 2.11

$$\frac{\left(\left(x^2\left(\frac{\left(48Da^{18}b^6-279Ca^{17}b^7+790Ba^{16}b^8-1686Aa^{15}b^9\right)x^2}{a^{22}b^3}+\frac{7\left(24Da^{19}b^5-132Ca^{18}b^6+365Ba^{17}b^7-768Aa^{16}b^8\right)}{a^{22}b^3}\right)+\frac{35\left(6Da^{20}b^4-30Ca^{19}b^5+80Ba^{18}b^6-165Aa^{17}b^7\right)}{a^{22}b^3}\right)x^2+\frac{105\left(Da^{21}b^3-4Ca^{20}b^4+10Ba^{19}b^5-20Aa^{18}b^6\right)}{a^{22}b^3}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca^2\sqrt{b}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ca^3\sqrt{b}+270\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}-720\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Aab^{\frac{5}{2}}+90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca^4\sqrt{b}-430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3b^{\frac{3}{2}}+1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{5}{2}}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ca^5\sqrt{b}+290\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4b^{\frac{3}{2}}-840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{5}{2}}+15Ca^6\sqrt{b}-70Ba^5b^{\frac{3}{2}}+198Aa^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^15
*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7 -
 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18
*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 1
0*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
- 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

maple [A] time = 0.01, size = 253, normalized size = 0.90

$$-\frac{3072 A\,b^{6} x^{12}-1280 B a\,b^{5} x^{12}+384 C\,a^{2} b^{4} x^{12}-48 D a^{3} b^{3} x^{12}+10752 A a\,b^{5} x^{10}-4480 B\,a^{2} b^{4} x^{10}+1344 C\,a^{3} b^{3} x^{10}-168 D a^{4} b^{2} x^{10}+13440 A\,a^{2} b^{4} x^{8}-5600 B\,a^{3} b^{3} x^{8}+1680 C\,a^{4} b^{2} x^{8}-210 D a^{5} b\,x^{8}+6720 A\,a^{3} b^{3} x^{6}-2800 B\,a^{4} b^{2} x^{6}+840 C\,a^{5} b\,x^{6}-105 D a^{6} x^{6}+840 A\,a^{4} b^{2} x^{4}-350 B\,a^{5} b\,x^{4}+105 C\,a^{6} x^{4}-84 A\,a^{5} b\,x^{2}+35 B\,a^{6} x^{2}+21 A\,a^{6}}{105 \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{7} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(3072*A*b^6*x^12-1280*B*a*b^5*x^12+384*C*a^2*b^4*x^12-48*D*a^3*b^3*x
^12+10752*A*a*b^5*x^10-4480*B*a^2*b^4*x^10+1344*C*a^3*b^3*x^10-168*D*a^4*b^
2*x^10+13440*A*a^2*b^4*x^8-5600*B*a^3*b^3*x^8+1680*C*a^4*b^2*x^8-210*D*a^5*
b*x^8+6720*A*a^3*b^3*x^6-2800*B*a^4*b^2*x^6+840*C*a^5*b*x^6-105*D*a^6*x^6+8
40*A*a^4*b^2*x^4-350*B*a^5*b*x^4+105*C*a^6*x^4-84*A*a^5*b*x^2+35*B*a^6*x^2+
21*A*a^6)/(b*x^2+a)^(7/2)/x^5/a^7
```

maxima [A] time = 1.55, size = 398, normalized size = 1.42

$$\frac{16\,Dx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Dx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{6\,Dx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{Dx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}-\frac{128\,Cbx}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{64\,Cbx}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{4}}-\frac{48\,Cbx}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{3}}-\frac{8\,Cbx}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}}+\frac{256\,Bb^{2}x}{21\,\sqrt{bx^{2}+a}\,a^{6}}+\frac{128\,Bb^{2}x}{21\left(bx^{2}+a\right)^{\frac{3}{2}}a^{5}}+\frac{32\,Bb^{2}x}{7\left(bx^{2}+a\right)^{\frac{5}{2}}a^{4}}+\frac{80\,Bb^{2}x}{21\left(bx^{2}+a\right)^{\frac{7}{2}}a^{3}}-\frac{1024\,Ab^{3}x}{35\,\sqrt{bx^{2}+a}\,a^{7}}-\frac{512\,Ab^{3}x}{35\left(bx^{2}+a\right)^{\frac{3}{2}}a^{6}}-\frac{384\,Ab^{3}x}{35\left(bx^{2}+a\right)^{\frac{5}{2}}a^{5}}-\frac{64\,Ab^{3}x}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a^{4}}-\frac{C}{\left(bx^{2}+a\right)^{\frac{7}{2}}ax}+\frac{10\,Bb}{3\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}x}-\frac{8\,Ab^{2}}{\left(bx^{2}+a\right)^{\frac{7}{2}}a^{3}x}-\frac{B}{3\left(bx^{2}+a\right)^{\frac{7}{2}}ax^{3}}+\frac{4\,Ab}{5\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}x^{3}}-\frac{A}{5\left(bx^{2}+a\right)^{\frac{7}{2}}ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*D
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^2*x/
```

```
(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*B*b^2
*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) - 1024/3
5*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*a^6) -
384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^(7/2)*a^
4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*A*b
^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*A*b/((
b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

mupad [B]   time = 2.40, size = 405, normalized size = 1.44

$$\frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}+\frac{x\,D}{{\left(b\,x^2+a\right)}^{9/2}}-\frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}+\frac{18\,b^2\,x^5\,D}{5\,a^2\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{72\,b^3\,x^7\,D}{35\,a^3\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{16\,b^4\,x^9\,D}{35\,a^4\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{A\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,B\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{B\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,A\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{3\,b\,x^3\,D}{a\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{29\,C\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,C\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{C\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D)/
(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5
/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A*b^2
)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x
^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*D)/(5*a^2*(a + b*x^2)^(9/2)) + (72*b^3
*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) + (16*b^4*x^9*D)/(35*a^4*(a + b*x^2)^(9/
2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^
(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) + (27*A*b^2)/(7*a^5*x*(a + b*x
^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a^4*(a + b*
x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7*a^2*(a + b
*x^2)^(7/2))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=334

$$-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}+\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(\right.\right.}{105a^7}$$

Rubi [A] time = 0.48, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 12, 271, 192, 191}

$$\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{35a^6\left(a+bx^2\right)^{5/2}}+\frac{8bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{21a^5\left(a+bx^2\right)^{7/2}}+\frac{48Ab^3-a\left(3a^2D-10abC+24b^2B\right)}{3a^4x\left(a+bx^2\right)^{7/2}}-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(7*a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/2)
) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*A*
b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8*b
*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7/2
)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a + b
*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(105
*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2
*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{14Ab-7a\left(B+Cx^2+Dx^4\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{12b(14Ab-7aB)-5a\left(-7aC-7aDx^2\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{35a^2}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10b\left(168Ab^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^3-a \cdots\right.}{\cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(\cdots\right.}{3a \cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(\cdots\right.}{3a \cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(\cdots\right.}{3a \cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(\cdots\right.}{3a \cdots}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a\left(\cdots\right.}{3a \cdots}$$

Mathematica [A] time = 0.16, size = 234, normalized size = 0.70

$$\frac{-a^7\left(15A+21Bx^2+35x^4\left(C+3Dx^2\right)\right)+14a^6bx^2\left(3A+6Bx^2+25Cx^4-60Dx^6\right)-56a^5b^2x^4\left(3A+15Bx^2-50Cx^4+30Dx^6\right)+112a^4b^3x^6\left(15A-60Bx^2+50Cx^4-12Dx^6\right)+128a^3b^4x^8\left(105A-105Bx^2+35Cx^4-3Dx^6\right)+256a^2b^5x^{10}\left(105A-42Bx^2+5Cx^4\right)-3072ab^6x^{12}\left(Bx^2-7A\right)+6144Ab^7x^{14}}{105a^8x^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*A
 - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^6)
 + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4*x^
8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2
 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.72, size = 304, normalized size = 0.91

$$\frac{-15a^7A - 21a^7Bx^2 - 35a^7Cx^4 - 105a^7Dx^6 + 42a^6Abx^2 + 84a^6bBx^4 + 350a^6bCx^6 - 840a^6bDx^8 - 168a^5Ab^2x^4 - 840a^5b^2Bx^6 + 2800a^5b^2Cx^8 - 1680a^5b^2Dx^{10} + 1680a^4Ab^3x^6 - 6720a^4b^3Bx^8 + 5600a^4b^3Cx^{10} - 1344a^4b^3Dx^{12} + 13440a^3Ab^4x^8 - 13440a^3b^4Bx^{10} + 4480a^3b^4Cx^{12} - 384a^3b^4Dx^{14} + 26880a^2Ab^5x^{10} - 10752a^2b^5Bx^{12} + 1280a^2b^5Cx^{14} + 21504aAb^6x^{12} - 3072ab^6Bx^{14} + 6144Ab^7x^{14}}{105a^8x^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^7*A + 42*a^6*A*b*x^2 - 21*a^7*B*x^2 - 168*a^5*A*b^2*x^4 + 84*a^6*b*B
*x^4 - 35*a^7*C*x^4 + 1680*a^4*A*b^3*x^6 - 840*a^5*b^2*B*x^6 + 350*a^6*b*C*
x^6 - 105*a^7*D*x^6 + 13440*a^3*A*b^4*x^8 - 6720*a^4*b^3*B*x^8 + 2800*a^5*b
^2*C*x^8 - 840*a^6*b*D*x^8 + 26880*a^2*A*b^5*x^10 - 13440*a^3*b^4*B*x^10 +
5600*a^4*b^3*C*x^10 - 1680*a^5*b^2*D*x^10 + 21504*a*A*b^6*x^12 - 10752*a^2*
b^5*B*x^12 + 4480*a^3*b^4*C*x^12 - 1344*a^4*b^3*D*x^12 + 6144*A*b^7*x^14 -
3072*a*b^6*B*x^14 + 1280*a^2*b^5*C*x^14 - 384*a^3*b^4*D*x^14)/(105*a^8*x^7*
(a + b*x^2)^(7/2))
```

fricas [A] time = 2.90, size = 311, normalized size = 0.93

$$-\frac{\left(128\left(3Da^3b^4 - 10Ca^2b^5 + 24Bab^6 - 48Ab^7\right)x^{14} + 448\left(3Da^4b^3 - 10Ca^3b^4 + 24Ba^2b^5 - 48Aab^6\right)x^{12} + 560\left(3Da^5b^2 - 10Ca^4b^3 + 24Ba^3b^4 - 48Aa^2b^5\right)x^{10} + 280\left(3Da^6b - 10Ca^5b^2 + 24Ba^4b^3 - 48Aa^3b^4\right)x^8 + 15Aa^7 + 35\left(3Da^7 - 10Ca^6b + 24Ba^5b^2 - 48Aa^4b^3\right)x^6 + 7\left(5Ca^7 - 12Ba^6b + 24Aa^5b^2\right)x^4 + 21\left(Ba^7 - 2Aa^6b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^8b^4x^{15} + 4a^9b^3x^{13} + 6a^{10}b^2x^{11} + 4a^{11}bx^9 + a^{12}x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 448
*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*D*a
^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*D*a^6*b
- 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*a^7
- 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^6*b +
 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8*b^4*x
^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

giac [B] time = 0.71, size = 938, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*a
^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^20*
b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 + 16
5*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10*C*a
^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/2) +
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) +
 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*
b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 42840*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 2
8560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*
sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 21966*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 30
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 8652*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^6*b^(5/2) + 19404*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(7/2) + 105*D*a^9*sqrt(b) - 490*C*a^8*b^(3/2) + 1386*B*a^7*b^(5/2) - 3072*A
*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^7)
```

maple [A] time = 0.01, size = 301, normalized size = 0.90

$$-\frac{-6144 A b^7 x^{14}+3072 B a b^6 x^{14}-1280 C a^2 b^5 x^{14}+384 D a^3 b^4 x^{14}-21504 A a b^6 x^{12}+10752 B a^2 b^5 x^{12}-4480 C a^3 b^4 x^{12}+1344 D a^4 b^3 x^{12}-26880 A a^2 b^5 x^{10}+13440 B a^3 b^4 x^{10}-5600 C a^4 b^3 x^{10}+1680 D a^5 b^2 x^{10}-13440 A a^3 b^4 x^8+6720 B a^4 b^3 x^8-2800 C a^5 b^2 x^8+840 D a^6 b x^8-1680 A a^4 b^3 x^6+840 B a^5 b^2 x^6-350 C a^6 b x^6+105 D a^7 x^6+168 A a^5 b^2 x^4-84 B a^6 b x^4+35 C a^7 x^4-42 A a^6 b x^2+21 B a^7 x^2+15 A a^7}{105 \left(b x^2+a\right)^{\frac{7}{2}} a^8 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(-6144*A*b^7*x^14+3072*B*a*b^6*x^14-1280*C*a^2*b^5*x^14+384*D*a^3*b^
4*x^14-21504*A*a*b^6*x^12+10752*B*a^2*b^5*x^12-4480*C*a^3*b^4*x^12+1344*D*a
^4*b^3*x^12-26880*A*a^2*b^5*x^10+13440*B*a^3*b^4*x^10-5600*C*a^4*b^3*x^10+1
680*D*a^5*b^2*x^10-13440*A*a^3*b^4*x^8+6720*B*a^4*b^3*x^8-2800*C*a^5*b^2*x^
8+840*D*a^6*b*x^8-1680*A*a^4*b^3*x^6+840*B*a^5*b^2*x^6-350*C*a^6*b*x^6+105*
D*a^7*x^6+168*A*a^5*b^2*x^4-84*B*a^6*b*x^4+35*C*a^7*x^4-42*A*a^6*b*x^2+21*B
*a^7*x^2+15*A*a^7)/(b*x^2+a)^(7/2)/x^7/a^8
```

maxima [A] time = 1.52, size = 489, normalized size = 1.46

$$-\frac{128 Dbx}{35 \sqrt{bx^2+a}\, a^5}-\frac{64 Dbx}{35 \left(bx^2+a\right)^{\frac{3}{2}} a^4}-\frac{48 Dbx}{35 \left(bx^2+a\right)^{\frac{5}{2}} a^3}-\frac{8 Dbx}{7 \left(bx^2+a\right)^{\frac{7}{2}} a^2}+\frac{256 Cb^2x}{21 \sqrt{bx^2+a}\, a^6}+\frac{128 Cb^2x}{21 \left(bx^2+a\right)^{\frac{3}{2}} a^5}+\frac{32 Cb^2x}{7 \left(bx^2+a\right)^{\frac{5}{2}} a^4}+\frac{80 Cb^2x}{21 \left(bx^2+a\right)^{\frac{7}{2}} a^3}-\frac{1024 Bb^3x}{35 \sqrt{bx^2+a}\, a^7}-\frac{512 Bb^3x}{35 \left(bx^2+a\right)^{\frac{3}{2}} a^6}-\frac{384 Bb^3x}{35 \left(bx^2+a\right)^{\frac{5}{2}} a^5}-\frac{64 Bb^3x}{7 \left(bx^2+a\right)^{\frac{7}{2}} a^4}+\frac{2048 Ab^4x}{35 \sqrt{bx^2+a}\, a^8}+\frac{1024 Ab^4x}{35 \left(bx^2+a\right)^{\frac{3}{2}} a^7}+\frac{768 Ab^4x}{35 \left(bx^2+a\right)^{\frac{5}{2}} a^6}+\frac{128 Ab^4x}{7 \left(bx^2+a\right)^{\frac{7}{2}} a^5}-\frac{D}{\left(bx^2+a\right)^{\frac{7}{2}} ax}+\frac{10 Cb}{3 \left(bx^2+a\right)^{\frac{7}{2}} a^2x}-\frac{8 Bb^2}{\left(bx^2+a\right)^{\frac{7}{2}} a^3x}+\frac{16 Ab^3}{\left(bx^2+a\right)^{\frac{7}{2}} a^4x}-\frac{C}{3 \left(bx^2+a\right)^{\frac{7}{2}} ax^3}+\frac{4 Bb}{5 \left(bx^2+a\right)^{\frac{7}{2}} a^2x^3}-\frac{8 Ab^2}{5 \left(bx^2+a\right)^{\frac{7}{2}} a^3x^3}-\frac{B}{5 \left(bx^2+a\right)^{\frac{7}{2}} ax^5}+\frac{2 Ab}{5 \left(bx^2+a\right)^{\frac{7}{2}} a^2x^5}-\frac{A}{7 \left(bx^2+a\right)^{\frac{7}{2}} ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*D*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*D*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*D*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*D*b*x/((b*x^2 + a)^(7/2)*a^2) +
256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*A*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*A
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - D/((b*x^2 + a)^(7/2)*a*x) + 10/3*C*b/((b*x
^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b^3/((b*x^2
 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/((b*x^2 + a)
^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B/((b*x^2 + a
)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/((b*x^2 + a)^(
7/2)*a*x^7)
```

mupad [B] time = 2.84, size = 421, normalized size = 1.26

$$\frac{\frac{61\,B\,b}{35\,a^3}+\frac{78\,B\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,C\,b}{21\,a^5}+\frac{256\,C\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{C}{3\,a^2}+\frac{19\,C\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{167\,A\,b^2}{35\,a^4}+\frac{191\,A\,b^3\,x^2}{35\,a^5}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{1024\,A\,b^3}{35\,a^7}+\frac{2048\,A\,b^4\,x^2}{35\,a^8}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,B\,b^2}{35\,a^6}+\frac{1024\,B\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7}-\frac{B\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}-\frac{{\left(\frac{a}{b\,x^2}+1\right)}^{9/2}\,D\,{}_2\mathrm{F}_1\left(\frac{9}{2},5;\ 6;\ -\frac{a}{b\,x^2}\right)}{10\,x\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{34\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5}-\frac{B\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,C\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{C\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{58\,A\,b^3}{7\,a^6\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{A\,b^2}{7\,a^3\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,B\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*B*b)/(35*a^3) + (78*B*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*C*b)/(21*a^5) + (256*C*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (C/(3*
a^2) + (19*C*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - ((167*A*b^2)/(35*a^
4) + (191*A*b^3*x^2)/(35*a^5))/(x^3*(a + b*x^2)^(5/2)) + ((1024*A*b^3)/(35*
a^7) + (2048*A*b^4*x^2)/(35*a^8))/(x*(a + b*x^2)^(1/2)) - ((512*B*b^2)/(35*
a^6) + (1024*B*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x^2)^(1
/2))/(7*a^5*x^7) - (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - ((a/(b*x^2) + 1)^(9/
2)*D*hypergeom([9/2, 5], 6, -a/(b*x^2)))/(10*x*(a + b*x^2)^(9/2)) + (34*A*b
*(a + b*x^2)^(1/2))/(35*a^6*x^5) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (3
2*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (C*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) -
 (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(3/2)) + (A*b^2)/(7*a^3*x^3*(a + b*x^2)^(7
/2)) + (27*B*b^2)/(7*a^5*x*(a + b*x^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{256b^2x\left(128Ab^3-3a\left(5a^2D-12abC+24b^2B\right)\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(128A\right.}{}$$

Rubi [A] time = 0.55, antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1803, 12, 271, 192, 191}

$$-\frac{256b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{105a^7\left(a+bx^2\right)^{5/2}}-\frac{16b^2x\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{63a^6\left(a+bx^2\right)^{7/2}}-\frac{2b\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)}{9a^5x\left(a+bx^2\right)^{7/2}}+\frac{-15a^3D-36ab(2bB-aC)+128Ab^3}{45a^4x^3\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -A/(9*a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)^(
7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (12
8*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(45*a^4*x^3*(a + b*x^2)^(7/2)) -
 (2*b*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)^(
7/2)) - (16*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*(a
 + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)
/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) -
 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*b^3 - 36*a*b*(2
*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{16Ab-9a\left(B+Cx^2+Dx^4\right)}{x^8\left(a+bx^2\right)^{9/2}}\,dx}{9a}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{14b(16Ab-9aB)-7a\left(-9aC-9aDx^2\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{63a^2}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12b\left(224A \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} - \frac{\left(128Ab^3 - \cdots\right.}{\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

$$= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{128Ab^3 - \cdots}{4\cdots}$$

Mathematica [A] time = 0.18, size = 270, normalized size = 0.69

$$\frac{-a^8\left(35A+45Bx^2+63Cx^4+105Dx^6\right)+2a^7bx^2\left(40A+21\left(3Bx^2+6Cx^4+25Dx^6\right)\right)-56a^6b^2x^4\left(4A+9Bx^2+45Cx^4-150Dx^6\right)+112a^5b^3x^6\left(8A+45Bx^2-180Cx^4+150Dx^6\right)+4480a^4b^4x^8\left(-2A+9Bx^2-9Cx^4+3Dx^6\right)+256a^3b^5x^{10}\left(-280A+315Bx^2-126Cx^4+15Dx^6\right)-1024a^2b^6x^{12}\left(140A-63Bx^2+9Cx^4\right)+2048ab^7x^{14}\left(9Bx^2-56A\right)-32768Ab^8x^{16}}{315a^9x^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12*
(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 - 1
50*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*a^3
*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^
2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 1
50*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^
9*x^9*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.83, size = 352, normalized size = 0.90

$$\frac{-35a^8A-45a^8Bx^2-63a^8Cx^4-105a^8Dx^6+80a^7Abx^2+126a^7bBx^4+252a^7bCx^6+1050a^7bDx^8-224a^6Ab^2x^4-504a^6b^2Bx^6-2520a^6b^2Cx^8+8400a^6b^2Dx^{10}+896a^5Ab^3x^6+5040a^5b^3Bx^8-20160a^5b^3Cx^{10}+16800a^5b^3Dx^{12}-8960a^4Ab^4x^8+40320a^4b^4Bx^{10}-40320a^4b^4Cx^{12}+13440a^4b^4Dx^{14}-71680a^3Ab^5x^{10}+80640a^3b^5Bx^{12}-32256a^3b^5Cx^{14}+3840a^3b^5Dx^{16}-143360a^2Ab^6x^{12}+64512a^2b^6Bx^{14}-9216a^2b^6Cx^{16}-114688aAb^7x^{14}+18432ab^7Bx^{16}-32768Ab^8x^{16}}{315a^9x^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^8*A + 80*a^7*A*b*x^2 - 45*a^8*B*x^2 - 224*a^6*A*b^2*x^4 + 126*a^7*b*
B*x^4 - 63*a^8*C*x^4 + 896*a^5*A*b^3*x^6 - 504*a^6*b^2*B*x^6 + 252*a^7*b*C*
x^6 - 105*a^8*D*x^6 - 8960*a^4*A*b^4*x^8 + 5040*a^5*b^3*B*x^8 - 2520*a^6*b^
2*C*x^8 + 1050*a^7*b*D*x^8 - 71680*a^3*A*b^5*x^10 + 40320*a^4*b^4*B*x^10 -
20160*a^5*b^3*C*x^10 + 8400*a^6*b^2*D*x^10 - 143360*a^2*A*b^6*x^12 + 80640*
a^3*b^5*B*x^12 - 40320*a^4*b^4*C*x^12 + 16800*a^5*b^3*D*x^12 - 114688*a*A*b
^7*x^14 + 64512*a^2*b^6*B*x^14 - 32256*a^3*b^5*C*x^14 + 13440*a^4*b^4*D*x^1
4 - 32768*A*b^8*x^16 + 18432*a*b^7*B*x^16 - 9216*a^2*b^6*C*x^16 + 3840*a^3*
b^5*D*x^16)/(315*a^9*x^9*(a + b*x^2)^(7/2))
```

fricas [A] time = 3.81, size = 354, normalized size = 0.90

$$\frac{\left(256\left(15Da^3b^5-36Ca^2b^6+72Bab^7-128Ab^8\right)x^{16}+896\left(15Da^4b^4-36Ca^3b^5+72Ba^2b^6-128Aab^7\right)x^{14}+1120\left(15Da^5b^3-36Ca^4b^4+72Ba^3b^5-128Aa^2b^6\right)x^{12}+560\left(15Da^6b^2-36Ca^5b^3+72Ba^4b^4-128Aa^3b^5\right)x^{10}-35Aa^8+70\left(15Da^7b-36Ca^6b^2+72Ba^5b^3-128Aa^4b^4\right)x^8-7\left(15Da^8-36Ca^7b+72Ba^6b^2-128Aa^5b^3\right)x^6-7\left(9Ca^8-18Ba^7b+32Aa^6b^2\right)x^4-5\left(9Ba^8-16Aa^7b\right)x^2\right)\sqrt{bx^2+a}}{315\left(a^9b^4x^{17}+4a^{10}b^3x^{15}+6a^{11}b^2x^{13}+4a^{12}bx^{11}+a^{13}x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 89
6*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*(1
5*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(15*D
*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8 + 7
0*(15*D*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(15*D*
a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 18*B*a^
7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 + a)/(a^
9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^13*x^9)
```

giac [B] time = 0.74, size = 1162, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A*
a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a^2
3*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b^7
+ 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5 - 2
0*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a)^(7/
2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 3150*(sqr
t(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(b*x^2 +
 a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 10
710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a
^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 39270*(s
qrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b
^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 81270*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(
7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 103950*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2)
 - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 84210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*D*a^8*b^(3/2) + 243432*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*C*a^7*b^(5/2) - 551124*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 10
73856*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 42210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*D*a^9*b^(3/2) - 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a
^8*b^(5/2) + 275076*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^7*b^(7/2) - 533124*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 11970*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*D*a^10*b^(3/2) + 34272*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^9*b^(
5/2) - 76644*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^8*b^(7/2) + 147456*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^7*b^(9/2) + 1470*D*a^11*b^(3/2) - 4158*C*a^10*
b^(5/2) + 9216*B*a^9*b^(7/2) - 17609*A*a^8*b^(9/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^9*a^8)
```

maple [A] time = 0.01, size = 349, normalized size = 0.89

$$-\frac{32768 A b^{8} x^{16}-18432 B a b^{7} x^{16}+9216 C a^{2} b^{6} x^{16}-3840 D a^{3} b^{5} x^{16}+114688 A a b^{7} x^{14}-64512 B a^{2} b^{6} x^{14}+32256 C a^{3} b^{5} x^{14}-13440 D a^{4} b^{4} x^{14}+143360 A a^{2} b^{6} x^{12}-80640 B a^{3} b^{5} x^{12}+40320 C a^{4} b^{4} x^{12}-16800 D a^{5} b^{3} x^{12}+71680 A a^{3} b^{5} x^{10}-40320 B a^{4} b^{4} x^{10}+20160 C a^{5} b^{3} x^{10}-8400 D a^{6} b^{2} x^{10}+8960 A a^{4} b^{4} x^{8}-5040 B a^{5} b^{3} x^{8}+2520 C a^{6} b^{2} x^{8}-1050 D a^{7} b x^{8}-896 A a^{5} b^{3} x^{6}+504 B a^{6} b^{2} x^{6}-252 C a^{7} b x^{6}+105 D a^{8} x^{6}+224 A a^{6} b^{2} x^{4}-126 B a^{7} b x^{4}+63 C a^{8} x^{4}-80 A a^{7} b x^{2}+45 B a^{8} x^{2}+35 A a^{8}}{315 \left(b x^{2}+a\right)^{\frac{7}{2}} a^{9} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/315*(32768*A*b^8*x^16-18432*B*a*b^7*x^16+9216*C*a^2*b^6*x^16-3840*D*a^3*
b^5*x^16+114688*A*a*b^7*x^14-64512*B*a^2*b^6*x^14+32256*C*a^3*b^5*x^14-1344
0*D*a^4*b^4*x^14+143360*A*a^2*b^6*x^12-80640*B*a^3*b^5*x^12+40320*C*a^4*b^4
*x^12-16800*D*a^5*b^3*x^12+71680*A*a^3*b^5*x^10-40320*B*a^4*b^4*x^10+20160*
C*a^5*b^3*x^10-8400*D*a^6*b^2*x^10+8960*A*a^4*b^4*x^8-5040*B*a^5*b^3*x^8+25
20*C*a^6*b^2*x^8-1050*D*a^7*b*x^8-896*A*a^5*b^3*x^6+504*B*a^6*b^2*x^6-252*C
*a^7*b*x^6+105*D*a^8*x^6+224*A*a^6*b^2*x^4-126*B*a^7*b*x^4+63*C*a^8*x^4-80*
A*a^7*b*x^2+45*B*a^8*x^2+35*A*a^8)/(b*x^2+a)^(7/2)/x^9/a^9
```

maxima [A] time = 1.57, size = 579, normalized size = 1.48

$$\frac{256\,Db^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Db^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Db^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Db^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{1024\,Cb^3x}{35\,\sqrt{bx^2+a}\,a^7}-\frac{512\,Cb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6}-\frac{384\,Cb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5}-\frac{64\,Cb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4}+\frac{2048\,Bb^4x}{35\,\sqrt{bx^2+a}\,a^8}+\frac{1024\,Bb^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7}+\frac{768\,Bb^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6}+\frac{128\,Bb^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5}-\frac{32768\,Ab^5x}{315\,\sqrt{bx^2+a}\,a^9}-\frac{16384\,Ab^5x}{315\,(bx^2+a)^{\frac{3}{2}}a^8}-\frac{4096\,Ab^5x}{105\,(bx^2+a)^{\frac{5}{2}}a^7}-\frac{2048\,Ab^5x}{63\,(bx^2+a)^{\frac{7}{2}}a^6}+\frac{10\,Db}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{8\,Cb^2}{(bx^2+a)^{\frac{7}{2}}a^3x}+\frac{16\,Bb^3}{(bx^2+a)^{\frac{7}{2}}a^4x}-\frac{256\,Ab^4}{9\,(bx^2+a)^{\frac{7}{2}}a^5x}-\frac{D}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}+\frac{4\,Cb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3}-\frac{8\,Bb^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3}+\frac{128\,Ab^3}{45\,(bx^2+a)^{\frac{7}{2}}a^4x^3}-\frac{C}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}+\frac{2\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5}-\frac{32\,Ab^2}{45\,(bx^2+a)^{\frac{7}{2}}a^3x^5}-\frac{B}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}+\frac{16\,Ab}{63\,(bx^2+a)^{\frac{7}{2}}a^2x^7}-\frac{A}{9\,(bx^2+a)^{\frac{7}{2}}ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*D*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*D*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*D*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*D*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*B
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*a^9) -
16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^2 + a)^(
5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*D*b/((b*x^2 + a)
^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*x^2 + a)^(
7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*D/((b*x^2 + a)^(7
/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((b*x^2 + a)^(
7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5*C/((b*x^2 +
a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45*A*b^2/((b*x^2
 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/63*A*b/((b*x^2
+ a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=214

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be\right.}{3b^6}$$

Rubi [A] time = 0.22, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1811, 1799, 1850}

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2be-10a^3f-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(4a^2be-5a^3f-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx &= \int \frac{x\left(cx^4+dx^6+ex^8+fx^{10}\right)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{cx^2+dx^3+ex^4+fx^5}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^2\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^5\sqrt{a+bx}}+\frac{a\left(-2b^3c+3ab^2d-4a^2be+5a\right.}{b^5}\right.\right. \\
&= \frac{a^2\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^6}-\frac{a\left(2b^3c-3ab^2d+4a^2be-5a^3f\right)\left(a+\right.}{3b^6}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 158, normalized size = 0.74

$$
\frac{\sqrt{a+bx^2}\left(-1280a^5f+128a^4b\left(11e+5fx^2\right)-16a^3b^2\left(99d+44ex^2+30fx^4\right)+8a^2b^3\left(231c+99dx^2+66ex^4+50fx^6\right)-2ab^4x^2\left(462c+297dx^2+220ex^4+175fx^6\right)+b^5x^4\left(693c+5\left(99dx^2+77ex^4+63fx^6\right)\right)\right)}{3465b^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

IntegrateAlgebraic [A] time = 0.11, size = 196, normalized size = 0.92

$$
\frac{\sqrt{a+bx^2}\left(-1280a^5f+1408a^4be+640a^4bfx^2-1584a^3b^2d-704a^3b^2ex^2-480a^3b^2fx^4+1848a^2b^3c+792a^2b^3dx^2+528a^2b^3ex^4+400a^2b^3fx^6-924ab^4cx^2-594ab^4dx^4-440ab^4ex^6-350ab^4fx^8+693b^5cx^4+495b^5dx^6+385b^5ex^8+315b^5fx^{10}\right)}{3465b^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*e - 1280*a^5
*f - 924*a*b^4*c*x^2 + 792*a^2*b^3*d*x^2 - 704*a^3*b^2*e*x^2 + 640*a^4*b*f*
x^2 + 693*b^5*c*x^4 - 594*a*b^4*d*x^4 + 528*a^2*b^3*e*x^4 - 480*a^3*b^2*f*x
^4 + 495*b^5*d*x^6 - 440*a*b^4*e*x^6 + 400*a^2*b^3*f*x^6 + 385*b^5*e*x^8 -
350*a*b^4*f*x^8 + 315*b^5*f*x^10))/(3465*b^6)
```

fricas [A] time = 0.96, size = 177, normalized size = 0.83

$$
\frac{\left(315\,b^5fx^{10}+35\left(11\,b^5e-10\,ab^4f\right)x^8+5\left(99\,b^5d-88\,ab^4e+80\,a^2b^3f\right)x^6+1848\,a^2b^3c-1584\,a^3b^2d+1408\,a^4be-1280\,a^5f+3\left(231\,b^5c-198\,ab^4d+176\,a^2b^3e-160\,a^3b^2f\right)x^4-4\left(231\,ab^4c-198\,a^2b^3d+176\,a^3b^2e-160\,a^4bf\right)x^2\right)\sqrt{bx^2+a}}{3465\,b^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

giac [A] time = 0.45, size = 264, normalized size = 1.23

$$\frac{\left(a^2b^3c - a^3b^2d - a^5f + a^4be\right)\sqrt{bx^2+a}}{b^6} + \frac{693\left(bx^2+a\right)^{\frac{5}{2}}b^3c - 2310\left(bx^2+a\right)^{\frac{3}{2}}ab^3c + 495\left(bx^2+a\right)^{\frac{7}{2}}b^2d - 2079\left(bx^2+a\right)^{\frac{5}{2}}ab^2d + 3465\left(bx^2+a\right)^{\frac{3}{2}}a^2b^2d + 315\left(bx^2+a\right)^{\frac{11}{2}}f - 1925\left(bx^2+a\right)^{\frac{9}{2}}af + 4950\left(bx^2+a\right)^{\frac{7}{2}}a^2f - 6930\left(bx^2+a\right)^{\frac{5}{2}}a^3f + 5775\left(bx^2+a\right)^{\frac{3}{2}}a^4f + 385\left(bx^2+a\right)^{\frac{9}{2}}be - 1980\left(bx^2+a\right)^{\frac{7}{2}}abe + 4158\left(bx^2+a\right)^{\frac{5}{2}}a^2be - 4620\left(bx^2+a\right)^{\frac{3}{2}}a^3be}{3465\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

maple [A] time = 0.01, size = 193, normalized size = 0.90

$$-\frac{\sqrt{b\,x^2+a}\,\left(-315f\,x^{10}b^5 + 350a\,b^4f\,x^8 - 385b^5e\,x^8 - 400a^2b^3f\,x^6 + 440a\,b^4e\,x^6 - 495b^5d\,x^6 + 480a^3b^2f\,x^4 - 528a^2b^3e\,x^4 + 594a\,b^4d\,x^4 - 693b^5c\,x^4 - 640a^4bf\,x^2 + 704a^3b^2e\,x^2 - 792a^2b^3d\,x^2 + 924a\,b^4c\,x^2 + 1280a^5f - 1408a^4be + 1584a^3b^2d - 1848a^2b^3c\right)}{3465b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3465*(b*x^2+a)^(1/2)*(-315*b^5*f*x^10+350*a*b^4*f*x^8-385*b^5*e*x^8-400*
a^2*b^3*f*x^6+440*a*b^4*e*x^6-495*b^5*d*x^6+480*a^3*b^2*f*x^4-528*a^2*b^3*e
*x^4+594*a*b^4*d*x^4-693*b^5*c*x^4-640*a^4*b*f*x^2+704*a^3*b^2*e*x^2-792*a^
2*b^3*d*x^2+924*a*b^4*c*x^2+1280*a^5*f-1408*a^4*b*e+1584*a^3*b^2*d-1848*a^2
*b^3*c)/b^6
```

maxima [A] time = 1.43, size = 347, normalized size = 1.62

$$\frac{\sqrt{bx^2+a}\,fx^{10}}{11\,b} + \frac{\sqrt{bx^2+a}\,ex^8}{9\,b} - \frac{10\,\sqrt{bx^2+a}\,afx^8}{99\,b^2} + \frac{\sqrt{bx^2+a}\,dx^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}\,aex^6}{63\,b^2} + \frac{80\,\sqrt{bx^2+a}\,a^2fx^6}{693\,b^3} + \frac{\sqrt{bx^2+a}\,cx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}\,adx^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}\,a^2ex^4}{105\,b^3} - \frac{32\,\sqrt{bx^2+a}\,a^3fx^4}{231\,b^4} - \frac{4\,\sqrt{bx^2+a}\,acx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}\,a^2dx^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}\,a^3ex^2}{315\,b^4} + \frac{128\,\sqrt{bx^2+a}\,a^4fx^2}{693\,b^5} + \frac{8\,\sqrt{bx^2+a}\,a^2c}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}\,a^3d}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}\,a^4e}{315\,b^5} - \frac{256\,\sqrt{bx^2+a}\,a^5f}{693\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
```

```
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

mupad [B] time = 1.20, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5 + d*x^7 + e*x^9 + f*x^11)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

sympy [A] time = 9.37, size = 442, normalized size = 2.07

$$\begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6}+\frac{128a^4e\sqrt{a+bx^2}}{315b^5}+\frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5}-\frac{16a^3d\sqrt{a+bx^2}}{35b^4}-\frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4}-\frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4}+\frac{8a^2c\sqrt{a+bx^2}}{15b^3}+\frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3}+\frac{16a^2ex^4\sqrt{a+bx^2}}{105b^3}+\frac{80a^2fx^6\sqrt{a+bx^2}}{693b^3}-\frac{4acx^2\sqrt{a+bx^2}}{15b^2}-\frac{6adx^4\sqrt{a+bx^2}}{35b^2}-\frac{8aex^6\sqrt{a+bx^2}}{63b^2}-\frac{10afx^8\sqrt{a+bx^2}}{99b^2}+\frac{cx^4\sqrt{a+bx^2}}{5b}+\frac{dx^6\sqrt{a+bx^2}}{7b}+\frac{ex^8\sqrt{a+bx^2}}{9b}+\frac{fx^{10}\sqrt{a+bx^2}}{11b} & \text{for}\ b \neq 0 \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.166 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2\right.}{b^5}$$

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1811, 1799, 1850}

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2be-4a^3f-2ab^2d+b^3c\right)}{3b^5}-\frac{a\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^5}+\frac{\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{\left(a+bx^2\right)^{7/2}(be-4af)}{7b^5}+\frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx &= \int \frac{x\left(cx^2+dx^4+ex^6+fx^8\right)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{cx+dx^2+ex^3+fx^4}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a\left(-b^3c+ab^2d-a^2be+a^3f\right)}{b^4\sqrt{a+bx}} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\sqrt{a}}{b^4}\right.\right. \\
&= -\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^5} + \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)\left(a+bx\right.}{3b^5}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(128a^4f-16a^3b\left(9e+4fx^2\right)+24a^2b^2\left(7d+3ex^2+2fx^4\right)-2ab^3\left(105c+42dx^2+27ex^4+20fx^6\right)+b^4x^2\left(105c+63dx^2+45ex^4+35fx^6\right)\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 148, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(128a^4f-144a^3be-64a^3bfx^2+168a^2b^2d+72a^2b^2ex^2+48a^2b^2fx^4-210ab^3c-84ab^3dx^2-54ab^3ex^4-40ab^3fx^6+105b^4cx^2+63b^4dx^4+45b^4ex^6+35b^4fx^8\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-210*a*b^3*c + 168*a^2*b^2*d - 144*a^3*b*e + 128*a^4*f +
105*b^4*c*x^2 - 84*a*b^3*d*x^2 + 72*a^2*b^2*e*x^2 - 64*a^3*b*f*x^2 + 63*b^4
*d*x^4 - 54*a*b^3*e*x^4 + 48*a^2*b^2*f*x^4 + 45*b^4*e*x^6 - 40*a*b^3*f*x^6
+ 35*b^4*f*x^8))/(315*b^5)
```

fricas [A] time = 1.11, size = 134, normalized size = 0.80

$$\frac{\left(35\,b^4fx^8+5\left(9\,b^4e-8\,ab^3f\right)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\left(21\,b^4d-18\,ab^3e+16\,a^2b^2f\right)x^4+\left(105\,b^4c-84\,ab^3d+72\,a^2b^2e-64\,a^3bf\right)x^2\right)\sqrt{bx^2+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

giac [A] time = 0.50, size = 197, normalized size = 1.18

$$-\frac{\left(ab^3c - a^2b^2d - a^4f + a^3be\right)\sqrt{bx^2+a}}{b^5} + \frac{105\left(bx^2+a\right)^{\frac{3}{2}}b^3c + 63\left(bx^2+a\right)^{\frac{5}{2}}b^2d - 210\left(bx^2+a\right)^{\frac{3}{2}}ab^2d + 35\left(bx^2+a\right)^{\frac{9}{2}}f - 180\left(bx^2+a\right)^{\frac{7}{2}}af + 378\left(bx^2+a\right)^{\frac{5}{2}}a^2f - 420\left(bx^2+a\right)^{\frac{3}{2}}a^3f + 45\left(bx^2+a\right)^{\frac{7}{2}}be - 189\left(bx^2+a\right)^{\frac{5}{2}}abe + 315\left(bx^2+a\right)^{\frac{3}{2}}a^2be}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

maple [A] time = 0.01, size = 145, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}\,\left(35f\,x^8b^4 - 40a\,b^3f\,x^6 + 45b^4e\,x^6 + 48a^2b^2f\,x^4 - 54a\,b^3e\,x^4 + 63b^4d\,x^4 - 64a^3bf\,x^2 + 72a^2b^2e\,x^2 - 84a\,b^3d\,x^2 + 105b^4c\,x^2 + 128a^4f - 144a^3be + 168a^2b^2d - 210a\,b^3c\right)}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(1/2)*(35*b^4*f*x^8-40*a*b^3*f*x^6+45*b^4*e*x^6+48*a^2*b^2*
f*x^4-54*a*b^3*e*x^4+63*b^4*d*x^4-64*a^3*b*f*x^2+72*a^2*b^2*e*x^2-84*a*b^3*
d*x^2+105*b^4*c*x^2+128*a^4*f-144*a^3*b*e+168*a^2*b^2*d-210*a*b^3*c)/b^5
```

maxima [A] time = 1.37, size = 263, normalized size = 1.57

$$\frac{\sqrt{bx^2+a}\,fx^8}{9\,b} + \frac{\sqrt{bx^2+a}\,ex^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}\,afx^6}{63\,b^2} + \frac{\sqrt{bx^2+a}\,dx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}\,aex^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}\,a^2fx^4}{105\,b^3} + \frac{\sqrt{bx^2+a}\,cx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}\,adx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}\,a^2ex^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}\,a^3fx^2}{315\,b^4} - \frac{2\,\sqrt{bx^2+a}\,ac}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}\,a^2d}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}\,a^3e}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}\,a^4f}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

mupad [B] time = 1.14, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^3 + d*x^5 + e*x^7 + f*x^9)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

sympy [A] time = 5.41, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5}-\frac{16a^3e\sqrt{a+bx^2}}{35b^4}-\frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4}+\frac{8a^2d\sqrt{a+bx^2}}{15b^3}+\frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3}+\frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3}-\frac{2ac\sqrt{a+bx^2}}{3b^2}-\frac{4adx^2\sqrt{a+bx^2}}{15b^2}-\frac{6aex^4\sqrt{a+bx^2}}{35b^2}-\frac{8afx^6\sqrt{a+bx^2}}{63b^2}+\frac{cx^2\sqrt{a+bx^2}}{3b}+\frac{dx^4\sqrt{a+bx^2}}{5b}+\frac{ex^6\sqrt{a+bx^2}}{7b}+\frac{fx^8\sqrt{a+bx^2}}{9b} & \text{for}\ b\neq 0 \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

3.167 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7}}{7b^4}$$

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1811, 1799, 1850}

$$\frac{\sqrt{a+bx^2}\left(a^2be+a^3(-f)-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\left(a+bx^2\right)^{5/2}(be-3af)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1811

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \int \frac{x\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}} + \frac{\left(b^2d-2abe+3a^2f\right)\sqrt{a+bx}}{b^3} + \frac{(be-3a\right.$$

$$= \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4} + \frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(be-3\right.$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+8a^2b\left(7e+3fx^2\right)-2ab^2\left(35d+14ex^2+9fx^4\right)+b^3\left(105c+35dx^2+21ex^4+15fx^6\right)\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 102, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-48a^3f+56a^2be+24a^2bfx^2-70ab^2d-28ab^2ex^2-18ab^2fx^4+105b^3c+35b^3dx^2+21b^3ex^4+15b^3fx^6\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(105*b^3*c - 70*a*b^2*d + 56*a^2*b*e - 48*a^3*f + 35*b^3*d
*x^2 - 28*a*b^2*e*x^2 + 24*a^2*b*f*x^2 + 21*b^3*e*x^4 - 18*a*b^2*f*x^4 + 15
*b^3*f*x^6))/(105*b^4)
```

fricas [A] time = 0.71, size = 94, normalized size = 0.78

$$\frac{\left(15\,b^3fx^6+3\left(7\,b^3e-6\,ab^2f\right)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+\left(35\,b^3d-28\,ab^2e+24\,a^2bf\right)x^2\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

giac [A] time = 0.40, size = 130, normalized size = 1.07

$$\frac{\left(b^3c - ab^2d - a^3f + a^2be\right)\sqrt{bx^2 + a}}{b^4} + \frac{35\left(bx^2 + a\right)^{\frac{3}{2}}b^2d + 15\left(bx^2 + a\right)^{\frac{7}{2}}f - 63\left(bx^2 + a\right)^{\frac{5}{2}}af + 105\left(bx^2 + a\right)^{\frac{3}{2}}a^2f + 21\left(bx^2 + a\right)^{\frac{5}{2}}be - 70\left(bx^2 + a\right)^{\frac{3}{2}}abe}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

maple [A] time = 0.01, size = 99, normalized size = 0.82

$$-\frac{\sqrt{b\,x^2 + a}\,\left(-15f\,x^6b^3 + 18a\,b^2f\,x^4 - 21b^3e\,x^4 - 24a^2bf\,x^2 + 28a\,b^2e\,x^2 - 35b^3d\,x^2 + 48a^3f - 56a^2be + 70a\,b^2d - 105b^3c\right)}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-15*b^3*f*x^6+18*a*b^2*f*x^4-21*b^3*e*x^4-24*a^2*b*
f*x^2+28*a*b^2*e*x^2-35*b^3*d*x^2+48*a^3*f-56*a^2*b*e+70*a*b^2*d-105*b^3*c)
/b^4
```

maxima [A] time = 1.36, size = 180, normalized size = 1.49

$$\frac{\sqrt{bx^2 + a}\,fx^6}{7\,b} + \frac{\sqrt{bx^2 + a}\,ex^4}{5\,b} - \frac{6\,\sqrt{bx^2 + a}\,afx^4}{35\,b^2} + \frac{\sqrt{bx^2 + a}\,dx^2}{3\,b} - \frac{4\,\sqrt{bx^2 + a}\,aex^2}{15\,b^2} + \frac{8\,\sqrt{bx^2 + a}\,a^2fx^2}{35\,b^3} + \frac{\sqrt{bx^2 + a}\,c}{b} - \frac{2\,\sqrt{bx^2 + a}\,ad}{3\,b^2} + \frac{8\,\sqrt{bx^2 + a}\,a^2e}{15\,b^3} - \frac{16\,\sqrt{bx^2 + a}\,a^3f}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

mupad [B] time = 1.08, size = 103, normalized size = 0.85

$$\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-18\,a\,b^2\,f\right)}{105\,b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + d*x^3 + e*x^5 + f*x^7)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

sympy [A] time = 3.41, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+\frac{fx^6\sqrt{a+bx^2}}{7b} & \text{for } b\neq 0 \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.168 $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=261

$$\frac{x^3\left(a\left(162a^3F-71a^2bD+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A\right.}{}$$

Rubi [A] time = 0.72, antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1804, 1800, 1585, 1263, 1584, 455, 388, 217, 206}

$$\frac{x^3\left(a\left(-71a^2bD+162a^3F+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(17a^2bD-24a^3F-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(\frac{A}{a}-\frac{a^2bD+a^3(-F)-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{x(bD-4aF)}{b^5\sqrt{a+bx^2}}+\frac{(2bD-9aF)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2))
 + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)/(35*a
^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*
D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b
^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1263

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, -Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(2*d*
f*(q + 1)), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &&
GtQ[m, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1800

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/c, Int[(c*x)^(m + 1)*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(\left(4Ab+\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}\right)x\right)-\frac{7a\left(b^2C-a\cdots\right)}{b\cdots}\cdots\right)}{\left(a+bx^2\right)^{7/2}}\cdots}{7ab}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}-\frac{7a\left(b^2C-abD+a\cdots\right)}{b^2}\cdots\right)}{\left(a+bx^2\right)^{7/2}}\cdots}{7ab}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^4+a\left(3b^3B-10ab^2C+17a^2bD-\cdots\right.\right.}{35a^2b^4\left(a+bx^2\right)^{5/2}}\cdots$$

Mathematica [A] time = 0.59, size = 221, normalized size = 0.85

$$\frac{105a^{7/2}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,(2bD-9aF)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(945a^7F-210a^6b\left(D-15Fx^2\right)+14a^5b^2x^2\left(261Fx^2-50D\right)+4a^4b^3x^4\left(396Fx^2-203D\right)+a^3b^4x^6\left(105Fx^2-352D\right)+2a^2b^5x^2\left(35A+21Bx^2+15Cx^4\right)+4ab^6x^4\left(14A+3Bx^2\right)+16Ab^7x^6\right)}{210a^3b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a
^6*b*(D - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-5
0*D + 261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A
 + 21*B*x^2 + 15*C*x^4)) + 105*a^(7/2)*(2*b*D - 9*a*F)*(a + b*x^2)^3*Sqrt[1
 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(210*a^3*b^(11/2)*(a + b*x^2)^(
7/2))
```

IntegrateAlgebraic [A] time = 0.84, size = 224, normalized size = 0.86

$$\frac{945a^7Fx-210a^6bDx+3150a^6bFx^3-700a^5b^2Dx^3+3654a^5b^2Fx^5-812a^4b^3Dx^5+1584a^4b^3Fx^7-352a^3b^4Dx^7+105a^3b^4Fx^9+70a^2Ab^5x^3+42a^2b^5Bx^5+30a^2b^5Cx^7+56aAb^6x^5+12ab^6Bx^7+16Ab^7x^7}{210a^3b^5\left(a+bx^2\right)^{7/2}}+\frac{(9aF-2bD)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9
/2),x]
```

```
[Out] (-210*a^6*b*D*x + 945*a^7*F*x + 70*a^2*A*b^5*x^3 - 700*a^5*b^2*D*x^3 + 3150
*a^6*b*F*x^3 + 56*a*A*b^6*x^5 + 42*a^2*b^5*B*x^5 - 812*a^4*b^3*D*x^5 + 3654
*a^5*b^2*F*x^5 + 16*A*b^7*x^7 + 12*a*b^6*B*x^7 + 30*a^2*b^5*C*x^7 - 352*a^3
*b^4*D*x^7 + 1584*a^4*b^3*F*x^7 + 105*a^3*b^4*F*x^9)/(210*a^3*b^5*(a + b*x^
2)^(7/2)) + ((-2*b*D + 9*a*F)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(11
/2))
```

fricas [A] time = 1.52, size = 705, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] [-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*
F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a
^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*
b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2
*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 +
```

```
105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9
*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^8 - 2*D*
a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6
 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 + 2*(792*F*a^
4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F
*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 -
 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^
2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7
*b^6)]
```

giac [A] time = 0.56, size = 224, normalized size = 0.86

$$\frac{\left(\left(\left(\left(\frac{105\,Fx^2}{b}+\frac{2\left(792\,Fa^4b^7-176\,Da^3b^8+15\,Ca^2b^9+6\,Bab^{10}+8\,Ab^{11}\right)}{a^3b^9}\right)x^2+\frac{14\left(261\,Fa^5b^6-58\,Da^4b^7+3\,Ba^2b^9+4\,Aab^{10}\right)}{a^3b^9}\right)x^2+\frac{70\left(45\,Fa^6b^5-10\,Da^5b^6+Aa^2b^9\right)}{a^3b^9}\right)x^2+\frac{105\left(9\,Fa^7b^4-2\,Da^6b^5\right)}{a^3b^9}\right)x}{210\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{\left(9\,Fa-2\,Db\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 + 6
*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 + 3
*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^6 +
 A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(
b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(11/2)
```

maple [B] time = 0.01, size = 478, normalized size = 1.83

$$\frac{F x^9}{2\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{D x^7}{7\left(b x^2+a\right)^{\frac{7}{2}} b}+\frac{9 F a x^7}{14\left(b x^2+a\right)^{\frac{7}{2}} b^2}-\frac{C x^5}{2\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{D x^5}{5\left(b x^2+a\right)^{\frac{5}{2}} b^2}+\frac{9 F a x^5}{10\left(b x^2+a\right)^{\frac{5}{2}} b^3}-\frac{B x^3}{4\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{5 C a x^3}{8\left(b x^2+a\right)^{\frac{7}{2}} b^2}-\frac{A x}{7\left(b x^2+a\right)^{\frac{7}{2}} b}-\frac{3 B a x}{28\left(b x^2+a\right)^{\frac{7}{2}} b^2}-\frac{15 C a^2 x}{56\left(b x^2+a\right)^{\frac{7}{2}} b^3}-\frac{D x^3}{3\left(b x^2+a\right)^{\frac{3}{2}} b^3}+\frac{3 F a x^3}{2\left(b x^2+a\right)^{\frac{3}{2}} b^4}+\frac{A x}{35\left(b x^2+a\right)^{\frac{5}{2}} a b}+\frac{3 B x}{140\left(b x^2+a\right)^{\frac{5}{2}} b^2}+\frac{3 C a x}{56\left(b x^2+a\right)^{\frac{5}{2}} b^3}+\frac{4 A x}{105\left(b x^2+a\right)^{\frac{3}{2}} a^2 b}+\frac{B x}{35\left(b x^2+a\right)^{\frac{3}{2}} a b^2}+\frac{C x}{14\left(b x^2+a\right)^{\frac{3}{2}} b^3}+\frac{8 A x}{105\sqrt{b x^2+a}\, a^3 b}+\frac{2 B x}{35\sqrt{b x^2+a}\, a^2 b^2}+\frac{C x}{7\sqrt{b x^2+a}\, a b^3}-\frac{D x}{\sqrt{b x^2+a}\, b^4}+\frac{9 F a x}{2\sqrt{b x^2+a}\, b^5}+\frac{D \ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{b^{\frac{9}{2}}}-\frac{9 F a \ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{2 b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] 2/35/(b*x^2+a)^(1/2)*B/a^2/b^2*x-1/(b*x^2+a)^(1/2)*D/b^4*x-9/2*F*a/b^(11/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*F*x^9/b/(b*x^2+a)^(7/2)-1/7/(b*x^2+a)^(7
/2)*D/b*x^7-1/5/(b*x^2+a)^(5/2)*D/b^2*x^5-1/3/(b*x^2+a)^(3/2)*D/b^3*x^3-1/2
/(b*x^2+a)^(7/2)*C/b*x^5+1/14/(b*x^2+a)^(3/2)*C/b^3*x-1/4/(b*x^2+a)^(7/2)*B
/b*x^3+3/140/(b*x^2+a)^(5/2)*B/b^2*x-1/7/(b*x^2+a)^(7/2)*A/b*x+8/105/(b*x^2
+a)^(1/2)*A/a^3/b*x+4/105/(b*x^2+a)^(3/2)*A/a^2/b*x+3/56/(b*x^2+a)^(5/2)*C*
a/b^3*x+1/7/(b*x^2+a)^(1/2)*C/a/b^3*x-3/28/(b*x^2+a)^(7/2)*B*a/b^2*x+1/35/(
b*x^2+a)^(3/2)*B/a/b^2*x+1/35/(b*x^2+a)^(5/2)*A/a/b*x+9/14*F*a/b^2*x^7/(b*x
^2+a)^(7/2)-15/56/(b*x^2+a)^(7/2)*C*a^2/b^3*x+9/2*F*a/b^5*x/(b*x^2+a)^(1/2)
+9/10*F*a/b^3*x^5/(b*x^2+a)^(5/2)+3/2*F*a/b^4*x^3/(b*x^2+a)^(3/2)-5/8/(b*x^
2+a)^(7/2)*C*a/b^2*x^3+D/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.75, size = 826, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] 1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*F*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*F*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*F*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*F*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b
*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 9/2*F*a*arcsinh(b*x/
sqrt(a*b))/b^(11/2) + D*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(A+B\,x^2+C\,x^4+F\,x^8+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

3.169 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=214

$$\frac{x\left(Ab^4-a^4F\right)}{ab^4\left(a+bx^2\right)^{7/2}}+\frac{x^5\left(a\left(-58a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{15a^3b^2\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(-10a^4F+ab^3B+6Ab^4\right)}{3a^2b^3\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(-176a^3F+15a\right.\right.}{105}$$

Rubi [A] time = 0.41, antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1814, 1157, 385, 217, 206}

$$\frac{x\left(\frac{15a^2bD-176a^3F+6ab^2C+8b^3B}{b^4}+\frac{48A}{a}\right)}{105a^3\sqrt{a+bx^2}}+\frac{x\left(a\left(-45a^2bD+122a^3F+3ab^2C+4b^3B\right)+24Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x\left(\frac{15a^2bD-22a^3F-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{35a\left(a+bx^2\right)^{5/2}}+\frac{x\left(\frac{A}{a}-\frac{a^2bD+a^3(-F)-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{F\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2)) +
 (((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(35*a*(a +
 b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F))*x)/(105*a^3*b^4*(a + b*x^2)^(3/2)) + (((48*A)/a + (8*b^3*B + 6*a*b^2*C
 + 15*a^2*b*D - 176*a^3*F)/b^4)*x)/(105*a^3*Sqrt[a + b*x^2]) + (F*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^4}-\frac{7a\left(b^2C-abD+a^2F\right)x^2}{b^3}-\frac{7a(bD-aF}{b^2}}{\left(a+bx^2\right)^{7/2}}}{7a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{24Ab^4+4ab^3B}{}}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\right.}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\right.}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\right.}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} + \frac{\left(24Ab^4+a\right.}{}$$

Mathematica [A]   time = 0.51, size = 197, normalized size = 0.92

$$\frac{x\left(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4-176a^4b^3Fx^6+a^3b^4\left(105A+35Bx^2+21Cx^4+15Dx^6\right)+2a^2b^5x^2\left(105A+14Bx^2+3Cx^4\right)+8ab^6x^4\left(21A+Bx^2\right)+48Ab^7x^6\right)}{105a^4b^4\left(a+bx^2\right)^{7/2}}+\frac{\sqrt{a}F\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*a
^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^4*b
^4*(a + b*x^2)^(7/2)) + (Sqrt[a]*F*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(b^(9/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A]   time = 0.73, size = 204, normalized size = 0.95

$$\frac{-105a^7Fx-350a^6bFx^3-406a^5b^2Fx^5-176a^4b^3Fx^7+105a^3Ab^4x+35a^3b^4Bx^3+21a^3b^4Cx^5+15a^3b^4Dx^7+210a^2Ab^5x^3+28a^2b^5Bx^5+6a^2b^5Cx^7+168aAb^6x^5+8ab^6Bx^7+48Ab^7x^7}{105a^4b^4\left(a+bx^2\right)^{7/2}}-\frac{F\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (105*a^3*A*b^4*x - 105*a^7*F*x + 210*a^2*A*b^5*x^3 + 35*a^3*b^4*B*x^3 - 350
*a^6*b*F*x^3 + 168*a*A*b^6*x^5 + 28*a^2*b^5*B*x^5 + 21*a^3*b^4*C*x^5 - 406*
a^5*b^2*F*x^5 + 48*A*b^7*x^7 + 8*a*b^6*B*x^7 + 6*a^2*b^5*C*x^7 + 15*a^3*b^4
*D*x^7 - 176*a^4*b^3*F*x^7)/(105*a^4*b^4*(a + b*x^2)^(7/2)) - (F*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

fricas [A] time = 1.42, size = 567, normalized size = 2.65

$$\left[\frac{105\left(Fa^4b^4x^8+4Fa^5b^3x^6+6Fa^6b^2x^4+4Fa^7bx^2+Fa^8\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(\left(176Fa^4b^4-15Da^3b^5-6Ca^2b^6-8Bab^7-48Ab^8\right)x^7+7\left(58Fa^5b^3-3Ca^3b^5-4Ba^2b^6-24Aab^7\right)x^5+35\left(10Fa^6b^2-Ba^3b^5-6Aa^2b^6\right)x^3+105\left(Fa^7b-Aa^3b^5\right)x\right)\sqrt{bx^2+a}}{210\left(a^4b^9x^8+4a^5b^8x^6+6a^6b^7x^4+4a^7b^6x^2+a^8b^5\right)},-\frac{105\left(Fa^4b^4x^8+4Fa^5b^3x^6+6Fa^6b^2x^4+4Fa^7bx^2+Fa^8\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(\left(176Fa^4b^4-15Da^3b^5-6Ca^2b^6-8Bab^7-48Ab^8\right)x^7+7\left(58Fa^5b^3-3Ca^3b^5-4Ba^2b^6-24Aab^7\right)x^5+35\left(10Fa^6b^2-Ba^3b^5-6Aa^2b^6\right)x^3+105\left(Fa^7b-Aa^3b^5\right)x\right)\sqrt{bx^2+a}}{105\left(a^4b^9x^8+4a^5b^8x^6+6a^6b^7x^4+4a^7b^6x^2+a^8b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] [1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b*
x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*
(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*
b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^
2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*
b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*
a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((176*F*a^
4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*(58*F*a^
5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*b^2 - B*
a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^2 + a))/
(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*b^5)]
```

giac [A] time = 0.59, size = 204, normalized size = 0.95

$$-\frac{\left(\left(x^2\left(\frac{\left(176Fa^4b^6-15Da^3b^7-6Ca^2b^8-8Bab^9-48Ab^{10}\right)x^2}{a^4b^7}+\frac{7\left(58Fa^5b^5-3Ca^3b^7-4Ba^2b^8-24Aab^9\right)}{a^4b^7}\right)+\frac{35\left(10Fa^6b^4-Ba^3b^7-6Aa^2b^8\right)}{a^4b^7}\right)x^2+\frac{105\left(Fa^7b^3-Aa^3b^7\right)}{a^4b^7}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{F\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48*
A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*A*
a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7))*
x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [B] time = 0.01, size = 427, normalized size = 2.00

$$-\frac{F x^{7}}{7\left(b x^{2}+a\right)^{\frac{7}{2}} b}-\frac{D x^{5}}{2\left(b x^{2}+a\right)^{\frac{7}{2}} b}-\frac{F x^{5}}{5\left(b x^{2}+a\right)^{\frac{5}{2}} b^{2}}-\frac{C x^{3}}{4\left(b x^{2}+a\right)^{\frac{7}{2}} b}-\frac{5 D a x^{3}}{8\left(b x^{2}+a\right)^{\frac{7}{2}} b^{2}}+\frac{A x}{7\left(b x^{2}+a\right)^{\frac{7}{2}} a}-\frac{B x}{7\left(b x^{2}+a\right)^{\frac{7}{2}} b}-\frac{3 C a x}{28\left(b x^{2}+a\right)^{\frac{7}{2}} b^{2}}-\frac{15 D a^{2} x}{56\left(b x^{2}+a\right)^{\frac{7}{2}} b^{3}}-\frac{F x^{3}}{3\left(b x^{2}+a\right)^{\frac{3}{2}} b^{3}}+\frac{6 A x}{35\left(b x^{2}+a\right)^{\frac{5}{2}} a^{2}}+\frac{B x}{35\left(b x^{2}+a\right)^{\frac{5}{2}} a b}+\frac{3 C x}{140\left(b x^{2}+a\right)^{\frac{5}{2}} b^{2}}+\frac{3 D a x}{56\left(b x^{2}+a\right)^{\frac{5}{2}} b^{3}}+\frac{8 A x}{35\left(b x^{2}+a\right)^{\frac{3}{2}} a^{3}}+\frac{4 B x}{105\left(b x^{2}+a\right)^{\frac{3}{2}} a^{2} b}+\frac{C x}{35\left(b x^{2}+a\right)^{\frac{3}{2}} a b^{2}}+\frac{D x}{14\left(b x^{2}+a\right)^{\frac{3}{2}} b^{3}}+\frac{16 A x}{35 \sqrt{b x^{2}+a}\, a^{4}}+\frac{8 B x}{105 \sqrt{b x^{2}+a}\, a^{3} b}+\frac{2 C x}{35 \sqrt{b x^{2}+a}\, a^{2} b^{2}}+\frac{D x}{7 \sqrt{b x^{2}+a}\, a b^{3}}-\frac{F x}{\sqrt{b x^{2}+a}\, b^{4}}+\frac{F \ln\left(\sqrt{b}\, x+\sqrt{b x^{2}+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*F*x^7/b/(b*x^2+a)^(7/2)-1/5*F/b^2*x^5/(b*x^2+a)^(5/2)-1/3*F/b^3*x^3/(b
*x^2+a)^(3/2)-F/b^4*x/(b*x^2+a)^(1/2)+F/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))-1/2*D*x^5/b/(b*x^2+a)^(7/2)-5/8*D*a/b^2*x^3/(b*x^2+a)^(7/2)-15/56*D*a^2/
b^3*x/(b*x^2+a)^(7/2)+3/56*D*a/b^3*x/(b*x^2+a)^(5/2)+1/14*D/b^3*x/(b*x^2+a)
^(3/2)+1/7*D/a/b^3*x/(b*x^2+a)^(1/2)-1/4*C*x^3/b/(b*x^2+a)^(7/2)-3/28*C*a/b
^2*x/(b*x^2+a)^(7/2)+3/140*C/b^2*x/(b*x^2+a)^(5/2)+1/35*C/a/b^2*x/(b*x^2+a)
^(3/2)+2/35*C/a^2/b^2*x/(b*x^2+a)^(1/2)-1/7*B/b*x/(b*x^2+a)^(7/2)+1/35*B/a/
b*x/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+
a)^(1/2)+1/7*A*x/a/(b*x^2+a)^(7/2)+6/35*A/a^2*x/(b*x^2+a)^(5/2)+8/35*A/a^3*
x/(b*x^2+a)^(3/2)+16/35*A/a^4*x/(b*x^2+a)^(1/2)
```

maxima [B] time = 1.74, size = 597, normalized size = 2.79

$$-\frac{1}{35}\left(\frac{35 x^{6}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b}+\frac{70 a x^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}+\frac{56 a^{2} x^{2}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{3}}+\frac{16 a^{3}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{4}}\right) F x-\frac{F x\left(\frac{15 x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b}+\frac{20 a x^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}}+\frac{8 a^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{3}}\right)}{15 b}-\frac{D x^{5}}{2\left(bx^{2}+a\right)^{\frac{7}{2}} b}-\frac{F x\left(\frac{3 x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}} b}+\frac{2 a}{\left(bx^{2}+a\right)^{\frac{3}{2}} b^{2}}\right)}{3 b^{2}}-\frac{F a x^{3}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{3}}-\frac{5 D a x^{3}}{8\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}-\frac{C x^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}} b}+\frac{16 A x}{35 \sqrt{bx^{2}+a}\, a^{4}}+\frac{8 A x}{35\left(bx^{2}+a\right)^{\frac{3}{2}} a^{3}}+\frac{6 A x}{35\left(bx^{2}+a\right)^{\frac{5}{2}} a^{2}}+\frac{A x}{7\left(bx^{2}+a\right)^{\frac{7}{2}} a}+\frac{139 F x}{105 \sqrt{bx^{2}+a}\, b^{4}}+\frac{17 F a x}{105\left(bx^{2}+a\right)^{\frac{3}{2}} b^{4}}-\frac{29 F a^{2} x}{35\left(bx^{2}+a\right)^{\frac{5}{2}} b^{4}}+\frac{D x}{14\left(bx^{2}+a\right)^{\frac{3}{2}} b^{3}}+\frac{D x}{7 \sqrt{bx^{2}+a}\, a b^{3}}+\frac{3 D a x}{56\left(bx^{2}+a\right)^{\frac{5}{2}} b^{3}}-\frac{15 D a^{2} x}{56\left(bx^{2}+a\right)^{\frac{7}{2}} b^{3}}+\frac{3 C x}{140\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}}+\frac{2 C x}{35 \sqrt{bx^{2}+a}\, a^{2} b^{2}}+\frac{C x}{35\left(bx^{2}+a\right)^{\frac{3}{2}} a b^{2}}-\frac{3 C a x}{28\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}-\frac{B x}{7\left(bx^{2}+a\right)^{\frac{7}{2}} b}+\frac{8 B x}{105 \sqrt{bx^{2}+a}\, a^{3} b}+\frac{4 B x}{105\left(bx^{2}+a\right)^{\frac{3}{2}} a^{2} b}+\frac{B x}{35\left(bx^{2}+a\right)^{\frac{5}{2}} a b}+\frac{F \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x - 1/
15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*F*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C*x^3/(
(b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 +
 a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7
/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2 + a)^(3/2
)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2 + a)^(3/2
)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^
3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b
^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^
2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8
/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/
35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x^2 + C\,x^4 + F\,x^8 + x^6\,D}{\left(b\,x^2 + a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

3.170 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(384Ab^4-a\left(15a^3F\right.\right.}{105a^5}$$

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1803, 1813, 12, 264}

$$-\frac{x^7\left(384Ab^4-a\left(6a^2bD+15a^3F+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1803

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A
*x^(m + 1)*(a + b*x^2)^(p + 1))/(a*(m + 1)), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1813

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[(A*x*(a + b*
x^2)^(p + 1))/a, x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4+Fx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2-aFx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^4(4b\ldots}{\ldots}}{\ldots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192A\ldots}{\ldots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192A\ldots}{\ldots}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{(192A\ldots}{\ldots}$$

Mathematica [A] time = 0.19, size = 138, normalized size = 0.72

$$\frac{a^4\left(-105A+105Bx^2+35Cx^4+21Dx^6+15Fx^8\right)+2a^3bx^2\left(-420A+105Bx^2+14Cx^4+3Dx^6\right)+8a^2b^2x^4\left(-210A+21Bx^2+Cx^4\right)+48ab^3x^6\left(Bx^2-28A\right)-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) + a
^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a + b
*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.57, size = 169, normalized size = 0.88

$$\frac{-105a^4A+105a^4Bx^2+35a^4Cx^4+21a^4Dx^6+15a^4Fx^8-840a^3Abx^2+210a^3bBx^4+28a^3bCx^6+6a^3bDx^8-1680a^2Ab^2x^4+168a^2b^2Bx^6+8a^2b^2Cx^8-1344aAb^3x^6+48ab^3Bx^8-384Ab^4x^8}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/
2)),x]
```

```
[Out] (-105*a^4*A - 840*a^3*A*b*x^2 + 105*a^4*B*x^2 - 1680*a^2*A*b^2*x^4 + 210*a^
3*b*B*x^4 + 35*a^4*C*x^4 - 1344*a*A*b^3*x^6 + 168*a^2*b^2*B*x^6 + 28*a^3*b*
C*x^6 + 21*a^4*D*x^6 - 384*A*b^4*x^8 + 48*a*b^3*B*x^8 + 8*a^2*b^2*C*x^8 + 6
*a^3*b*D*x^8 + 15*a^4*F*x^8)/(105*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.92, size = 187, normalized size = 0.97

$$\frac{\left(\left(15\,Fa^4+6\,Da^3b+8\,Ca^2b^2+48\,Bab^3-384\,Ab^4\right)x^8+7\left(3\,Da^4+4\,Ca^3b+24\,Ba^2b^2-192\,Aab^3\right)x^6-105\,Aa^4+35\left(Ca^4+6\,Ba^3b-48\,Aa^2b^2\right)x^4+105\left(Ba^4-8\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{105\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] 1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(
C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*
x)
```

giac [A] time = 0.60, size = 220, normalized size = 1.14

$$\frac{\left(\left(x^2\left(\frac{\left(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right)x^2+\frac{105\left(Ba^{13}b^3-4\,Aa^{12}b^4\right)}{a^{14}b^3}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 -
 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11
```

```
*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a
^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b
*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 166, normalized size = 0.86

$$-\frac{384Ab^4x^8-48Bab^3x^8-8Ca^2b^2x^8-6Da^3bx^8-15Fa^4x^8+1344Aab^3x^6-168Ba^2b^2x^6-28Ca^3bx^6-21Da^4x^6+1680Aa^2b^2x^4-210Ba^3bx^4-35Ca^4x^4+840Aa^3bx^2-105Ba^4x^2+105Aa^4}{105\left(bx^2+a\right)^{\frac{7}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(384*A*b^4*x^8-48*B*a*b^3*x^8-8*C*a^2*b^2*x^8-6*D*a^3*b*x^8-15*F*a^4
*x^8+1344*A*a*b^3*x^6-168*B*a^2*b^2*x^6-28*C*a^3*b*x^6-21*D*a^4*x^6+1680*A*
a^2*b^2*x^4-210*B*a^3*b*x^4-35*C*a^4*x^4+840*A*a^3*b*x^2-105*B*a^4*x^2+105*
A*a^4)/(b*x^2+a)^(7/2)/x/a^5
```

maxima [B] time = 1.54, size = 421, normalized size = 2.18

$$-\frac{Fx^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5Fax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Dx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{16Bx}{35\sqrt{bx^2+a}\,a^4}+\frac{8Bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}a}+\frac{Fx}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{Fx}{7\sqrt{bx^2+a}\,ab^3}+\frac{3Fax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15Fa^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{3Dx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2Dx}{35\sqrt{bx^2+a}\,a^2b^2}+\frac{Dx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3Dax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8Cx}{105\sqrt{bx^2+a}\,a^3b}+\frac{4Cx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{Cx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}-\frac{128Abx}{35\sqrt{bx^2+a}\,a^5}-\frac{64Abx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48Abx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8Abx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{A}{\left(bx^2+a\right)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] -1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^
2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b)
- 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) -
 A/((b*x^2 + a)^(7/2)*a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^2\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)



GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```